Spare parts inventory pooling games

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Equipment-intensive high-tech companies such as airlines, nuclear power plants and medical equipment manufacturers are often confronted with the difficult task of maintaining high availability of their technically advanced systems. A random failure of just one critical component can cause a complex machine to break down. To prevent long and costly downtimes, spare parts are kept on stock, such that the failed component can be quickly replaced by a spare one from inventory.

Inventory pooling can be an effective strategy to improve system availability while considerably reducing total costs. Inventory pooling refers to an arrangement where demand at a stockpoint that is out of stock is satisfied from another stockpoint with a positive on-hand inventory.

We study a situation where n independent companies separately stock spare parts of the same item for a technically advanced machine. They may reduce expected joint holding and downtime costs by pooling inventory. A real life example is a situation with multiple airline companies that use the same type of aircrafts and independently stock spare engines at separate locations. These companies can cooperate by pooling their inventories. However, first the participating companies will have to be convinced that such an arrangement is beneficial for everyone and that no group of companies is merely subsidizing another. These cost allocation intricacies add another layer of difficulty to the spare parts pooling process.

In order to obtain insights into these issues, we will use cooperative game theory, which deals with joint profits or costs that can be obtained by groups of decision makers if they coordinate their actions. We examine the conditions under which such a game has a nonempty core, i.e. a stable cost allocation exists.

Spare parts inventory models with multiple stockpoints that pool their inventory by using lateral transshipments have been analyzed quite extensively in the literature. We refer to Paterson et al. (2009) for an overview. However, (almost) all authors consider centralized inventory systems, whereas we are interested in decentralized inventory systems with independent decision makers.

Our application of cooperative game theory to an inventory-based problem falls into a growing stream of literature. A number of authors have focused on newsvendor games (cf. Özen et al. (2008) and references therein), in which n independent retailers each face a

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newsvendor problem and groups of retailers can improve their expected joint profit by coordinating orders, followed by transshipments after demand realization is known. Inventory situations with an infinite time horizon, rather than a single period, have been studied by several other authors. For example, Anily and Haviv (2007) consider a setting with deterministic continuous demand, where firms can reduce ordering costs by cooperating through joint replenishments.

The specific characteristics of spare parts inventory systems, such as the focus on expensive low-demand items with high service requirements, make them distinctly different from the aforementioned newsvendor or continuous review inventory systems. There are, however, only few papers that have analyzed spare parts inventory systems from a game theoretic point of view. Wong et al. (2007) are the first to study spare parts inventory systems in the context of cooperative game theory, but they don't focus on structural properties of the games, such as non-emptiness of the core.

Our paper does investigate non-emptiness of the core for spare parts inventory pooling games in general. We analyze these situations by defining a cooperative cost game, in which the cost of a coalition is the sum of holding costs and expected downtime costs.

We assume that failed components are immediately sent into repair and are perfectly repairable. Hence, the inventory system at a company can be seen as being controlled by a base stock policy with a fixed base stock level. We also assume Poisson demand processes with constant rates, as well as the use of an emergency procedure in case of a stock-out, negligible transshipment times, and complete pooling of inventory between members of a pooling group.

Our analysis concentrates on core non-emptiness of these games. We first look at a base setting where companies are (almost) identical and subsequently look at several generalizations. We first prove that for this base setting, the core of the associated game will always be non-empty. Afterwards, we generalize this result by showing that for situations allowing companies to have non-identical demand rates and base stock levels and for situations allowing companies to have non-identical downtime costs, the core of the associated game is also non-empty. The managerial implication is that collaboration between independent spare parts stockpoints will be a win-win situation; it will always be possible to divide joint costs amongst the companies such that no group of companies has an incentive to split off. However, when companies have non-identical downtime costs along with non-identical base stock levels or demand rates, the associated game may have an empty core, essentially due to non-optimality of the full pooling approach.

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