# Short term entry barriers may be good for long term competition

*Jean-Pierre PONSSARD*
CNRS and Ecole Polytechnique,
Paris, France
mail address: jean-pierre.ponssard@polytechnique.edu

January 2008
(preliminary and incomplete, comments welcome)

**Abstract**

Entry barriers encourage competition "for" the market as opposed to "in" the market. Efficient entrants use penetrating strategies while inefficient incumbents harvest the market before leaving. These phenomenon are explored in an infinite horizon game in which history matters. Under some circumstances, higher entry barriers induce entry of efficient firms while lower entry barriers would not. This comes from the expected benefit of future rents. Social welfare may be enhanced as well. This result suggests that a rule of reason should be applied and that entry barriers should not be considered per se anticompetitive.

## 1 Introduction

In many markets the consumer buys from only one producer (i.e. for goods such as cars, electronic sets, banking accounts, insurance contracts, contracts with utility or internet providers...) and rarely changes from one producer to another one when the good (or the contract) needs be renewed. That incumbent producer enjoys some extra profit due to the short term barrier (frictions of various sorts, psychological bias, contract clauses, switching costs...) that limits competition. Is it to say that such markets are immune to potential competition?

Consider the video game industry. The early developments illustrate a period of intense short term competition with many bankruptcies followed by the emergence of Nintendo as a dominant player. Nintendo deployed various strategies to "lock in" the consumer (Sheff, 1993). However, in a judgment pronounced in 1992, the FTC declared that "lawyers would be more productively occupied playing Super Mario 3 that bringing cases like this case" (see Nintendo HBS case study, 1996). Further developments demonstrated that this judgment was well founded. Indeed new competitors such as Sony and Microsoft appeared in that industry.

In markets such as these, competition is "for the market" rather than "in the market". The best way to compete is through innovation relative to the current incumbent. Once a breakthrough occurs, a "penetrating" strategy is devised leaving some opportunity for the current incumbent to "harvest" its market share. One may argue that it is the expectation of the future profits that will be generated by the short term entry barrier that creates the motivation of potential entrants. If this is so it may be that some short term entry barrier triggers dynamic efficiency and that this is good for social welfare?

The balance between short term and long term competition is a recurrent topic in industrial organization. For instance, it has been argued by d'Aspremont and Motta (2000) that tough price competition may trigger a higher concentration level in the industry, thus inducing a welfare loss. There the degree of toughness is associated to either Bertrand or Cournot competition. A similar point is made by Norman and Thisse (2000) comparing discriminatory pricing with mill pricing in the context of spatial competition. In such models the degree of concentration is endogenously determined through a free entry condition. Historically, it seems that the enforcement of price competition indeed induced concentration moves in many industries. This prompted the attention of antitrust authorities to the analysis of mergers and market dominance.

There are also many models that explore the impacts of switching costs and network effects on short term and long term competition (see Farrell and Klemperer, 2001, for a survey). Such models exhibit the fact that long term competition is often more intense than it seems: firms may fiercely compete to lock in the customers. The models can be explored to discuss the overall impact on pricing, entry and social welfare. The general idea is that, not necessarily but often, firms dissipate resources in creating and defending switching costs and network effects.

This paper is concerned with similar issues but with a major difference. The social welfare analysis is on dynamic productive efficiency: what are the incentives for the firms to compete through relative cost advantage rather than through defensive moves? It will be shown that the existence of short term entry barrier may under some circumstances encourage dynamic productive efficiency. Increasing the short term entry barrier makes the market potentially more profitable thus triggers more long term competition to fight for the market. These two effects balance each other so that the incumbency rent appears as a bell shaped function. Decreasing the short term entry barrier may then increase the incumbent profit, and make it more difficult to be dislodged by a more efficient competitor, which eventually discourages competition through cost advantage. This result suggests that a rule of reason should be applied and that short term entry barriers should not be considered per se anticompetitive.

That cost asymmetry matters in practice has been exemplified by a number of illuminating case studies by Scherer (1992). There, whenever import barriers are lowered, the domestic industry may either fight back or harvest its market share depending on its relative cost position. At the more macro level, Aghion et ali. (2006) provide evidence that, as the rate of imports increases, the profitability of the domestic firm may either increase or decrease depending again on its position relative to the industry efficiency frontier. While Sherer analysis is purely empirical, Aghion et ali. use a Schumpeterian model to justify their result, which is a line of reasoning similar to the one developed here.

The argument of this paper is based on a model in which firms repeatedly compete for a market. Price is used as the strategic variable. Firms can only make short term price commitments. They maximize their total discounted profits. Due to fixed cost relative to market size, only one firm may operate with positive profit at any given stage. The analysis can be limited to only two firms: the two most efficient firms in the industry which repeatedly fight for incumbency. The stage game has two pure price equilibria in anyone of which one of the two firms features as an incumbent and the other one as an entrant. The game is solved using a forward induction approach for extensive games originally developed in Ponssard (1991). Forward induction has also been used to study entry in other circumstances, see for instance Bagwell and Ramey (1996) who investigate the role of excess capacity.

In a recent paper on switching costs, Biglaiser et ali. (2007) show that reducing the level of switching cost may increase the incumbency rent. In their stage model there are no fixed cost so that the equilibrium requires randomized strategies. The solution concept used to select the equilibrium in the infinite horizon game is also specific. The result is obtained whenever customers may have either zero or an exogenously given switching cost. To the best of my knowledge these are the only two papers in which it is shown that this bell shaped property of the incumbency rent carries over from a two stage model (which is known) to an infinite horizon model. That this can be the case provides a strong ground to discuss the social welfare of short term entry barrier relative to dynamic productive efficiency.

The paper is organized as follows. Section 2 details the competition model under study, gives the equilibrium of the one stage game and provides the intuition about the impact of introducing a longer time horizon. Section 3 details the solution of the infinitely repeated game using forward induction as a selection principle. The economic properties of that solution as a function of the short term entry barriers are discussed in section 4. Differences with an approach based on perfect Markov equilibra (Maskin and Tirole, 1988) are pointed out.

# 2 A simple economic model of dynamic competition with short term entry barriers

## 2.1 The model: an infinitely repeated stage game with price competition and high fixed costs

The paper is mostly conceptual, the analysis will be carried on using a simple competition model.

Consider a situation in which two firms repeatedly compete for a market. Let $i \in \{1, 2\}$ be anyone of the two firms and $j \in \{1, 2\}, j \neq i$ be the other one. Each firm $i$ incurs a fixed cost $f_i$ at each stage but only if it is active on the market, i.e. if it delivers a strictly positive quantity. The fixed cost is not a set up cost incurred once and for all at the beginning of the infinite stage game. Firms differ only in their fixed costs with $f_1 \leq f_2$, so that firm 1 is more efficient than firm 2. If $f_1 = f_2$, this is the *symmetric* case, otherwise it is the *asymmetric* case. Price is the decision variable and noted $p_i \in [0, 1]$. Prices are set simultaneously at each stage, no long term price commitments are allowed. For a given price $p_j$ of firm $j$ the demand function of firm $i$, to be noted $d_i(p_i, p_j)$, is kinked using a positive constant parameter denoted $s$ to be interpreted as the short term entry barrier. More precisely, for $i \in \{1, 2\}$, we have:

$$
\begin{array}{cc}
0 \leq p_i \leq p_j - s & d_i(p_i, p_j) = 1 - p_i \\
p_j - s \leq p_i \leq p_j + s & d_i(p_i, p_j) = (p_j - p_i + s)(1 - p_j + s)/2s \\
p_j + s \leq p_i \leq 1 & d_i(p_i, p_j) = 0
\end{array}
$$

and the stage profit function writes:

$$\pi_i(p_i, p_j) = p_i d_i(p_i, p_j) - (1_{d_i \neq 0}) f_i$$

The fixed costs are sufficiently high so that only one firm may generate positive profit at anyone stage. Firms maximize their discounted profits using the same discount factor $\delta$. The paper focuses on the role of the short term entry barrier $s$ and not on the role of the time commitment induced by the discount factor $\delta$. It is assumed that $\delta$ is close to 1.

## 2.2 Analysis of the one stage game: the equilibrium set as a function of the short term entry barrier

For $i \in \{1, 2\}$ denote the pure monopoly profit as $\pi_i^m(p_i) = p_i(1 - p_i) - f_i$. Let $p_i^{ac}$ be the minimal price that solves $\pi_i^m(p_i) = 0$. This price will referred to as the *average cost* of firm $i$.

Define the best entry price of firm $j$ to a price $p_i$ as the price that maximizes its revenue $p_j d_j(p_i, p_j)$. Denote $p_j^{BE}(p_i)$ this price. Simple calculations show that:

$$
\begin{array}{cc}
0 \leq p_i \leq 3s & p_j^{BE}(p_i) = (p_i + s)/2 \\
3s \leq p_i \leq s + 1/2 & p_j^{BE}(p_i) = p_i - s \\
s + 1/2 \leq p_i & p_j^{BE}(p_i) = 1/2
\end{array}
$$

Define the *one stage limit price* $p_i^{\max}$ as the highest $p_i$ such that:

$$\pi_j(p_i, p_j^{BE}(p_i)) \leq 0.$$

Let $p_i^{\min} = Max(p_i^{ac}, 2s)$.

It is a simple matter to check that $p_i^{\min} \leq p_i^{\max}$ as long as $f_j \geq 9s(1 - s)/8$ (or $s \leq .5 - (.25 - 8f_j/9)^{.5}$).

**Proposition 1** *For $i \in \{1,2\}$ , if $f_j \geq 9s(1-s)/8$, for all $p_i \in [p_i^{\min}, p_i^{\max}]$ the set of prices $[p_i, p_j = p_i + s]$ is a pure Nash equilibrium.*

For $0 \leq s \leq p_i^{ac}/2$ it is clear that $[p_i^{\min}, p_i^{\max}] = [p_i^{ac}, p_j^{ac} + s]$. For $p_i^{ac}/2 \leq s \leq .5 - (.25 - 8f_j/9)^{.5}$ then $[p_i^{\min}, p_i^{\max}] = [2s, p_i^{\max}]$ and the determination of $p_i^{\max}$ requires some numerical calculations.

As firm 2 becomes less efficient relative to firm 1, $f_2$ increases, $p_2^{\min}$ increases but $p_2^{\max}$ remains unaffected. For a large enough $f_2$, $p_2^{\min} > p_2^{\max}$ , firm 2 can no longer be an incumbent.

For a given pair of fixed costs, $f_1 > f_2$, if $s$ is small enough there is only one set of pure Nash equilibria in which firm 1 is the incumbent ($p_1^{\min} \leq p_1^{\max}$), if $s$ is large enough there are two sets of pure Nash equilibria ($p_1^{\min} \leq p_1^{\max}$ and $p_2^{\min} \leq p_2^{\max}$). The larger $s$ the larger the incumbency rents. But, if $s$ gwere too large the regime of competition in the stage game would change to a randomized equilibrium and, ultimately, to a duopoly in pure strategies. In the paper it is assumed at all times that this does not occur, $s$ remains small enough so that at least $p_1^{\min} \leq p_1^{\max}$.

### 2.3 The multi stage game: the competitive pressure increases, the less efficient firm harvests the market and the more efficient firm moves in

Suppose the game is repeated twice and investigate an equilibrium path in which firm $i$ would select $p_i^1$ at the first stage and then $p_i^2$ at the second stage. If time matters the successive prices should be interdependent. Forward induction captures very well this idea. It requires that firm $j$ cannot deviate through $p_j^{BE}(p_i^1)$ at the first stage to capture the second stage incumbency rent associated to its preferred equilibrium payoff $\pi_j^m(p_j^{\max})$. The competitive pressure in a two stage is higher than in a one stage game ($p_i^1 \leq p_i^2$).

The general results are as follows (Ponssard, 1991 and Gromb et ali., 1997):

- The range of equilibrium prices for $p_i^1$ is now $[p_i^{\min}, p_i^{2,\max}]$ in which $p_i^{2,\max}$ is the highest $p_i$ such that $\pi_j(p_i, p_j^{BE}(p_i)) + \delta\pi_j^m(p_j^{\max}) \leq 0$.

- Even if $p_2^{\min} \leq p_2^{\max}$ it may very well be the case that $p_2^{\min} > p_2^{2,\max}$ , the less efficient firm may be an incumbent in a one stage game but not in a two stage game; if this is the case, there is an equilibrium path in which firm 2 first selects $p_2 = p_2^{\max}$ and then it is firm 1 that is the incumbent at the second stage with $p_1 = p_1^{\max}$ . It is important to note that this equilibrium is not the outcome of tacit collusion but the outcome of the inability of the less efficient firm to sustain long term incumbency when facing a more efficient entrant. In a qualitative sense it means that less efficient firm benefits enough from the short term entry barrier to extract some rent through a harvesting strategy, but not enough to secure a long term incumbency advantage.

In this paper, equilibrium paths in the infinite horizon game are selected so that such phenomenon are captured. The impacts of the short term entry barrier are analyzed in details. Intuitively, the less short term competition ($s$

large) the more intense long term competition, and the easier for a more efficient entrant to penetrate the market. If these intuitions are correct they would imply that short term entry barriers would, under some circumstances, be beneficial to trigger productive efficiency, i.e. to have the more efficient firm as the long term incumbent. The circumstances are important to be determined since, of course, the best of all words would be to have no short term entry barriers, i.e. $s = 0$.

# 3 The solution of the infinitely repeated game

Some preliminary comments are in order. Thanks to the folk theorem, the set of perfect Nash equilibria of the repeated game is extremely large. As motivated in the preceding section, the idea is to use forward induction as a selection process. This will be done in a loose way through the direct proposal of a subset of perfect equilibria that captures the idea, rather than from a deductive process from general principles. This is so because there is no formal definition of forward induction for finite extensive games and, a fortiori, for infinite extensive ones.

The two firms are arbitrarily distinguished as a *long term* firm, denoted as firm $L$, and a *short term* firm, denoted as firm $S$. At this point no assumption is made regarding the relative efficiency of the two firms, though one should associate the long term firm with the more efficient firm and the short term firm with the less efficient one.

The proposed solution selects two equilibria that generate two equilibrium paths. One equilibrium path is such that firm $L$ stays as a permanent incumbent with a price $p_L = q_L$. The other equilibrium path is such that firm $S$ stays as an incumbent for a finite number of stages, $n$, with a sequence of prices $(p_S^t = q_S^t)_{t=1,..n}$ and then firm $L$ becomes the incumbent for ever with a price $p_L = q_L$. Clearly, firm $L$ prefers the first path and firm $S$ the second path. These two paths reinforce each other through a forward induction argument in the following sense. Suppose the first path is played and firm $S$ is unhappy. It could play $p_S^{BE}(q_L)$ to signal that the game should from now on proceed along the second path. Along the equilibrium path it should be that such a deviation be unprofitable. And vice versa for firm $L$. In this way the two paths are constructed through a unique interdependent procedure.

The complete the strategy for each firm one needs to precise what to do in case of deviation. The following rule is adopted. If an incumbent prices at a higher price than the one corresponding to the proposed path or the entrant at a price below its stage best response then the game actually proceeds along the path in which it is the entrant which becomes the incumbent. Other deviations such as pricing below the proposed price for the incumbent or selecting any price above its stage best response for the entrant have no influence on the path, it stays exactly on track.

Altogether, the game may start with either one of the two firms as an incumbent but, after at most $n$ stages, how it started is irrelevant, firm $L$ is the incumbent for ever.

It is now shown that the proposed equilibria can be obtained through the solution of a system to be denoted $\Phi_\delta^n$. For an integer $n$, if they exist, define the real numbers $q_L$ and $(q_S^t)_{t=0,..n}$ that solves the following system:

$$
\begin{array}{lll}
\text{for } t \in \{0,1,2...n\} & \pi_L(p_L^{BE}(q_S^t), q_S^t) + \delta(1-\delta^{n-t})\pi_L^m(q_L)/(1-\delta) = 0 & (1) \\
 & \pi_S(p_S^{BE}(q_L), q_L) + \Sigma_{t=1}^{t=n}\delta^t \pi_S^m(q_S^t) = 0 & (2) \\
 & \pi_L^m(q_L) \geq 0 & (3) \\
\text{for } t \in \{1,2...n\} & \Sigma_{t'=t}^{t'=n}\delta^{t'-t}\pi_S^m(q_S^{t'}) \geq 0 & (4) \\
 & \Sigma_{t'=0}^{t'=n}\delta^{t'}\pi_S^m(q_S^{t'}) < 0 & (5)
\end{array}
$$

The interpretation is the following:

Equation (1) means that firm $L$ is patient enough to let firm $S$ harvest the market before penetrating it. It would cost $-\pi_L(p_L^{BE}(q_S^t), q_S^t)$ to move the game to the preferred path of firm $L$ generating an incremental discounted profit exactly equal to $\delta(1-\delta^{n-t})\pi_L^m(q_L)/(1-\delta)$ so that such a deviation is not profitable. The term $\pi_L^m(q_L)$ will be called the *incumbency rent*.

Equation (2) takes the point of view of firm $S$ . There is no point to challenge the incumbency of firm $L$ to capture the total discounted profit associated with a harvesting strategy. The term $\Sigma_{t=1}^{t=n}\delta^t \pi_S^m(q_S^t)$ will be called the *harvesting benefit*.

Inequations (3) and (4) stands for individual rationality: the incumbency rent is positive and the harvesting benefit remains positive at all stages.

Inequation (5) means that the strategy of firm $S$ is indeed a harvesting strategy of duration $n$. It would not be profitable for firm $S$ to harvest on a duration $n+1$.

**Proposition 2** *If it exists, the solution of system $\Phi_\delta^n$ defines two perfect equilibria for the infinitely repeated game.*

System $\Phi_\delta^n$ is precisely constructed to make any deviation from the paths unprofitable. The proof is omitted.

A number of general propositions can be proved and generalized to other demand functions than the ones introduced in section 2.1 (these proofs are in the appendix).

**Proposition 3** *The system $\Phi_\delta^n$ has at most one solution for $\delta$ close enough to 1.*

More can be said about this solution. Condition (1) implies that the sequence $(q_S^t)_{t=0}^{t=n}$ is strictly increasing in $t$. The following property holds.

**Proposition 4** *If, for $m > n$, $\Phi_\delta^m$ and $\Phi_\delta^n$ have both one solution, the longer the duration of the harvesting strategy, the higher its associated benefit.*

This implies that the system $\Phi_\delta^n$ indeed defines what one would intuitively expect from a harvesting strategy: an increasing sequence of prices and direct links between the pressure of the entrant, the duration of the harvesting strategy and the benefit that can be associated with that strategy. In the context of this game it is natural to focus on the most profitable harvesting strategy (i.e. the longest duration $n$ such that $\Phi_\delta^n$ has one solution).

It remains to be seen whether the proposed solution is not empty. Because the solution involves integers, it may not exist for very small values of $n$. Numerical analysis shows for instance that for a discount factor equal to 1 and not too much asymmetry in costs the system $\Phi_1^n$ has one solution for $n \geq 3$. The existence of an upper bound for $n$ can be precisely characterized.

**Proposition 5** *Define* $q_S^*$ *such that* $\int_{p=q_S^*}^{p=q_S^{\max}} \pi_S^m(p) d[\pi_L(p_L^{BE}(p), p)] = 0$, *the system* $\Phi_1^n$ *has one solution for all values of* $n$ *if only if* $\pi_S^m(q_S^*) + \pi_S(p_S^{BE}(p_L^{ac}), p_L^{ac}) \leq 0$. *Then* $\lim_{n\to\infty} q_L = p_L^{ac}$ *and* $\lim_{n\to\infty} q_S^1 = q_S^*$.

However, as soon as $\delta < 1$, the is always an upper bound for $n$.

**Proposition 6** *If* $\delta < 1, \exists n_\delta$ *such that* $\forall n > n_\delta, \Phi_\delta^n$ *has no solution.*

These last two propositions provide a link with the traditional rent dissipation result for symmetric entry games (Wilson, 1992). This point will be discussed later on.

# 4 Discussion: more short term entry barrier may be better than less

## 4.1 Main results

It is now shown that the incumbency rent is bell shaped as a function of the short term entry barrier. The discussion will proceed in two steps. Firstly the solution for $\delta = 1$ is considered. Secondly, the role of the discount factor is introduced. System $\Phi_\delta^n$ cannot be solved analytically. However it can be solved numerically.

Fig. 1 gives the incumbency rent as a function of $s$ for three cost configurations: one symmetric and two asymmetric.

- case 1: $f_L = f_S = .16$

The relevant range for $s$ is (see proposition 1): $0 \leq s \leq .5 - (.25 - 8f/9)^{.5} = .17$

The fact that the rent is bell shaped illustrates that this solution provides a balance between short term and long term competition: for $s$ small short term competition is tough, the incumbency position is not worth very much and the pressure to get it is moderate, altogether there is room for some rent. For $s$ large short term competition is smooth, the incumbency position is worth a lot and the pressure to get it is intense, altogether there is no room for rent.

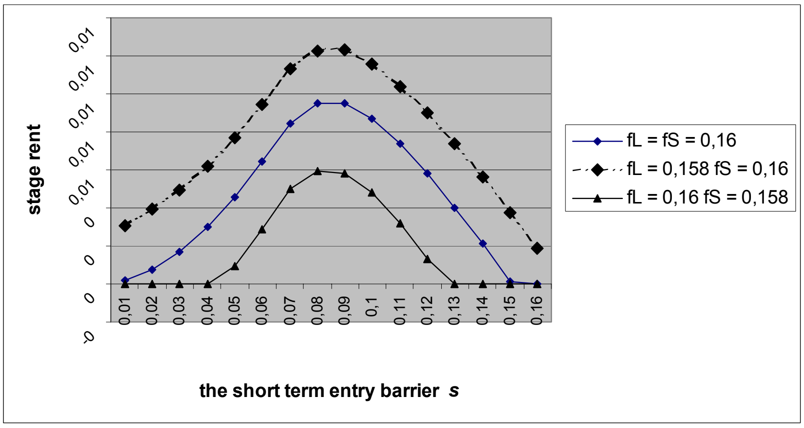


Figure 1: The incumbency rent is bell shaped

- case 2: $f_L = .158; f_S = .16$

The incumbency rent is higher, it pays for the long term firm to be more efficient.

- case 3: $f_L = .16; f_S = .158$

The incumbency rent of the long term firm decreases when the short term firm becomes more efficient. It becomes zero for $s$ small or for $s$ large: the short term firm would stay indefinitely if it were to enter.

From a policy perspective, a situation in which $s = .14$ is preferable to a situation in which $s = .08$. Indeed with $s = .14$ the rent for a historical inefficient incumbent is zero so that it would certainly prefer to use a harvesting strategy. This is not the case with $s = .08$, there its equilibrium price is $p = .210$. With $s = .14$ the efficient incumbent will select an equilibrium price that can be computed to be $p = .208$. This proves that the consumer is better off. The model illustrates a nice Shumpeterian process: the less efficient incumbent is eliminated by the more efficient one and both the more efficient firm and the consumer benefit from the process.

The bell shaped nature of the incumbency rent provides some protection for an inefficient historical incumbent. This protection is the highest the closest $s$ is to the value that gives the maximal rent. In industries in which innovation comes from breakthrough, this protection may not be so important, a much more efficient entrant will make its way to the market, but in industries in which innovation is more incremental it could pay to increase $s$!

By continuity, the arguments remain true for $\delta$ close to 1. It is interesting to quantify this continuity property. It has been proved that as soon as $\delta < 1$ the incumbency rent can no longer be equal to zero ($n$ is bounded). Fig. 2 depicts this rent for $\delta = .98$. It can be observed that enlarging the commitment period (i.e. decreasing $\delta$) strongly mitigates the power of long term competition. As might be expected it is for large values of $s$ that the impact is the largest because

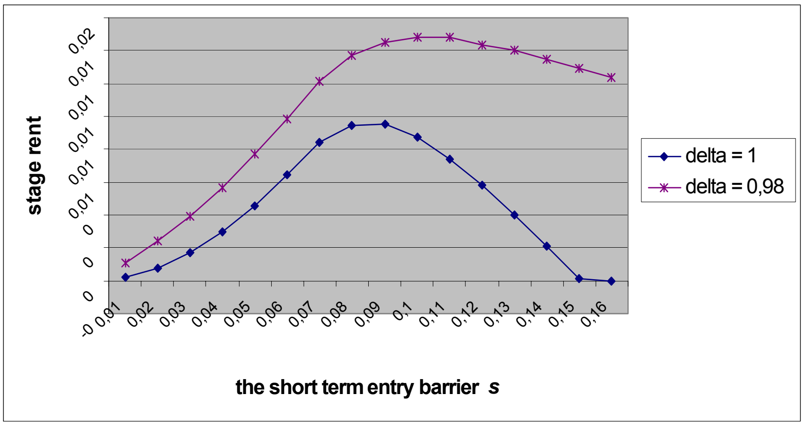


Figure 2: The role of the discount factor

this coincides with the zone in which the power of long term competition is at its peak.

This model also provides a theory of limit pricing which is consistent for all values of cost asymmetry. This is illustrated with $\delta = 1$. Take the one stage game with $s = .1$, $f_L = .16$ and let $f_S$ vary from .1 to .23. Firm $L$ is blockaded from entry with $f_S = .1$ but no longer is with $f_S = .11$, in which case $p_L^{\max} = .215 > p_L^{ac} = .2$. The static limit price increases as a function of $f_S$. Observe that for $f_S > .21$, it is firm $S$ which is blockaded from entry. Fig. 3 depicts the equilibrium price of firm $L$ as $f_S$ increases from .11 to .21. One can see that it increases from firm $L$ average cost to its static limit price $p_L^{\max}$.

## 4.2 Long term competition is over weighted with perfect Markov equilibria

These results are very different from what would be obtained using a perfect Markov equilibria. To construct such an equilibria one needs to modify the game form. Suppose that at each stage firms move in sequence: firstly the incumbent at the previous stage if it were unchallenged, in the reverse order if it were challenged (the results would remain true with an alternate game form such as the one used in Maskin and Tirole, 1988). Let $p_1^*$ and $p_2^*$ be the solution the following system of two equations:

$$\begin{array}{c} -C_1(p_2) + \delta\pi_1^m(p_1)/(1-\delta) = 0 \\ -C_2(p_1) + \delta\pi_2^m(p_2)/(1-\delta) = 0 \end{array}$$

**Proposition 7** *The perfect Markov equilibrium strategies are as follows: any of the two firms, say firm $i$, remains the incumbent for ever with $p_i = p_i^*$; the reaction function of the outsider, say firm $j \neq i$, is such that if $p_i > p_i^*$ then $p_j = p_j^{BE}(p_i^*)$ but if $p_i \leq p_i^*$ then $p_j = p_i^* + s$.*

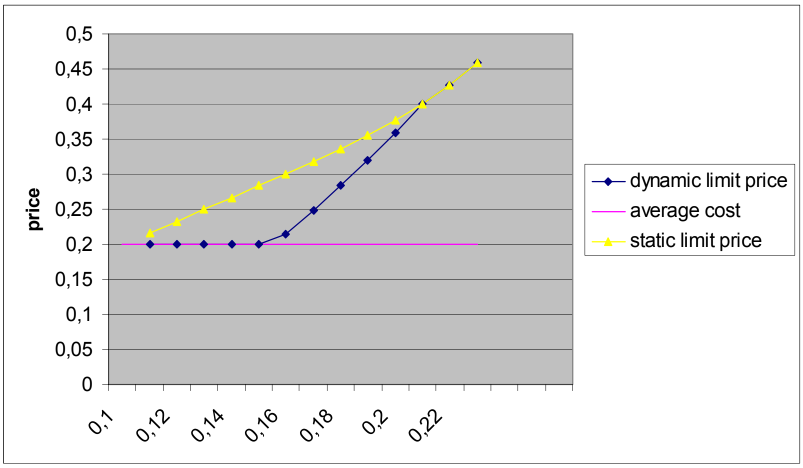


Figure 3: The dynamic limit price of firm $L$ as a function of $f_S$

This solution has two striking features which make it unappealing from an economic point of view: firstly, $\lim_{\delta\to 1} p_i^* = p_i^{ac}$ , the pressure of long term competition eliminates the role of cost asymmetry, secondly, for $\delta$ close to 1, the incumbency rent of the less efficient firm is greater than the one the more efficient one would get as an incumbent (for more details, see Lahmandi et ali., 1996).

This solution concept puts too much pressure on long term competition (see Fig. 4).

# 5 REFERENCES


Aghion Ph., Blundell R., Griffith R., Howitt P. and Prantl S. (2006). The effects of entry on incumbent innovation and productivity. Working paper Harvard University.

d'Aspremont, C. and Motta, M. (2000). *Tougher price competition or lower competition: a trade-off for anti-trust authorities?* in Market structure and competition policy, G. Norman and JF Thisse eds., Cambridge University Press.

Bagwell K. and Ramey G. (1996). *Capacity, entry, and forward induction*, Rand Journal of Economics, 27-4, 660:680.

Biglaiser, G., Crémer, J. and Dobos, G. (2007). The value of switching costs, unpublished working paper.

Eaton, B. and R. Lipsey (1980). Exit barriers are entry barriers : the durability of capital as an entry barrier, *Bell Journal of Economics*, 10:20-32.

Farrell, J. and Klemperer, P. (2001). Coordination and Lock-in: competition with switching costs and network effects, working paper

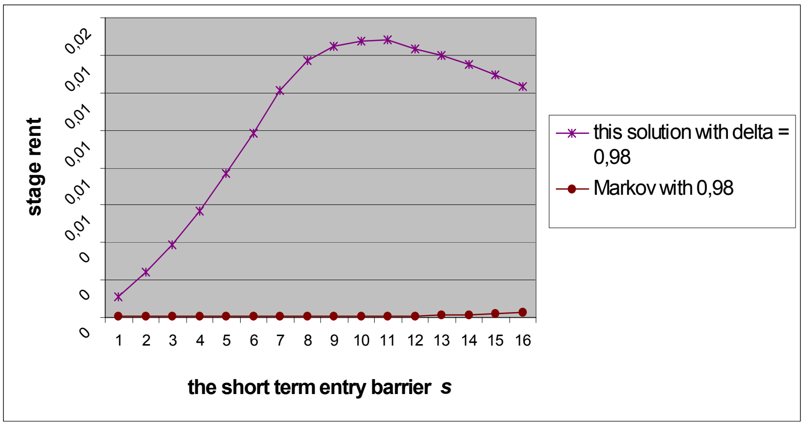


Figure 4: This solution versus the perfect Markov solution

Gromb, D., Ponssard J-P. and D. Sevy (1997). Selection in dynamic entry games, *Games and Economic Behavior*, 21:62-84.

Lahmandi, R.,Ponssard J-P. and D. Sevy (1996). Efficiency of dynamic quantity competition : a remark on Markov equilibria, *Economic Letters*, 50:213-221.

Maskin, E. and J. Tirole (1988). A theory of dynamic oligopoly, Part I : overview and quantity competition with large fixed costs, *Econometrica*, 56-3:549-569.

Nintendo, (1995). Power Play (A): Nintendo in 8-bit Video Games, *Harvard Business School case study.*

Norman G. and Thisse J.-F. (2000). *Should pricing policies be regulated when firms may tacitly collude?* in Market structure and competition policy, G. Norman and JF Thisse eds., Cambridge University Press.

Ponssard, J-P. (1991). Forward Induction and Sunk Costs Give Average Cost Pricing, *Games Economic Behavior*, 3:221-236.

Sheff, D. (1993). *Game over: how Nintendo conquered the world.* Random House, New York.

Scherer, F.M. (1992) *International High-Technology Competition,* Harvard University Press.

Wilson, R. (1992). Strategic Models of Entry Deterrence, in *Handbook of Game Theory*, vol. 1 (R.J. Aumann and S. Hart, eds.), Amsterdam, Elsevier Science.

# 6 Appendix : Proofs

**Proof.** [1] (Proposition 3) The proof is given for $\delta = 1$. The argument can be extended to $\delta$ close to 1 by continuity. Define the system $\Phi_1^n$ as the limit of $\Phi_\delta^n$ as $\delta$ goes to 1. It is easily seen that the system $\Phi_1^n$ is

$$\begin{array}{lll}
\text{for } t \in \{0,1,2...n\} & \pi_L(p_L^{BE}(q_S^t), q_S^t) + (n-t)\pi_L^m(q_L) = 0 & (1') \\
 & \pi_S(p_S^{BE}(q_L), q_L) + \Sigma_{t=1}^{t=n} \pi_S^m(q_S^t) = 0 & (2') \\
 & \pi_L^m(q_L) \geq 0 & (3') \\
\text{for } t \in \{1,2...n\} & \Sigma_{t'=t}^{t'=n} \pi_S^m(q_S^{t'}) \geq 0 & (4') \\
 & \Sigma_{t'=0}^{t'=n} \pi_S^m(q_S^{t'}) < 0 & (5')
\end{array}$$

The proof runs as follows. Firstly prove that conditions (1'-2'-3'-4') of $\Phi_1^n$ have a unique solution. Secondly, check whether condition (5') is satisfied: if it is, the unique solution of $\Phi_1^n$ is obtained; if it is not, $\Phi_1^n$ has no solution. To simplify notations, for $i \in \{1,2\}$, define the function $C_i(p_j) = -\pi_i(p_i^{BE}(p_j), p_j)$. This function $C_i$ may be interpreted as the entry cost for firm $i$. It will be convenient to assume that the fixed costs are small enough so that the functions $\pi_i^m$ and $C_i$ are respectively strictly increasing and strictly decreasing over the relevant values of prices, i.e. one never needs to consider prices above 1/2.To prove the first part, for all $p_L \in [p_L^{ac}, p_L^{\max}]$, define the function $W(p_L) = C_S(p_L) - \Sigma_1^n \pi_S^m(p_S^t)$ in which the sequence $(p_S^t)$ is derived from $p_L$ through (1') that is,

$$-C_L(p_S^t) + (n-t)\pi_L^m(p_L) = 0 \qquad \text{for } t \in \{0,1,2...n\}$$

then, show that $W(p_L)$ is negative (step 1) then positive (step 2) and that its derivative is strictly positive (step 3) so that there is a unique solution to the equation $W(p_L) = 0$.

Step 1: if $p_L = p_L^{ac}$ then $W(p_L) < 0$. In that case $p_S^t = C_L^{-1}(0) = p_S^{\max}$ for all $t$ so that $W(p_L^{ac}) = C_S(p_L^{ac}) - n\pi_S^m(p_S^{\max})$. By assumption $\pi_S^m(p_S^{\max}) > 0$ so that for $n$ large enough $W(p_L^{ac}) < 0$.

Step 2: if $p_L = p_L^{\max}$ then $W(p_L) > 0$. Since $C_L$ is strictly decreasing, the sequence $(p_S^t)$ is a strictly increasing sequence bounded by $p_S^{\max}$. Since $\pi_S^m$ is strictly increasing this implies that $\Sigma_1^n \pi_S^m(p_S^t)$ is certainly negative for $n$ large enough so that $W(p_L^{\max}) = -\Sigma_1^n \pi_S^m(p_S^t)$ is certainly positive.

Step 3: $\frac{dW}{dp_L} > 0$. We have $\frac{dW}{dp_L} = \frac{dC_S}{dp_L} - \Sigma_{t=1}^{t=n}(\frac{d\pi_S^m}{dp_S^t} \cdot \frac{dp_S^t}{dp_L})$. Using (1') we get:

$$\frac{dp_S^t}{dp_L} = (n-t)\frac{d\pi_L^m}{dp_L} / \frac{dC_L}{dp_S^t}$$

By substitution it follows that:

$$\frac{dW}{dp_L} = \frac{dC_S}{dp_L} - \frac{d\pi_L^m}{dp_L}\Sigma_{t=1}^{t=n}((n-t)\frac{d\pi_S^m}{dp_S^t} / \frac{dC_L}{dp_S^t})$$

By assumption $-\frac{d\pi_S^m}{dp_S^t} / \frac{dC_L}{dp_S^t}$ is uniformly bounded away from zero by $\varepsilon$ so that

[1] I am indebted to Rida Laraki for providing the argument for this proof.

$$\frac{dW}{dp_L} \geq \frac{dC_S}{dp_L} + \frac{d\pi_L^m}{dp_L}\frac{n(n-1)}{2}\varepsilon$$

Since $\dfrac{d\pi_L^m}{dp_L}$ is bounded away from zero and since $\dfrac{dC_S}{dp_L}$ is bounded away from $-\infty$ we certainly have $\dfrac{dW}{dp_L} > 0$ for $n$ large enough. Hence for a given $n$ large enough there is a unique solution to $W(p_L) = 0$ that is, to (2'). This solution is in $]p_L^{ac}, p_L^{\max}[$ so that (3') is also satisfied. Denote $q_L$ this solution and $(q_S^t)$ for $t \in \{0, 1, 2...n\}$ the associated sequence obtained through (1'). Observe that (4') is satisfied as well : since $\pi_S^m$ is increasing the function $\Sigma_{t'=t}^{t'=n}\pi_S^m(q_S^{t'})$ is bell shaped with respect to $t$ so for all $t$ we have:

$$\Sigma_{t'=t}^{t'=n}\pi_S^m(q_S^{t'}) \geq Min(\Sigma_1^n\pi_S^m(q_S^t), \pi_S^m(q_S^n)) = Min(C_S(q_L), \pi_S^m(q_S^{\max}) > 0$$

because $q_L < p_L^{\max}$ implies $C_S(q_L) > 0$ and $\pi_L^m(p_L^{\max}) > 0$ by construction. It is now a simple matter to check whether (5') holds or not. If it does a complete solution to $\Phi_1^n$ is obtained, if it does not there cannot be a solution for that value of $n$ since conditions (1') through (4') have a unique solution. ■

**Proof.** (Proposition 4) Suppose $q_L(m, \delta) > q_L(n, \delta)$ then $\pi_L^m(y_L(m, \delta)) > \pi_L^m(q_L(n, \delta))$. Since $C_L$ is strictly decreasing this implies for all $t \in \{0, 1, 2...n\}$:

$$q_S^{m-t}(m, \delta) < q_S^{n-t}(n, \delta)$$

so that

$$\Sigma_{t=n}^{t=0}\delta^{n-t}\pi_S^m(q_S^{m-t}(m, \delta)) < \Sigma_{t=n}^{t=0}\delta^{n-t}\pi_S^m(q_S^{n-t}(n, \delta)).$$

For $t \in \{n+1, ..m\}$ we still have $q_S^{m-t}(m, \delta) < q_S^0(n, \delta)$ and, because of (5) we also certainly have $\pi_S^m(q_S^0(n, \delta)) < 0$ then

$$\Sigma_{t=m-1}^{t=0}\delta^{m-1-t}\pi_S^m(q_S^{m-t}(m, \delta)) \leq \Sigma_{t=n}^{t=0}\delta^{n-t}\pi_S^m(q_S^{m-t}(m, \delta)).$$

Then

$$\Sigma_{t=m-1}^{t=0}\delta^{m-1-t}\pi_S^m(q_S^{m-t}(m, \delta)) < \Sigma_{t=n}^{t=0}\delta^{n-t}\pi_S^m(q_S^{n-t}(n, \delta)).$$

By construction the left hand side should be greater or equal to zero while the right hand side should be strictly negative thus a contradiction. ■

**Proof.** (Proposition 5) The intuition for this proof is the following. If $n$ can be arbitrarily large it must be that the corresponding $q_L$ be close to average cost. Then the sequence $(q_S^t)_{t=1...n}$ generate a total payoff that is proportional to $\int_{q_S^1}^{p_S^{\max}}\pi_S^m(p)\frac{dC_L}{dp}(p)dp$.Conditions (4) and (5) imply that this integral be zero. This allows the precise determination of $\lim q_S^1$ and the condition $\pi_S^m(q_S^*) + \pi_S(p_S^{BE}(p_L^{ac}), p_L^{ac}) \leq 0$ holds.

If $\Phi_1^n$ has a solution for arbitrarily large $n$, clearly $\lim_{n\to\infty}\pi_L^m(q_L(n)) = 0$.

Combining this result with (2') we get $\lim_{n\to\infty}\Sigma_1^n\pi_S^m(q_S^t(n)) = C_S(p_L^{ac})$.

Given that $\frac{dC_L}{dp}$ is bounded away from infinity and from zero and that $\pi_S^m(p)$ is bounded away from zero, there exists a unique $q^* < p_S^{ac}$ such that

$$\int_{p^*}^{p_S^{\max}} \pi_S^m(p)\frac{dC_L}{dp}(p)dp = 0$$

For all $p \leq p_S^{1,\max}$ define $F(p) = \int_p^{p_S^{1,\max}} \pi_S^m(u)\frac{dC_L}{du}(u)du$, the function $F$ is such that $F(x) > 0$ iff $p < q^*$.

We now show the convergence of $q_S^1(n)$ to $q^*$.

Using (1') and (2') we get :

$$\begin{aligned} C_S(q_L(n))\pi_L^m(q_L(n)) &= \Sigma_{t=1}^{t=n}\pi_S^m(q_S^t(n))\pi_L^m(q_L(n)) \\ &= \Sigma_{t=1}^{t=n}\pi_S^m(q_S^t(n))\left[C_L\left(q_S^{t-1}(n)\right) - C_L\left(\ q_S^t(n)\right)\right] \end{aligned}$$

When $\pi_L^m(q_L(n)$ is small this non negative expression is close to $F(q_S^1(n))$.

To see this, make the change of variable from $p_S$ to $u = C_L(p_S)$. As $t$ goes from 1 to $n$, $p_S$ increases from $q_S^1(n)$ to $q_S^n(n)$ and $u$ from $u^1(n) = C_L\left(q_S^1(n)\right)$ to $u^n(n) = C_L(q_S^n(n)) = 0$ but $u^{t-1}(n) - u^t(n)$ remains $t$ independent and equals $\pi_L^m(q_L(n))$, let $\Delta u(n) = \pi_L^m(q_L(n))$.

We may then write

$$\pi_L^m(q_L(n))\Sigma_{t=1}^{t=n}\pi_S^m(q_S^t(n)) = \Sigma_{t=1}^{t=n}\pi_S^m(C_L^{-1}(u^t(n)))\Delta u(n)$$

For large values of $n$ we have

$$\Sigma_{t=1}^{t=n}\pi_S^m(C_L^{-1}(u^t(n)))\Delta u(n) \approx \int_{u^1(n)}^0 \pi_S^m(C_L^{-1}(u))du = \int_{q_S^1(n)}^{p_S^{\max}} \pi_S^m(p)\frac{dC_L}{dp}(p)dp.$$

This proves that $q_S^1(n)$ cannot be far below $q^*$. Using (2') and (5') for the two sequences $n$ and $n+1$, it is clear that $q_S^1(n+1)$ and $q_S^1(n)$ cannot be far apart either. More precisely:

$$\left|q_S^1(n+1) - q_S^1(n)\right| \leq -Min(\frac{dC_L}{dp}(q_S^1(n)), \frac{dC_L}{dp}(q_S^0(n)))\pi_L^m(q_L(n))$$

Since $\frac{dC_L}{dp}$ is bounded away from infinity, $\lim_{n\to\infty}\left|q_S^1(n+1) - q_S^1(n)\right| = 0$, this is enough to prove that $q_S^1(n)$ converges to some limit and this limit can only be $q^*$ since

$\lim_{n\to\infty} C_S(q_L(n))\pi_L^m(q_L(n)) = \lim_{n\to\infty} C_S(q_L(n)) \lim_{n\to\infty} \pi_L^m(q_L(n))$
$= C_S(0).0 = 0.$ ■

**Proof.** (Proposition 6) Consider the symmetric case with $\delta < 1$ and assume the system $\Phi_\delta^n$ has a solution for arbitrarily large values of $n$. Using conditions (1) and (2), it is easily seen that, as $n \to \infty$, $\lim q_S^1 = \lim q_S^0 = \delta\pi_L^m(q_L)/(1-\delta)$. Hence conditions (4) and (5) cannot be simultaneously satisfied. ■