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6. Thierry BRECHET et Marc GERMAIN. Les affres de la modélisation, May 2002.

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Constraining Equitable Allocations of Tradable CO_2 Emission Quotas by Acceptability

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Abstract. Since the signing of the Kyoto Protocol, future commitments are likely to be framed in terms of tradable quotas. The discussions on the allocation of the quotas among countries will be based – at least partly – on rules corresponding to a certain conception of equity. For instance, allocating quotas in direct proportion to population, in relation to GDP or according to past emissions has been advocated. Taking a long term perspective, we compute such allocations of tradable quotas with a dynamic (closed-loop) model. The total amount of quotas to be distributed at each period corresponds to the world optimal amount of emissions to be realized at each period. We observe that most "equitable" allocation rules do not make the agreement individually rational for every country along the entire time path. We then propose a mechanism which determines allocations of quotas that are as close as possible to any "equitable" allocation while satisfying individual rationality.

Key words: acceptability, climate change, dynamic games, equity, tradable quotas

JEL classification: C73, H23, Q25, Q28

1. Introduction

Like any other international environmental agreements, treaties on climate change cannot be enforced by any supranational authority and therefore have to be signed on a voluntary basis. The formation and the stability of such agreements have been investigated by the game-theoretical literature. As pointed out in the surveys by Folmer, Hanley and Missfeldt (1998) and Finus (2002), a key question that has been addressed in this literature is the one of free-riding incentives. Indeed, countries always have an incentive not to join the agreement on emissions abatement and to benefit from the efforts of the other countries taking part to the agreement. Furthermore, even if it signs the treaty, a country also has an incentive not to comply with it and to emit more pollutants than allowed by the treaty. Hence, it emerges that, «*since accession to an agreement is voluntary, treaties must be individually rational*» (Finus (2002)). Each country must receive a higher payoff under the cooperative than under the status quo situation. In such a context, individual rationality is a necessary (although not a sufficient) condition for an agreement to be stable.

At the same time, a second stream of the literature has empirically evaluated the long run welfare implications of various agreements on climate change. The signing of the Kyoto Protocol in 1997 has fostered such analyses and has established tradable emissions quotas as the leading policy instrument. Although the Protocol has not yet been ratified by enough countries in order to come into force, discussions on future commitments are already taking place. Most discussions assume that tradable quotas will keep being the main instrument. In these circumstances, the allocation of the quotas to the countries are likely to be inspired by some considerations of equity, as shown by the proposals of Parties to the United Nations Framework Convention on Climate Change. A short review of the literature on the cost impact of different 'equitable' allocation rules has been made by Rose, Stevens, Edmonds and Wise (1998). They conclude that the reviewed papers have one or several limitations – (i) the choice of a set of regions which does not fully covers the globe, (ii) a too short time horizon, (iii) identical cost functions for the regions and (iv) the impossibility to model or to compute all types of allocation rules – which are overcome in their paper. However, benefits to climate change mitigation are usually not included and the dynamic aspects have not been fully accounted for in this literature. Our paper improves upon these issues by explicitly taking into account the damages caused by climate change in a dynamic (closed-loop) setting.

The damage costs originate from the RICE model (Nordhaus and Yang (1996)). They allow to develop a cost-benefit analysis determining an optimal trajectory of world emissions and to compute the gains from the cooperation by taking avoided damages into account. This setting permits us to *check the individual rationality of various equitable rules* that might be used to allocate the quotas corresponding to the optimal level of world emissions at each period.

Furthermore, if an equitable allocation rule is not individually rational, we propose a method which determines a *new allocation satisfying individual rationality while being as close as possible to the initial equitable rule*. The equitable allocation rule is thus constrained by individual rationality. Hence, we make a link with the game-theoretical stream of the literature by taking stability aspects into consideration. However, our notion of stability is limited here to individual rationality and does not prevent free-riding in the traditional sense.

The dynamic setting is based on previous work by Germain, Toint, Tulkens and de Zeeuw (2003). It is aimed at checking the stability (here, individual rationality) of an allocation rule *along the entire time path* of the agreement. Indeed, it is not sufficient to require the agreement to be individually rational by considering the total discounted costs from the first to last period. Such a form of rationality is rather weak. It implies that, even if the agreement is rational in the first period, there is no guarantee that it will still be so in a subsequent period, i.e., when considering total discounted cost from this subsequent period to the last period for instance. On the contrary, the dynamic framework of Germain et al. (2003)

prevents such an issue by requiring a stronger form of rationality in the spirit of subgame perfectness.

The structure of the paper is the following. Section 2 introduces and discusses the main so-called equitable allocation rules. Following Rose et al. (1998), a distinction is made between rules which are based on the allocation itself (allocation-based rules) and rules that take the outcome of the allocation into account (outcome-based rules). The dynamic model that we use is presented in section 3. It explains how tradable quotas are introduced in the model and defines the international optimum as well as the cooperative and non-cooperative scenarios. The welfare implications of the different allocation rules are then analyzed in section 4. We check whether the main equitable rules are individually rational or not. We find that participation constraints are strong, as no pure allocation-based equitable rule does satisfy individual rationality along the entire time path. In section 5, we then describe the method by which equitable allocations are constrained by individual rationality and we compute such allocations. Finally, section 6 summarizes the results and concludes.

2. "Equitable" Allocation Rules

The issue of equity in the global climate change problem has already been investigated by several authors. A couple of equity principles relevant to that debate has been put forward by Kverndockk (1995), Grubb (1997), Rowland (1997) and others. Based on some observable characteristics like population, GDP and historical emissions, proposals to differentiate nations' emissions reductions have been introduced (see AGBM (1996, 1997)). Each of them may then be somehow related to a principle of equity.

Hence, a certain amount of quotas are allocated among the nations according to a certain rule. As they can be related to principles of equity, they are called "equitable" allocation rules. Following Rose et al. (1998), the literature distinguishes allocation-based from outcome-based quotas allocation rules. Under an allocation-based rule, the differentiation criterion bears directly on the allocation of the total amount of quotas, while under an outcome-based rule, the criterion applies to the outcome of the allocation, that is to the total net costs or gains resulting from the implementation of the policy. For instance, if the differentiation criterion is the population, the allocation-based rule provides the same amount of quotas per capita to each nation while the outcome-based rule allocates an amount of quotas such that net costs or gains per head are equalized across countries.

Note that an infinite number of allocation rules may be built on a mixture of several criteria. For instance, Cline (1992) advocates a rule based on a weighted sum of population, GDP and historical emissions. Numerous differentiation proposals by nations, like the Australian, French and Norwegian ones, also combine several criteria (see AGBM (1996, 1997) or Torvanger and Godal (1999), Torvanger and Ringius (2000) and Reiner and Jacoby (1998)). Since our aim is not

Table I. Equitable allocation rules

Criteria	Equitable allocation rules	
	Allocation-based	*Outcome-based*
Population	Egalitarian	Horizontal equity – population
GDP	GDP	Horizontal equity – GDP
GDP per capita	Ability to pay	Vertical equity
Historical emissions	Grandfathering	–
Population and historical emissions	Convergence	–

to analyze the impact of all possible quota allocation rules but rather to concentrate on the link between such rules and the requirement of individual rationality, only the main rules are considered. See also Manne and Richels (1998) and Richels, Edmonds, Gruenspecht and Wigley (1996). They are presented in Table I.

The (i) population and (ii) GDP criteria lead to straightforward definitions of the corresponding allocation rules.

The (iii) GDP per capita criterion defines for country i at time t a share λ_{it} of either the total amount of quotas to be allocated (Ability to pay rule) or the total net costs or gains (Vertical equity rule) in that period which is given by

$$\lambda_{it} = \frac{POP_{it}\left(\frac{GDP_{it}}{POP_{it}}\right)^{-\gamma}}{\sum_j POP_{jt}\left(\frac{GDP_{jt}}{POP_{jt}}\right)^{-\gamma}}$$

with $\gamma < 1$.

Finally, (iv) historical emissions are those of the year 1990. Under the Grandfathering rule, the share of quotas received by any country keeps therefore the same over the whole planning horizon.

However, under the convergence rule, this share is initially based on the historical emissions criterion but converges gradually to the share of the population. More formally, we assume a linear transition from the historical emissions to the population criteria up to the convergence period ($tconv$) that we set in 2100. Shares are thus given by

$$\lambda_{it} = (1 - \frac{t}{tconv})\frac{E90_i}{\sum_j E90_j} + \frac{t}{tconv}\frac{POP_{it}}{\sum_j POP_{jt}}$$

where $E90$ is the vector of emissions in the year 1990.

Let us now turn to the set up of the dynamic model used to simulate the optimal abatement trajectories and the countries' behavior under these various quotas allocation rules.

3. The Model

3.1. PRELIMINARIES

Consider the following model developed by Germain and van Ypersele (1999) and Germain et al. (2003). n countries or regions indexed by $i \in N = \{1, 2, \dots, n\}$ decide on levels of CO$_2$ emissions abatement over some planning period $\Theta = \{1, 2, \dots, T\}$, T being a positive and finite integer. At each period $t \in \Theta$, country i's emissions of CO$_2$ are a proportion ν_{it} of its output Y_{it}. The ability of country i to control its emissions in period t is represented by the choice of a positive value for the rate μ_{it} $(0 \leq \mu_{it} \leq 1)$ which affects the emissions-output ratio. Accordingly, emissions of country i at period t are given by:

$$E_{it} = \nu_{it}\,[1 - \mu_{it}]\,Y_{it}. \tag{1}$$

Due to the dynamic structure of the model, computations require the use of a sophisticated algorithm which has been developed only for a single control variable (see Germain et al. (2003)). Hence, for computability reasons, output is considered as exogenous. This assumption might somehow affect the abatement costs. However, these costs will prove to be very low relative to output. This tends to indicate that investment decisions are not much influenced by the level of abatement. Accordingly, the impact of this assumption on abatement costs should be limited.

Wherever they are emitted, these pollutants accumulate in the atmosphere. The change in the concentration of CO$_2$ at time $t + 1$, M_{t+1}, with respect to its preindustrial level, M_0, is written

$$M_{t+1} - M_0 = [1 - \delta]\,[M_t - M_0] + \beta \sum_{i=1}^{n} E_{it} \tag{2}$$

where $\delta (0 < \delta < 1)$ is the rate of decay of CO$_2$ in the atmosphere and $\beta (0 < \beta < 1)$ is the marginal atmospheric retention ratio of CO$_2$. This concentration modifies the radiative forcing which then influences the atmospheric temperature w.r.t. its preindustrial level, ΔT_t. This is the so-called 'greenhouse effect' which is summarized by

$$\Delta T_t = \eta \ln \left(\frac{M_t}{M_0} \right) \tag{3}$$

where $\eta (0 < \eta)$ is an exogenous parameter.

Each country therefore bears two types of costs. On the one hand, the increase of the temperature causes damages to country i:

$$D_{it}(\Delta T_t) = b_{i1} \Delta T_t^{b_{i2}} Y_{it} \tag{4}$$

where b_{i1} and b_{i2} are positive parameters with $b_{i2} > 1\ \forall i$. By (3), damages in period t are a function of the stock of CO$_2$ which is influenced by all countries' emissions from period 1 to $t-1$. Increasing the control rate (μ_{ik}) of a country at a given period $k < t$ – while keeping constant all other control rates – decreases damages of all countries in period t since (1) is strictly decreasing in μ_{it} and (2), (3) and (4) are strictly increasing in respectively E_{it}, M_t and ΔT_t.

On the other hand, the control of CO_2 emissions by country i at a time t, through the choice of $\mu_{it} > 0$, requires either the use of less polluting technologies which are more expensive, or a reduction in the output. Using equation (1), these costs are expressed by a twice continuously differentiable and strictly decreasing convex function of the emissions

$$C_{it}\left(E_{it}\right) = a_{i1}\left[1 - \frac{E_{it}}{\nu_{it}Y_{it}}\right]^{a_{i2}} Y_{it} \tag{5}$$

where a_{i1} and $a_{i2}(i \in N)$ are positive parameters with $a_{i2} > 1\ \forall i$ and where $1 - \frac{E_{it}}{\nu_{it}Y_{it}} = \mu_{it}$.

Countries must therefore balance the abatement costs with the damages when deciding on a level of emissions. However, since the emissions of a country contribute to damages for all countries, the chosen abatement rate will be different according to whether each country does take into account the impact of its emissions on the other countries or not. If each country takes this externality into account, an international optimum is reached.

3.2. THE INTERNATIONAL OPTIMUM

An international optimum at period t is defined as the solution of the minimization of an unweighted sum of all countries' abatement and damage costs borne from that period t until the last one, T. Hence, over the whole planning horizon, the optimal policy is given by the solution of, *at each period of time* $t \in \Theta$:

$$\min_{\{E_s\}_{s\in\{t,\ldots,T\}}} \left\{ \sum_{s=t}^{T} \sum_{i=1}^{n} \alpha^s \left[C_{is}\left(E_{is}\right) + \alpha D_{i,s+1}\left(M_{s+1}\right)\right] \right\} \tag{6}$$

subject to (1)–(5) and to $0 < E_{it} \leq \nu_{it}Y_{it}\ \forall i \in N,\ \forall t \in \Theta$ and where $\alpha(0 < \alpha \leq 1)$ is the discount factor. Such a definition implicitly assumes that countries' marginal utilities are linear. This assumption implies that distributive concerns are left aside. However, we do not exclude the possibility of lump sum transfers aimed at taking care of such concerns. The initial allocation of the tradable quotas could even play such a role.

Call E_{it}^* the optimal level of emissions for country i at period t. The optimal rates of abatement at time t, μ_t^*, are therefore given by (1) and the optimal stock of CO_2, M_t^*, at the same period is determined by (2). At each period t, the optimal policy satisfies the usual Samuelson condition for public goods:

$$C'_{i,t}\left(E_{i,t}^*\right) = -\alpha\beta \sum_{j=1}^{n} \sum_{s=t}^{T} \alpha^{s-t}\left[1-\delta\right]^{s-t} D'_{j,s+1}\left(M_{s+1}^*\right) \tag{7}$$

that is the marginal abatement cost of every country is equal to the sum of the marginal damages across the counties and across time.

From a collective point of view, we know that the optimum is better than any other (non-cooperative in particular) policy. Nothing ensures however that this is also verified at the individual level. Indeed, countries being different, some of them may, at some periods, be better off at the non-cooperative equilibrium than at the optimum, so that cooperation is not profitable for those countries, at least in those periods.

3.3. ALLOCATION OF TRADABLE QUOTAS TO SUSTAIN COOPERATION

Based on theoretical work by Chander and Tulkens (1995, 1997), Germain et al. (2003) propose for the climate change framework a system of financial compensation between countries that ensures that all of them (and even all subgroups of them) have an interest in adopting the internationally optimal policy. We adapt their framework to the context of tradable quotas.

As suggested in subsection 3.1, at every period t each country chooses between, on the one hand, cooperating (signing the agreement) and receiving $\widetilde{E}_{i,t}$ tradable quotas satisfying

$$\sum_i \widetilde{E}_{i,t} = \sum_i E^*_{i,t}, \quad \forall t \tag{8}$$

and, on the other hand, adopting a non-cooperative policy. These cooperative and non-cooperative scenarios are described below.

It is assumed that the market for tradable quotas is perfectly competitive and that countries are neither allowed to borrow quotas, nor to bank some of them. In such a context it is well-known that when the total amount of quotas allocated in period t corresponds to the world optimal level of emissions in that period, every country chooses a level of emissions corresponding to their optimal level of emissions defined above in section 3.2. Each country abates – and buys or sells quotas according to its level of abatement – in such a way that its marginal cost of abatement, i.e., the opportunity cost of buying or the opportunity gain of selling a quota, equals the price the quotas σ_t, which is given by

$$\sigma_t = -C'_{i,t}\left(E^*_{i,t}\right) = \alpha\beta \sum_{j=1}^{n} \sum_{s=t}^{T} \alpha^{s-t} [1-\delta]^{s-t} D'_{j,s+1}\left(M^*_{s+1}\right), \quad \forall i \in I. \tag{9}$$

This price is independent of the allocation of the quotas among countries.

3.3.1. *The last period*

Let us first analyze the cooperative scenario with tradable quotas in the last period. Let $W_{i,T}$ be the cost borne by country i in period T if every country adopts its optimal level of emissions:

$$W_{i,T} = C_{i,T}\left(E^*_{i,T}\right) + \alpha D_{i,T+1}\left(M^*_{T+1}\right).$$

Given the allocation rule that is being used for the allocation of the quotas $\left\{\widetilde{E}_{1,T}, \ldots, \widetilde{E}_{n,T}\right\}$ which satisfies (8), the total cost of cooperating at period T for country i is thus:

$$\begin{aligned} \widetilde{W}_{i,T} &= C_{i,T}\left(E^*_{i,T}\right) + \alpha D_{i,T+1}\left(M^*_{T+1}\right) + \sigma_T\left[E^*_{i,T} - \widetilde{E}_{i,T}\right] \\ &= W_{i,T} + \sigma_T\left[E^*_{i,T} - \widetilde{E}_{i,T}\right]. \end{aligned} \tag{10}$$

We now turn to the non-cooperative equilibrium. At the last decision period – period T – and in the absence of cooperation, the fallback position of each country is supposed to be the non-cooperative Nash equilibrium. In that case, given an inherited stock M_T, it solves the following problem:

$$\min_{E_{iT}} \left\{C_{iT}\left(E_{iT}\right) + \alpha D_{i,T+1}\left(M_{T+1}\right)\right\} \tag{11}$$

subject to (1)–(5), $0 < E_{iT} \leq \nu_{iT} Y_{iT}$ and $E_{jT}(j \neq i)$ given. Call $V_{i,T}$ the resulting cost for country i of the simultaneous resolution of this problem by the n countries, $E^v_{i,T}$ the resulting emission level and $\mu^v_{i,T}$ the corresponding abatement rate.

Then, we impose a condition on the allocation of the quotas: it must be such that international cooperation – with tradable quotas – is individually rational for all countries, i.e., such that each country enjoys lower costs than at its fallback position, the non-cooperative equilibrium. Formally, $\widetilde{E}_{i,T}$ is such that[1]

$$\widetilde{W}_{i,T} \leq V_{i,T}, \quad \forall i \in N, \tag{12}$$

where $\widetilde{W}_{i,T} = W_{i,T} + \sigma_T\left[E^*_{i,T} - \widetilde{E}_{i,T}\right]$.

Following Germain et al. (2003), we then make the following assumption: since the allocation of quotas guarantees that the individual rationality constraints (12) are satisfied, countries will sign the agreement in period T.

3.3.2. *The preceding periods*

The same reasoning is then applied for the preceding periods. However, countries' expectations on the future must be taken into account. Assuming that, as in Germain et al. (2003), countries have rational expectations, they face the same alternative of cooperating or not, *knowing that they will cooperate in the future thanks to allocations of quotas that make cooperation individually rational.*

Formally, if each country cooperates at period t knowing that cooperation will also take place at the subsequent periods, country i bears the following cost:

$$\widetilde{W}_{i,t} = C_{i,t}\left(E^*_{i,t}\right) + \alpha D_{i,t+1}\left(M^*_{t+1}\right) + \sigma_t\left[E^*_{i,t} - \widetilde{E}_{i,t}\right] + \alpha \widetilde{W}_{i,t+1} \tag{13}$$

where $E^*_{i,t}$ is given by the resolution of problem (6) at period t, $\widetilde{E}_{i,t}$ is the amount of quotas distributed to country i at time t and the price of the tradable quotas at time t, σ_t, is given by (9).

If, on the other hand, they behave non-cooperatively at time t, each country calculates the following value function:

$$V_{it}(M_t) = \min_{\{E_{it}\}} \left\{ \left[C_{it}\left(E_{it}\right) + \alpha D_{i,t+1}\left(M_{t+1}\right) \right] + \alpha \widetilde{W}_{i,t+1}(M_{t+1}) \right\} \tag{14}$$

subject to (1)–(5), $0 < E_{it} \leq \nu_{it} Y_{it}$, $E_{jt}(j \neq i)$ given, where $\widetilde{W}_{i,t+1}$ is defined following (13) and the allocation of quotas are required to be such that cooperation is also individually rational at period t, that is $\widetilde{E}_{i,t}$ satisfies

$$\widetilde{W}_{i,t} \leq V_{i,t} \tag{15}$$

$\forall i \in N, \forall t \in \Theta$ with $\sum_{i=1}^{n} \widetilde{E}_{i,t} = \sum_{i=1}^{n} E^*_{i,t}$. Hence, we require individual rationality along the whole path. The simultaneous resolution of the n problems (14) leads to what is called – by Germain et al. (2003) – the *fallback non-cooperative equilibrium* at period t.[2]

Since $\widetilde{W}_{i,t} \leq V_{i,t}\ \forall i$, we again assume (as at period T) that countries will indeed cooperate in period t and that this is perfectly anticipated in the preceding periods. Cooperation extends then to all periods by backward induction.

4. Results

Data are based on the RICE model developed by Nordhaus and Yang (1996) and Nordhaus and Boyer (1999). Each period corresponds to ten years, the first one being decade 1991–2000. The world is divided into six regions, namely 1) USA, 2) Japan, 3) European Union (EU), 4) China, 5) Former Soviet Union (FSU) and 6) Rest of the World (ROW). For each region, data are shown in the appendix. Under each of the allocation rules described in section 2, the model is run assuming a time horizon of 300 years. Results are however shown only for the first twelve decades in order to avoid boundary effects.[3]

The choice of a discount rate is a very much debated issue. Since we work in the very long term, our simulations will use a rather low annual discount rate that we set at 1%. Such a choice is based on two arguments. Firstly, according to IPCC (2002), most recent studies analyze rates which vary with the time period considered. For instance, Cropper et al. (1994) show that individuals tend to use a lower discount rate for longer horizons and, based on a survey on 1700 professional economists, Weitzman (1998) shows that «*the discount rate falls progressively, from 4% to 0% as the perspective shifts from the immediate (up to 5 years hence) to the far distant future (beyond 300 years)*». Secondly, one of our aims is to include avoided damages in the gains from the cooperation. The use of a high discount rate would then completely erode this effect since damages are likely to become important only in the far future.

Table II. Optimal emissions and abatament rates (%) for selected periods

Period	World optimal emissions (GtC per year)	Optimal abatement rates (%)					
		USA	JPN	EU	CHI	FSU	ROW
2000–2010	6.291	26.0	18.8	21.6	50.5	38.5	38.6
2030–2040	8.688	26.9	18.8	21.8	43.1	33.6	40.2
2060–2070	11.516	27.5	18.6	21.9	39.7	30.7	41.4
2090–2100	14.574	27.8	18.4	21.8	37.9	28.8	42.1

4.1. THE OPTIMAL PATH

The optimal policy leads to a significant decrease in CO2 emissions with respect to the laissez-faire situation. In 2100, world emissions are cut by almost 40% with respect to their business-as-usual (BAU) trend. However, world optimal emissions still increase very much, reaching 14.6 GtC per year in 2090–2100 as shown in Table II. Furthermore, these reductions are not the same across regions. Indeed, optimal emissions abatement rates are such that marginal abatement costs equalize across regions, as shown by relation (7). Since marginal abatement costs are lower in CHI and ROW than in USA, EU and JPN, it is not surprising to observe much higher abatement rates in those regions (see Table II).

At the beginning of the next century, the optimal temperature increase, resulting from the optimal world level of emissions, is around 0.75 °C lower than the BAU temperature which lies in the average of IPCC's projections, i.e., around 2.5 °C higher in 2090–2100 than in 1990–2000.

4.2. ARE EQUITABLE ALLOCATION RULES ACCEPTABLE?

In this section, we present the results of an agreement defined by (i) the optimal world level of CO_2 emissions at each period and (ii) an "equitable" rule to allocate the quotas corresponding to these optimal world emissions. It is provisionally assumed that the allocation of quotas according to any allocation rule induces cooperation. Individual rationality is then checked ex-post.

Under an outcome-based allocation rule, countries share the surplus resulting from cooperation. Since the sharing factors are all positive, every country is assured to enjoy a level of welfare that is at least as large as the one it would enjoy if no agreement was reached. Thus, by definition, every **outcome-based** allocation rule is individually rational.

However, simulations show that none of the **allocation-based** allocation rules does satisfy individual rationality for all countries at the same time. For every allocation rule, the amount of quotas per head received by each country and the resulting trades of quotas are shown for selected periods in Table III. Total costs due to cooperation, i.e., the difference between total costs borne under the cooperative and under the non-cooperative scenarios (see $\widetilde{W}_{i,t} - V_{i,t}$), are also presented. These total costs due to cooperation are indeed strictly positive for at least one region in at least one period. Accordingly, none of these allocation-based rules leads to a stable agreement on the whole path. Let us analyze the results in more details.

Under the *Egalitarian* allocation rule, all regions receive by definition the same amount of quotas per head at each period. The allocation is such that CHI receives more quotas than its BAU emissions, that is "hot air", between decades 1990–2000 and 2060–2070. All countries sell (purchase) quotas as long as their marginal abatement cost is larger (lower) than the price of the quotas. At the equilibrium price, marginal abatement costs are equalized across regions and each of them emits its optimal level of CO$_2$ as explained in section 3. The discounted price of the quotas and the optimal, i.e., actual, emissions per head in each region are presented in Table IV. In CHI and ROW, per capita emissions are much lower than in the other regions for two reasons: firstly, they abate more and, secondly, although their energy intensity (ν_{it}) is relatively higher, their output is lower. In FSU, the energy intensity is also very high but output is not so low, while the differences in per capita emissions between USA on the one hand and JPN and EU on the other hand come mainly from the much larger energy intensity in USA.

Given the allocation of quotas determined by the *Egalitarian* rule, CHI and ROW are net sellers of quotas while USA, JPN, EU and FSU are net buyers, except from 2080–2090 onwards for EU. Since its optimal level of emissions is particularly large, USA imports up to 3 times its allocation. This leads to large costs because the price of the quotas is significantly high, as shown in Table IV. These costs must be added to the abatement costs and the damage costs to form the total costs under the cooperative scenario in a given period. Under the *non-cooperative scenario*, USA would not have to bear the costs of purchasing quotas and would also bear lower abatement costs in that period. However, damage costs would be larger due to the lower world abatement leading to higher temperatures. Under the *cooperative scenario*, the costs of purchasing quotas and the increase in abatement costs are so large that they exceed the avoided damage costs at all periods. Hence, total costs due to cooperation are strictly positive for USA along the whole path under the *Egalitarian* allocation rule.

FSU stands in a similar situation, although its total costs due to cooperation are slightly lower than those of the USA. On the contrary, the sellers – CHI and ROW – enjoy important gains due to cooperation. The large gains of ROW are not only due to its revenues from the sales of quotas, but also from the avoided damages on which it sets a relatively high value, as suggested by the parameters of the damage functions. Finally, JPN and EUs' situations are similar. Although they start losing from cooperation, their total costs due to cooperation become negative from decade 2020–2030 for JPN and from decade 2040–2050 for EU, until the end of the planning horizon. Indeed, on the one hand, they do not have to purchase as many quotas as USA or FSU, and, on the other hand, they set a higher value than USA and FSU on the avoided damages.

Table III. Results for the five (unconstrained) rules

	Total costs due to cooperation: $\widetilde{W}_{it} - V_{it}$ (billion 1990 US$)						Quotas per head: $\widetilde{E}_{it}/POP_{it}$ (1 ton of C per quota)						Net sales per head: $\left[\widetilde{E}_{it} - E^*_{it}\right]/POP_{it}$ (1 ton of C per quota)					
								EGALITARIAN										
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	523	12	96	–479	324	–1005	1.1	1.1	1.1	1.1	1.1	1.1	–3.2	–0.9	–0.8	0.6	–1.2	0.2
2030–2040	590	–10	6	–540	272	–932	1.1	1.1	1.1	1.1	1.1	1.1	–3.3	–0.8	–0.4	0.6	–0.9	0.1
2060–2070	580	–42	–100	–541	224	–840	1.3	1.3	1.3	1.3	1.3	1.3	–3.3	–0.6	–0.1	0.5	–0.8	0.1
2090–2100	516	–72	–192	–474	180	–758	1.5	1.5	1.5	1.5	1.5	1.5	–3.3	–0.4	0.2	0.5	–0.7	0.1
								GDP										
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	–376	–448	–874	321	368	480	6.0	6.4	4.7	0.1	0.8	0.5	1.7	4.5	2.9	–0.3	–1.4	–0.4
2030–2040	–578	–567	–1039	414	281	875	6.9	7.4	4.7	0.2	1.1	0.6	2.5	5.5	3.2	–0.3	–0.9	–0.4
2060–2070	–739	–659	–1128	491	198	1118	7.8	8.5	4.8	0.4	1.4	0.8	3.2	6.6	3.4	–0.4	–0.7	–0.4
2090–2100	–826	–709	–1130	542	122	1201	8.8	9.6	5.0	0.6	1.8	1.1	3.9	7.7	3.7	–0.5	–0.4	–0.4
								ABILITY TO PAY										
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	665	79	293	–1196	396	–767	0.3	0.3	0.3	1.9	0.7	1.0	–4.0	–1.7	–1.5	1.5	–1.5	0.1
2030–2040	751	62	219	–1267	347	–727	0.3	0.3	0.4	1.8	0.8	1.1	–4.0	–1.6	–1.2	1.2	–1.2	0.1
2060–2070	758	35	125	–1226	304	–716	0.4	0.4	0.5	1.9	1.0	1.3	–4.2	–1.5	–0.9	1.2	–1.1	0.1
2090–2100	703	9	31	–1071	262	–733	0.5	0.5	0.7	2.1	1.2	1.5	–4.3	–1.5	–0.6	1.1	–1.0	0.0

Table III. Continued

	Total costs due to cooperation: $\widetilde{W}_{it} - V_{it}$ (billion 1990 US$)						Quotas per head: $\widetilde{E}_{it}/POP_{it}$ (1 ton of C per quota)						Net sales per head: $\left[\widetilde{E}_{it} - E^{*}_{it}\right]/POP_{it}$ (1 ton of C per quota)					
	GRANDFATHERING																	
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	–4	–49	–78	–64	–38	–296	4.0	1.8	1.7	0.6	2.8	0.8	–0.3	–0.2	–0.1	0.1	0.5	–0.1
2030–2040	–207	–126	–310	–53	–290	372	5.1	2.4	2.2	0.7	3.4	0.8	0.7	0.5	0.7	0.1	1.4	–0.2
2060–2070	–477	–207	–547	–2	–528	1041	6.5	3.2	2.8	0.8	4.3	0.9	1.9	1.3	1.4	0.1	2.3	–0.4
2090–2100	–726	–271	–734	92	–708	1548	8.2	4.1	3.6	1.0	5.4	1.0	3.4	2.1	2.2	0.0	3.2	–0.5
	CONVERGENCE																	
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000-2010	55	–42	–58	–110	2	–375	3.6	1.7	1.6	0.6	2.6	0.8	–0.6	–0.3	–0.2	0.2	0.3	0.0
2030–2040	147	–74	–170	–270	–40	–208	3.3	1.8	1.7	0.9	2.4	0.9	–1.1	–0.1	0.2	0.3	0.4	–0.1
2060–2070	345	–79	–199	–422	57	–422	2.5	1.7	1.6	1.2	2.0	1.2	–2.1	–0.2	0.2	0.4	–0.1	0.0
2090–2100	516	–72	–192	–474	180	–758	1.5	1.5	1.5	1.5	1.5	1.5	–3.3	–0.4	0.2	0.5	–0.7	0.1

Table IV. Discounted equilibrium quotas' price and emissions per head

Period	Discounted price ($1990 per quota)	Emissions per head (tons of C)					
		USA	JPN	EU	CHI	FSU	ROW
2000–2010	68.7	4.3	2.0	1.8	0.4	2.2	0.8
2030–2040	71.0	4.4	1.9	1.5	0.6	2.0	1.0
2060–2070	69.1	4.6	1.9	1.4	0.8	2.1	1.2
2090–2100	63.3	4.9	2.0	1.4	1.0	2.2	1.5

The same kind of analysis can be performed for the other allocation-based rules. As expected, the *GDP* rule would clearly provide gains from cooperation for US, JPN and EU and positive costs due to cooperation for CHI, FSU and ROW. Although abatement costs are slightly higher for the former, their energy intensity (emissions per unit of GDP) is much lower so that they are net sellers while the latter are net buyers.

The ability to pay (*ATP*) allocation rule (with $\gamma = 0.5$) gives results that are more similar to those obtained with the *Egalitarian* allocation rule, with US and FSU losing and ROW and CHI gaining from cooperation. However, EU and JPN would not gain from cooperation before the twelfth period.

Under the *Grandfathering* rule (with 1990 as the reference year), US, JPN, EU and FSU gain from cooperating. ROW would then enjoy gains by cooperating during the first three periods but would lose very much from the fourth period onwards. As far as CHI is concerned, it would gain from cooperation during 80 years as it is a net seller due to its low abatement costs and as it sets a relatively high value on the damages avoided under the cooperative scenario.

Finally, the *Convergence* rule, which is based on a progressive switch from the *Grandfathering* to the *Egalitarian* rule, leads to positive costs due to cooperation for USA and FSU. As JPN and EU enjoy gains during all periods under the *Grandfathering* rule and from 2020 or 2040 onwards under the *Egalitarian* rule, it is not surprising to observe that they gain from cooperation at all periods under the *Convergence* rule.

As a conclusion, the *Egalitarian* and the *ATP* rules tend to favor CHI and ROW relative to the *GDP* and *Grandfathering* rules which benefit to USA, FSU and, to a lower extent, to JPN and EU. Our results cannot readily be compared to those of recent studies on equitable allocation rules since our model (i) includes damages and (ii), due to its fully dynamic structure, measures at each period the costs to be borne during that period as well as during all the subsequent periods (see section 3). However, even in a non fully dynamic context and in the absence of damages, such studies lead to the same classification of the equitable allocation rules.

Since none of theses allocation rules does satisfy individual rationality for all countries at each period, we propose to build new allocations based on these rules but which necessarily lead to individual rationality.

5. Building Equitable Allocation Rules Constrained by Acceptability

The method proposed here allows to compute allocations of quotas that are acceptable (or, at least, individually rational) while taking as much as possible into account any equitable allocation rule. This method is applied to allocation-based rules – the outcome-based rules being individually rational by definition. The idea may be summarized as follows. If, in a given period, an allocation-based rule leads to an outcome such that it is not rational for some regions to cooperate, let us give those regions the amount of quotas such that they are indifferent between cooperating and not cooperating, and let us redistribute the rest of the quotas to the other regions *according to the initial allocation rule*. Some of the regions that were firstly induced to cooperate may not be willing to do so anymore because, by necessarily receiving fewer quotas than previously, they may now lose from cooperation. Those are then also just compensated in order to be induced to cooperate, and so on until every region enjoys negative costs due to cooperation.

5.1. THE METHOD: LOOKING FOR “CLOSE” ALLOCATIONS OF QUOTAS

In a given period t, let us distribute the quotas according to a certain allocation rule. If, given the computed outcome, it is not rational for some countries to cooperate, the method consists in finding another allocation of the quotas which (i) is feasible, (ii) satisfies individual rationality for every region and (iii) is as close as possible to the initial one in the sense of the allocation rule that has been used. Hence, the notion of distance between the initial allocation and the new one takes the allocation rule into account.

Recall that $\widetilde{E}_{it}$ is the amount of quotas received by country i at time t according to the chosen allocation rule and that $\widetilde{W}_{it}$ is the cost borne by the same country at the same time when every country cooperates at this period as at every subsequent period. Consider $\widehat{E}_{it}$ as an alternative allocation of quotas to country i at time t. Formally, the method consists in solving, at each period t:

$$\min_{\{\widehat{E}_{it}\}} \sum_i \frac{\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]^2}{\lambda_{it}} \tag{16}$$

where λ_{it} ($\sum_i \lambda_{it} = 1\ \forall t$) is the sharing factor used in the allocation rule, subject to the feasibility constraint

$$\sum_i \widehat{E}_{it} = \sum_i \widetilde{E}_{it} \tag{17}$$

and the individual rationality constraints

$$\widehat{W}_{it} \leq V_{it}, \quad \forall i, \tag{18}$$

where $\widehat{W}_{it}$ is given by

$$\widehat{W}_{it} = \widetilde{W}_{it} + \sigma_t^* \left[\widetilde{E}_{it} - \widehat{E}_{it}\right] \tag{19}$$

and where σ_t^* is the price of the quotas at time t. The dynamic (backwards) resolution of the optimal and fallback equilibrium scenarios must then consider $\widehat{W}_{it}$ instead of $\widetilde{W}_{it}$ as the future costs of cooperation at each period.

Let I ($I \subset N$ and $N \backslash I \neq \emptyset$) be the set of regions for which the participation constraint is *not* binding in period t for an allocation of quotas $\left[\widetilde{E}_{1t}, \ldots, \widetilde{E}_{it}, \ldots, \widetilde{E}_{nt}\right]$ determined by an equitable allocation rule. Then:

Proposition 1 *In period t, problem (16) subject to (17)–(19) leads to an allocation* $\left[\widehat{E}_{1t}, \ldots, \widehat{E}_{it}, \ldots, \widehat{E}_{nt}\right]$ *such that*

$$\frac{\widehat{E}_{it}}{\lambda_{it}} = \frac{\widehat{E}_{jt}}{\lambda_{jt}}, \forall i, j \in I, i \neq j \quad \text{with} \quad \widehat{E}_{kt} < \widetilde{E}_{kt} \quad \text{and} \quad \widehat{W}_{kt} \leq V_{kt} \quad \forall k \in I$$

$$\frac{\widehat{E}_{it}}{\lambda_{it}} \gtreqless \frac{\widehat{E}_{jt}}{\lambda_{jt}}, \forall i, j \notin I, i \neq j \quad \text{with} \quad \widehat{E}_{kt} > \widetilde{E}_{kt} \quad \text{and} \quad \widehat{W}_{kt} = V_{kt} \quad \forall k \notin I.$$

Proof. See appendix 2.

The aim of the method is thus to minimize the deviation from the allocation rule in order to satisfy the participation constraint. Thus all regions $i \in I$ receive fewer quotas than under the initial rule and their gains from cooperation are still positive. Furthermore, they contribute to the compensation of the other regions according to the (initial) allocation rule: the initial equitable allocation rule is then preserved *among the compensating regions*. On the other hand, the initial equitable allocation rule is not necessarily preserved among the compensated regions, which receive more quotas than under the initial rule. Their costs due to cooperation are then null, which makes them indifferent between cooperating and not cooperating.

5.2. THE CONSTRAINED EQUITABLE ALLOCATION RULES

Table V shows the total costs due to cooperation and the quotas allocated per head under the *Egalitarian*, the *Grandfathering* and the *Convergence* rules *constrained by individual rationality*. The results under the same *unconstrained* rules are also presented for the sake of comparison. Note that the results under the *ATP* and *GDP* rules are rather similar to those under, respectively, the *Egalitarian* and the *Grandfathering* rules. Due to limited space, they are not described below.

Under each of the constrained rules, all countries gain – or at least do not lose – from cooperation at each period. The unconstrained *Egalitarian* rule leads to positive costs due to cooperation for USA and FSU. Under the constrained allocation, these costs are driven down to zero in order to induce them to cooperate. They receive then much more quotas than under the initial *Egalitarian* rule: 3.9 instead of 1.1 for USA in 2000–2010 and 2.6 instead of 1.1 for FSU in the same period. JPN and EU also need to be compensated until 2030–2040 but they require a much lower increase of their initial allocation of quotas than USA and FSU.

Table V. Comparison of constrained and unconstrained rules

	Total costs due to cooperation (billion 1990 US$)						Quotas per head (1 ton of C per quota)					
EGALITARIAN												
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	523	12	96	–479	324	–1005	1.1	1.1	1.1	1.1	1.1	1.1
2030–2040	590	–10	6	–540	272	–932	1.1	1.1	1.1	1.1	1.1	1.1
2060–2070	580	–42	–100	–541	224	–840	1.3	1.3	1.3	1.3	1.3	1.3
2090–2100	516	–72	–192	–474	180	–758	1.5	1.5	1.5	1.5	1.5	1.5
EGALITARIAN CONSTRAINED												
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	0	0	0	–238	0	–290	3.9	1.2	1.4	0.8	2.6	0.8
2030–2040	0	0	0	–349	0	–265	4.0	1.0	1.1	0.9	2.2	0.9
2060–2070	0	–30	–58	–386	0	–244	4.2	1.2	1.2	1.2	2.2	1.2
2090–2100	0	–62	–159	–344	0	–232	4.3	1.4	1.4	1.4	2.3	1.4
GRANDFATHERING												
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	–4	–49	–78	–64	–38	–296	4.0	1.8	1.7	0.6	2.8	0.8
2030–2040	–207	–126	–310	–53	–290	372	5.1	2.4	2.2	0.7	3.4	0.8
2060–2070	–477	–207	–547	–2	–528	1041	6.5	3.2	2.8	0.8	4.3	0.9
2090–2100	–726	–271	–734	92	–708	1548	8.2	4.1	3.6	1.0	5.4	1.0
GRANDFATHERING CONSTRAINED												
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	0	–51	–82	–65	–43	–288	4.0	1.8	1.7	0.6	2.8	0.8
2030–2040	–79	–103	–239	0	–197	0	4.4	2.1	1.9	0.6	3.0	0.9
2060–2070	–78	–127	–308	0	–214	0	4.6	2.2	2.0	0.8	3.0	1.1
2090–2100	–102	–145	–354	0	–210	0	4.9	2.4	2.1	1.1	3.2	1.4
CONVERGENCE												
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	55	–42	–58	–110	2	–375	3.6	1.7	1.6	0.6	2.6	0.8
2030–2040	147	–74	–170	–270	–40	–208	3.3	1.8	1.7	0.9	2.4	0.9
2060–2070	345	–79	–199	–422	57	–422	2.5	1.7	1.6	1.2	2.0	1.2
2090–2100	516	–72	–192	–474	180	–758	1.5	1.5	1.5	1.5	1.5	1.5
CONVERGENCE CONSTRAINED												
Period	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*	*USA*	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
2000–2010	0	–40	–50	–102	0	–336	3.9	1.7	1.6	0.6	2.6	0.8
2030–2040	0	–69	–154	–248	–23	–120	4.0	1.8	1.7	0.8	2.3	0.9
2060–2070	0	–68	–167	–349	0	–135	4.2	1.6	1.5	1.1	2.2	1.1
2090–2100	0	–62	–159	–344	0	–232	4.3	1.4	1.4	1.4	2.3	1.4

Table VI. Quotas per head Grandf. and Egal. constrained rules and ratio in 2060–2070

	USA	*JPN*	*EU*	*CHI*	*FSU*	*ROW*
Quotas per head Grandf. Constrained *	4.57	2.24	1.98	0.80	3.03	1.10
Quotas per head Egal. Constrained **	4.19	1.16	1.16	1.16	2.20	1.16
Ratio (*)/(**)	1.09	1.93	1.71	0.69	1.38	0.95

The compensating regions, i.e., those that enjoyed gains from cooperation under the initial allocation rule, receive fewer quotas than previously. Accordingly, their gains decrease. Nevertheless, the initial *Egalitarian* rule still applies among them. In decades 2000–2010 and 2030–2040 for instance, CHI and ROW compensate all the other regions and receive both 0.8 quotas per head instead of 1.1 in 2000–2010 and 0.9 quotas per head instead of 1.1 in 2030–2040. From 2060 onwards, they are joined by JPN and EU which also contribute to compensate USA and FSU. JPN, EU, CHI and ROW receive then the same amount of quotas per head (1.2 in 2060–2070 and 1.4 in 2090–2100) while enjoying gains from cooperation.

The picture is almost the reverse under the constrained *Grandfathering* rule: CHI and ROW must be compensated from, respectively, 2030 and 2020 onwards. Note that USA is also compensated in the first two periods, that is up to 2010. However, they enjoy significant gains afterwards. Hence, among them, USA, JPN, EU and FSU still receive quotas according to the initial *Grandfathering* rule. However, their allocation is almost divided by a factor of 2 in order to compensate CHI and ROW.

Under the constrained *Convergence* rule, the *Grandfathering* component of the rule is dominated by the *Egalitarian* one. Exactly as under the constrained *Egalitarian* rule, USA and FSU must be compensated and do not enjoy any gains from cooperation, except for FSU in 2030–2040.

Let us finally compare the different constrained equitable allocation rules by analyzing two alternative constrained allocations of quotas, the *Grandfathering* and the *Egalitarian* rules for instance, in a given period, decade 2060–2070 for instance (see Table VI). The ratio of the allocations under the *Grandfathering* and the *Egalitarian* constrained rules are significantly different from 1. JPN receives almost two times more quotas under the *Grandfathering* constrained rule than under the *Egalitarian* one, while this factor drops to 0.69 for CHI. Accordingly, constrained allocations of quotas still vary considerably with the initial allocation rule under consideration. Such a comparison suggests therefore that there is some room for considering equity in the allocation of quotas while taking acceptability, defined here as individual rationality, into account.

6. Conclusion

Using a long term dynamic (closed loop) model, this paper has analyzed the welfare implications of different rules to allocate tradable CO_2 emissions quotas among the regions of the world. The total amount of quotas to be distributed at each period of time corresponds to the optimal amount of emissions to be realized during that period. Following Rose et al. (1998), we distinguish the rules which apply directly to the allocation of the quotas – *allocation-based* rules – from those which apply to the surplus resulting from cooperation – *outcome-based* rules. By definition, every *outcome-based* allocation rule leads to individual rationality since every country receives a payoff at least as large as under a non-cooperative situation. However, computations show that none of the envisaged *allocation-based* equitable rules (e.g., the Egalitarian, GDP, Ability to pay, Grandfathering and Convergence rules) satisfies individual rationality for every country along the entire time path.

Since individual rationality is a necessary condition for the stability of the agreement, we have developed a method consisting in finding an allocation of quotas which guarantees individual rationality for every country at each period and which is as close as possible to any given equitable allocation-based rule. Hence, the priority is given to satisfying participation constraints and the degree of freedom that is left is devoted to satisfying equity in the allocation of the quotas. The countries for which an equitable allocation-based rule is not acceptable are just compensated by receiving more quotas in order to be indifferent between cooperating and not cooperating. The unconstrained countries receive then fewer quotas. However, the equitable rule is preserved *among them*.

In the context of the discussions taking place on post-Kyoto commitments, three results of this analysis deserve particular attention. Firstly, equitable allocation-based rules need to be very much modified for the agreement to be individualy rational. For instance, constraining the *Egalitarian* rule (same amount of quotas per capita) by individual rationality leads the USA to receive 3 times the amount of quotas it would get under the unconstrained egalitarian rule. Such a departure from the *Egalitarian* rule is necessary in order to provide USA incentives to join the agreement. Under the constrained *Grandfathering* rule (quotas are allocated in proportion to emissions in 1990), the picture changes. Developing countries must be compensated and receive more quotas than under the unconstrained rule, while USA receives only half of the quotas it would get under this unconstrained rule.

Secondly, taking individual rationality constraints into account does *not* mean that there is no more room for satisfying equity principles. Indeed, once they are constrained by rationality, the allocation rules based on various conceptions of equity still lead to rather different amounts of quotas in each country. For instance, the ratio between the quotas allocated under the constrained *Grandfathering* and the constrained *Egalitarian* rules is 1.9 for Japan and 0.7 for China in 2060–2070.

Thirdly, our computations have shown that, for instance, an agreement based on the – unconstrained – *Grandfathering* rule leads to individual rationality in period

2000–2010, but not from 2010–2020 onwards. Therefore, allocation rules must be designed by keeping in mind that rationality (and, more generally, stability) in the first periods does not at all guarantee rationality (stability) along the entire time path. This highlights the necessity to use a fully dynamic setting such as the one developed here.

However, these features were only tractable in a model excluding economic interactions between the countries, with the level of emissions abatement as the only control variable of the countries and with a decomposition of the world in only six regions. These limitations could be addressed in a first extension of the paper. A second extension could be to broaden the concept of rationality to coalitions of countries rather than limiting it to individual countries as it is done in the present paper.

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Notes

1. This relation follows from the fact that the international optimum is collectively preferable to the Nash equilibrium – there is an ecological surplus –, so that $\sum_i V_{i,T} \geq \sum_i W_{i,T}$ with $\sum_i W_{i,T} = \sum_i \widetilde{W}_{i,T}$ by $\sum_i E^*_{i,T} = \sum_i \widetilde{E}_{i,T}$.
2. Given the non-convexity of the cost functions, we are not able to prove uniqueness of the Nash equilibrium. However, the algorithm developed by Germain et al. (2003) allows us to verify that, at each step, the objective function of each country is well behaved and yields existence and uniqueness of the fallback non-cooperative Nash equilibrium.
3. We refer the interested reader to Germain et al. (2003) for a description of the algorithm used. All computations were made with the MATLAB software.
4. For more details, see Nordhaus and Yang (1996), pp. 744–745.
5. According to any allocation-based sharing rule, each country i receives $\widetilde{E}_{it} = \lambda_{it} \sum_{j=1}^{n} E^*_{jt}$ quotas at period t.

References

AGBM (1996), *Quantified Emission Limitation and Reduction Objectives within Specified Time-Frames: Review of Possible Indicators to Define Criteria for Differentiation among Annex I Parties*, FCCC/AGBM/1996/7, June 21.

AGBM (1997), *Framework Compilation of Proposals from Parties for the Elements of a Protocol or Another Legal Instrument*, FCCC/AGBM/1997/2, January 31.

Chander, P. and H. Tulkens (1995), 'A Core-theoretic Solution for the Design of Cooperative Agreements on Transfrontier Pollution', *International Tax and Public Finance* **2**, 279–293.

Chander, P. and H. Tulkens (1997), ‘The Core of an Economy with Multilateral Environmental Externalities’, *International Journal of Game Theory* **26**, 379–401.

Cline, W. (1992), *The Economics of Global Warming*, Institute for International Economics, Washington, DC.

Cropper, M., S. Aydede and P. Portney (1994), ‘Preferences for Life Saving Programs: How the Public Discounts Time and Age’, *Journal of Risk and Uncertainty* **8**, 243–265.

Finus, M. (2002), ‘Game Theory and International Environmental Cooperation: Any Practical Application?’, in C. Böhringer, M. Finus and C. Vogt, eds., *Controlling Global Warming: Perspectives from Economics, Game Theory and Public Choice*. Cheltenham, UK: Edward Elgar.

Folmer, H., N. Hanley and F. Missfeldt (1998), ‘Game-theoretic Modelling of Environmental and Resource Problems: An Introduction’, in N. Hanley and H. Folmer, eds., *Game Theory and the Environment*. Edward Elgar Publishing Limited.

Germain, M., Ph. Toint, H. Tulkens and A. De Zeeuw (2003), *Transfers to Sustain Core-theoretic Cooperation in International Stock Pollutant Control*. CORE Discussion Paper n°9832, Université catholique de Louvain, Louvain-La-Neuve, forthcoming in *Journal of Economic Dynamics and Control*.

Germain, M., Ph. Tulkens, H. Tulkens and J. P. van Ypersele (2002), *Side Payments and International Cooperation in a Regionalised Integrated Assessment Model for Climate Change*. CORE Discussion Paper n°2002/19, Université catholique de Louvain, Louvain-La-Neuve.

Germain, M. and J. P. van Ypersele (1999), *Financial Transfers to Sustain International Cooperation in the Climate Change Framework*. CORE Discussion Paper n°9936 and CLIMNEG Working Paper n°19, CORE, Université catholique de Louvain, Louvain-La-Neuve.

Grubb, M. (1995), ‘Seeking Fair Weather: Ethics and the International Debate on Climate Change’, *International Affairs* **71**(3), 463–496.

Intergovernmental Panel on Climate Change (IPCC) (1996), *Climate Change 1995: Economic and Social Dimensions*. Cambridge University Press.

Intergovernmental Panel on Climate Change (IPCC) (2002), *Climate Change 2001: Economic and Social Dimensions*. Cambridge University Press.

Kverndokk, S. (1995), ‘Tradeable CO2 Emission Permits: Initial Distribution as a Justice Problem’, *Environmental Values* **4**, 129–148.

Manne, A. and R. Richels (1998), *Economic Impacts of Alternative Emission Reduction Scenarios*. Special Report ACCF Center for Policy Research, October 1998, Internet Edition.

Nordhaus, W. D. and J. Boyer (1999), *Roll the DICE Again: Economic Models of Global Warming*. Internet Edition (http://www.econ.yale.edu/~nordhaus/homepage/dicemodels.htm), October 25, 1999.

Nordhaus, W. D. and Z. Yang (1996), ‘A Regional Dynamic General-equilibrium Model of Alternative Climate-change Strategies’, *The American Economic Review* **86**(4), 741–765.

Richels, R., J. Edmonds, H. Gruenspecht and T. Wigley (1996), *The Berlin Mandate: The Design of Cost-effective Strategies*. Paper prepared for the Standford Energy Modeling Forum, February 3, 1996.

Reiner, D. and H. Jacoby (1998), *Annex I Differentiation Proposals: Implications for Welfare, Equity and Policy*. Report No. 27, Massachusetts Institute of Techonology, Joint Program on the Science and Policy of Global Change.

Rose, A., B. Stevens, J. Edmonds and M. Wise (1998), ‘International Equity and Differentiation in Global Warming Policy’, *Environmental and Resource Economics* **12**, 25–51.

Rowland, I. (1997), ‘International Fairness and Justice in Addressing Global Climate Change’, *Environmental Politics* **6**, 1–30.

Torvanger, A. and O. Godal (1999), *A Survey of Differentiation Methods for National Greenhouse Gas Reduction Targets*. Report 1999:5, Center for International Climate and Environmental Research (CICERO), Oslo, August 1999.

Torvanger, A. and L. Ringius (2000), *Burden Differentiation: Criteria for Evaluation and Development of Burden Sharing Rules*. Working Paper 2000:1, Center for International Climate and Environmental Research (CICERO), Oslo, February 2000.

Weitzman, M. (1998), *Gamma Discounting*. Discussion Paper 1843, Harvard University, Harvard, MA.

Appendix

DATA

The preindustrial level of the CO_2 atmospheric stock M_0 is equal to 590 billion tons of carbon equivalent. The rate of decay of CO_2 in the atmosphere, δ, is equal to 0.0833 per decade, while the marginal atmospheric retention ratio of CO_2, β, is equal to 0.64. Parameter η is equal to 2.5/ln(2) ; it is calibrated in such a way that a doubling of the CO_2 atmospheric concentration results in an increase of global temperature of 2.5 degrees Celsius with respect to its preindustrial level.

Output and emissions growth rates are taken from a model developed at CORE and which is based on the RICE'98 model (Nordhaus and Boyer, 1998) (see Germain, M., Ph. Tulkens, H. Tulkens and J.P. van Ypersele (2002)). Output, population and energy intensity are presented in Tables A.1 to A.3. Parameters characterizing damage and abatement cost functions (4) and (5) are given in Table A.4.[4]

Table A.1. Population (billion) – POP_{it}

	USA	JPN	EU	CHI	FSU	ROW	World
1990–2000	0.250	0.124	0.367	1.134	0.289	3.103	5.266
2000–2010	0.264	0.124	0.382	1.255	0.307	3.685	6.019
2010–2020	0.274	0.125	0.394	1.354	0.321	4.227	6.694
2020–2030	0.281	0.125	0.403	1.432	0.333	4.714	7.286
2030–2040	0.285	0.125	0.410	1.492	0.341	5.141	7.795
2040–2050	0.288	0.125	0.415	1.539	0.348	5.510	8.225
2050–2060	0.291	0.125	0.419	1.574	0.354	5.821	8.583
2060–2070	0.292	0.125	0.422	1.601	0.358	6.082	8.879
2070–2080	0.293	0.125	0.424	1.621	0.361	6.298	9.121
2080–2090	0.294	0.125	0.425	1.636	0.363	6.475	9.318
2090–2100	0.294	0.125	0.426	1.647	0.365	6.620	9.477
2100–2110	0.294	0.125	0.427	1.656	0.366	6.738	9.606
Total	3.400	1.493	4.914	17.941	4.105	64.414	

Table A.2. GDP (billions US$ of 1990 per decade/1000) – Y_{it}

	USA	JPN	EU	CHI	FSU	ROW	World
1990–2000	63.11	33.11	77.11	4.78	9.88	56.83	244.82
2000–2010	72.92	36.78	83.35	6.82	11.81	77.80	289.48
2010–2020	83.42	40.83	90.04	9.51	14.13	103.90	341.83
2020–2030	94.58	45.26	97.11	12.98	16.84	135.48	402.25
2030–2040	106.33	50.05	104.53	17.34	19.95	172.80	471.00
2040–2050	118.61	55.22	112.28	22.70	23.46	215.95	548.22
2050–2060	131.41	60.78	120.34	29.16	27.40	264.90	633.99
2060–2070	144.68	66.74	128.70	36.80	31.76	319.52	728.20
2070–2080	158.39	73.09	137.31	45.70	36.55	379.55	830.59
2080–2090	172.51	79.87	146.17	55.90	41.77	444.69	940.91
2090–2100	187.02	87.05	155.24	67.42	47.42	514.57	1058.72
2100–2110	201.89	94.67	164.50	80.26	53.47	588.76	1183.55
Total	1534.87	723.45	1416.68	389.37	334.44	3274.75	

Table A.3. Energy intensity (*1000) – ν_{it}

	USA	JPN	EU	CHI	FSU	ROW
1990–2000	0.231	0.092	0.118	2.069	1.199	0.693
2000–2010	0.209	0.081	0.105	1.573	0.944	0.632
2010–2020	0.190	0.072	0.094	1.239	0.760	0.578
2020–2030	0.174	0.064	0.085	1.006	0.624	0.531
2030–2040	0.160	0.058	0.077	0.838	0.522	0.490
2040–2050	0.148	0.053	0.070	0.714	0.444	0.454
2050–2060	0.137	0.048	0.064	0.620	0.384	0.422
2060–2070	0.128	0.044	0.059	0.547	0.336	0.395
2070–2080	0.120	0.040	0.055	0.490	0.298	0.370
2080–2090	0.112	0.037	0.051	0.443	0.267	0.348
2090–2100	0.106	0.035	0.048	0.405	0.241	0.329
2100–2110	0.100	0.032	0.045	0.374	0.220	0.312

Table A.4. Other parameters

	USA	JPN	EU	CHI	FSU	ROW
a_{i1}	0.07	0.05	0.05	0.15	0.15	0.1
a_{i2}	2.2887	2.2887	2.2887	2.2887	2.2887	2.2887
b_{i1}	0.01102	0.01174	0.01174	0.015523	0.00857	0.02093
b_{i2}	1.5	1.5	1.5	1.5	1.5	1.5

PROOF OF PROPOSITION 1

The langrangian of problem (16) writes

$$\begin{aligned} L_t &= \sum_i \frac{\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]^2}{\lambda_{it}} + \mu_t \sum_i \left[\widehat{E}_{it} - \widetilde{E}_{it}\right] \\ &\quad + \sum_i \pi_{it} \left[\widetilde{W}_{it} + \sigma_t^* \left[\widetilde{E}_{it} - \widehat{E}_{it}\right] - V_{it}\right] \end{aligned}$$

where μ_t is the multiplier associated to constraint (17) and π_{it} are the multipliers associated to constraints (18). The Kuhn-Tucker conditions of this problem are

$$\frac{\partial L_t}{\partial \widehat{E}_{it}} = \frac{2\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]}{\lambda_{it}} - \mu_t - \sigma_t^* \pi_{it} = 0, \quad \forall i, \tag{20}$$

$$\frac{\partial L_t}{\partial \mu_t} = \sum_i \widehat{E}_{it} - \sum_i \widetilde{E}_{it} = 0, \tag{21}$$

$$\begin{aligned} \frac{\partial L_t}{\partial \pi_{it}} &= \widetilde{W}_{it} + \sigma_t^* \left[\widetilde{E}_{it} - \widehat{E}_{it}\right] - V_{it} \leq 0, \quad \pi_{it} \geq 0 \text{ and} \\ &\quad \pi_{it} \left[\widetilde{W}_{it} + \sigma_t^* \left[\widetilde{E}_{it} - \widehat{E}_{it}\right] - V_{it}\right] = 0, \quad \forall i. \end{aligned} \tag{22}$$

Furthermore, $\pi_{it} = 0 \; \forall i \in I$ and these regions contribute to the compensation of the other regions according to the (initial) allocation rule. Indeed, (20) then leads to

$$\frac{\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]}{\lambda_{it}} = -\frac{\mu_t}{2}, \quad \forall i \in I, \tag{23}$$

with $\widehat{E}_{it} < \widetilde{E}_{it}$ as $\lambda_{it} > 0 \; \forall i$ and as $\mu_t > 0$ by combining (20) and (21). Note that this also gives $\widehat{W}_{it} \leq V_{it} \; \forall i \in I$ from (19) and (22).

The initial equitable allocation rule is then preserved *among the compensating regions*. Indeed, for *allocation-based allocation rules*, allocations of quotas are (initially) such that $\frac{\widetilde{E}_{it}}{\lambda_{it}} = \frac{\widetilde{E}_{jt}}{\lambda_{jt}}, \forall i \neq j$.[5] By (23), that is $\frac{\widehat{E}_{it}}{\lambda_{it}} = \frac{\widetilde{E}_{it}}{\lambda_{it}} - \frac{\mu_t}{2}, \forall i \in I$, the new allocation of quotas is thus also such that $\frac{\widehat{E}_{it}}{\lambda_{it}} = \frac{\widehat{E}_{jt}}{\lambda_{jt}}, \forall i, j \in I, i \neq j$ (that is only among the compensating regions).

On the other hand, $\widetilde{W}_{it} > V_{it}$ and $\widetilde{W}_{it} + \sigma_t^* \left[\widetilde{E}_{it} - \widehat{E}_{it}\right] - V_{it} = 0 \; \forall i \notin I$, which leads to $\widehat{E}_{it} > \widetilde{E}_{it} \; \forall i \notin I$. Thus

$$\frac{\left[\widehat{E}_{it} - \widetilde{E}_{it}\right]}{\lambda_{it}} = \frac{\left[\pi_{it}\sigma_t^* - \mu_t\right]}{2}, \quad \forall i \notin I, \tag{24}$$

with $\pi_{it}\sigma_t^* > \mu_t$. The initial equitable allocation rule is therefore not necessarily preserved among the compensated regions.

Environmental Economics & Management Memorandum

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