Choosing Among Rules of k names

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What is the rule of k names?

Given a set of candidates for office, a committee chooses k candidates from this set by voting, and makes a list with their names. Then a single individual from outside the committee selects one of the listed names for appointment.

Remarks: a family of rules, depending on screening rules. More general possibilities, with choosers also being several. Special cases: one proposer, v-votes screening rule. Later, we'll consider special cases based on restrictions of preference domains: single peaked proposers, polarized proposers.

Historical Examples:

- Bishops

"*Dual postulation seems to have been common between 1180 and 1191. It is described as "the way of elections in the land beyond the sea of patriarch, archbishop, bishop and abbots that they nominate two and present it to the king and the king takes one"*

Dual postulation may have received theoretical support from an analogy with Byzantine practice. In Constantinople the patriarch was selected by the Emperor from three candidates put forward by the Holy Synod. But dual postulation was decried in the decretal Cum terra, quae, by Pope Celestine between 15 April and 25 October 1191, as giving too much power to the patriarch or prince.

(Strategy and Manipulation in Medieval Elections, by Sara and Joel Uckelman)”

- Senate (France, Spain and Brazil first constitutions).

These rules are still very much in use, for certain types of appointments:

Examples:

- Rectors of Public Universities (Brazil and Turkey).
- Members of Courts of Justices (Brazil, Chile and Mexico).
- Members of European Court of Human Rights.
- Members of company's board administration (Brazil, new law of corporate governance).

Certainly, under some circumstances, the rule may just be a formality to hide the control of one of the sides:

Examples:

- always choose the most voted.
- "rule of one name".

Are these rules a subject of interest?

Sometimes yes, so much that they generate public polemics. Medieval times, Turkish universities, improvements in Colombia, the election of Suárez in the Spanish transition.

Example: The struggle to appoint bishops

"*By the end of the sixth century, seeing the Church's flourishing wealth and power, secular rulers desired to influence the selection of bishops. In the Byzantine East, the emperors restricted episcopal elections to clergy and nobility, claimed the right to nominate the bishop of Constantinople and began to require money in return for confirmations, a practice known as simony.*

In the Latin West, popular elections remained, but royal interference increased. Kings began to control the nominations and even started choosing laymen as bishops.

The issue of lay investiture came to a climax in the fifty year War of Investitures, as the Church and the empire fought for control of appointing bishops. It concluded in 1122 with the Concordat of Worms which ended lay investiture but still gave royalty much influence on elections.

By the end of the twelfth century, episcopal elections were further limited as the new bishops were elected solely by Cathedral canons, a bishop's elite group of administrative advisors.

From the thirteenth century onward, papal intervention in episcopal elections increased. The pope claimed the right of confirmation of any bishop-elect. Rulers were given the authority to nominate the candidates and some, such as the ruers of Spain, we given full power to appoint bishops." (excerpts from :"An intriguing story: elections of bishops in the Catholic Church")

From "rules of one name" to real effectivity.

Examples:

- The case of the Spanish transition
- The recent changes in Colombia

An interesting usage of the rules, as transitory methods:

- The Brazilian Law of Corporate Finance:

The Brazilian law of corporate finance, approved in 2001, states that the preferred stockholders who hold at least 10% of the company capital stock shall choose one name among the three names listed by the controller of the company to become their representative on the company´s board of administration (see Lei n. 10303, October 31th, 2001, Brasil). After 2006, the law states that there will be not such restriction.

Controversies around the proper use of the rule: the case Turkish rectors

“Turkey's top education board has altered the selection process of rectors following criticism from experts and President Abdullah Gül.

The Higher Education Board, or YÖK, came under fire in July over its controversial rector-nomination lists, which exclude rector candidates with high vote counts, and prioritize candidates who receive fewer votes in elections at their universities.

Under the new selection process, YÖK will send the names of the three rector candidates who receive the most votes at their universities.

The decision to change the selection process was announced following a recent incident at Giresun University, during which a candidate who received two votes was chosen over candidates who both received roughly 30 votes, prompting Gül to say, "This system is not right."

The new system will be officially adopted during the YÖK general assembly, to be headed by YÖK President Yusuf Ziya Özcan on November 3 and 4.

Under current law, university rectors are elected in a three-phase system. During the first phase voting is held among the academic staff, and the names of the six candidates who receive the highest number of votes are submitted to YÖK. The board then prepares a short-list of three candidates and submits it to the president, who makes the final selection. (Friday, October 29, 2010, ANKARA - Hürriyet Daily News)”

What kind of contributions can we make to their study?

Check the validity of our intuitions. An (incomplete) list of intuitions is the following:

- comparative statics: other things equal, proposers would prefer a small k, the chooser a large one; the chooser would prefer a small v, rather than a large one

- welfaristic comparisons: are rules of k names in some sense ex-ante optimal within some large class of alternative rules? Which rules would be ex ante optimal within a given class of rules of k names?

- efficiency considerations: are there reasons to think that these rules may provide an efficient way to use the special skills of different types of agents?

- dynamic considerations: when are rules of k names expected to be set in place, removed or changed?

- We shall provide some results, but also exhibit some of the difficulties that appear when trying to check whether these simple intuitions may or may not hold.

Modeling the game among agents. The special case of one proposer.

Characterization: The equilibrium outcome is the proposer´s best candidate among the chooser's (#**A**-*k*+1)-top candidates

Example 1:

A=$\{a, b, c, d\}$

1	Chooser
a	c
b	b
c	a
d	d

Let k=2, proposer 1 proposes $\{a; d\}$ and the chooser chooses according to his preferences.

So a is the unique equilibrium outcome.

<u>Equilibrium Outcomes</u>

k=1 $\{a\}$

k=2 $\{a\}$

k=3 $\{b\}$

k=4 $\{c\}$

Complications for the case N>1

•Controversy over which game and equilibrium concept are adequate.

•Equilibria depend on the screening rule;

•Equilibria may not exist;

•Multiciplicity of equilibria;

A simultaneous game with complete information.

The Constrained Chooser Game:

• The set of players is the set of proposers **N**.

• Players' strategy space is the space of admissible messages associated with the screening rule.

•The players only care about the identity of the winning candidate.

- Equilibria depend on the screening rule;
- Equilibria may not exist;
- Multiciplicity of equilibria;

In our previous work we have characterized the strong Nash equilibrium outcomes for the case where the screening rule is majoritarian. There may exist several equilibria (in case of even number of proposers) or may not exist any.

Majoritarian screening rule:

We say that a screening rule is majoritarian if and only if for every set with k candidates there exists an action such that every strict majority coalition of proposers can impose the choice of this set provided that all of its members choose this action.

Characterization of the Set of Strong Nash Equilibrium Outcomes for Majoritarian Screening Rules

Proposition 1: For any majoritarian screening rule and odd number of proposers, a candidate is a strong Nash equilibrium outcome of the Constrained Chooser Game if and only if he is the Condorcet winner over the chooser's (#**A**-*k*+1)-top candidates.

$$\text{Chooser´s}(\#\mathbf{A}-k+1)\text{ top-candidates} = \left\{\begin{array}{c} \text{Chooser} \\ y_1 \\ . \\ . \\ . \\ y_{\#A-k+1} \end{array}\right.$$

$$\begin{array}{c} y_{\#A-k+2} \\ . \\ . \\ . \\ y_{\#A} \end{array}$$

Corollary: For any majoritarian screening rule and odd number of proposers, the set of strong Nash equilibrium outcome is singleton or empty.

single peakedness ensures existence.

Comparative Statics

In the Constrained Chooser Game, the chooser cannot be worst off if:

k increases.

an undesirable candidate withdraws from the contest.

Majoritarian X Non-Majoritarian Screening Rules

We do not have a characterization for other, non-majoritarian rules. Again, equilibria may or may not exist.

We provide examples where the chooser can be better off when a majoritarian screening rule is replaced by non-majoritarian screening rule, but we also show that he can be worst off.

That's why from now on we concentrate on what we call v-votes screening rules for k names.

Three of our six rules in the previous paper are not majoritarian, and they all have the same structure: v- votes screening rules for k names.

v-votes screening rule for k names:
Each proposer votes for v candidates and the list has the names of the k most voted candidates, with a tie break when needed. The parameter v is smaller than the number of candidates and the tie break criterion is established by a strict ordering of alternatives.

v-rule of k names :
Given a set of candidates for office, a committee chooses k members from this set by using a v-votes screening rule for k names. Then a single individual from outside the committee selects one of the listed names for the office.

Examples of V-votes Screening Rules:

1. **3-votes screening rule for 3 names**: Each proposer votes for three candidates and the list has the names of the three most voted candidates, with a tie-break when needed. It is used in the election of Irish Bishops and that of Prosecutor-General in most of Brazilian states.(It is majoritarian)

2. **3-votes screening rule for 5 names**: Each proposer votes for three candidates and the list has the names of the five most voted candidates, with a tie break when needed. It is used in the election of the members of Superior Court of Justice in Chile.

3. **2-votes screening rule for 3 names**: Each proposer votes for two candidates and the list has the names of the three most voted candidates, with a tie break when needed. It is used in the election of the members of Court of Justice in Chile.

4. **1-vote screening rule for 3 names**: Compute the plurality score of the candidates and include in the list the names of the three most voted candidates, with a tie break when needed. It is used in the election of rectors of public universities in Brazil.

The other two are not v-votes screening rules but are majoritarians:

5. **1-vote sequential plurality:** The list is made with the names of the winning candidates in three successive rounds of plurality voting. It is used in the election of English Bishops.

6. **3-votes sequential strict plurality**: this is a sequential rule adopted by the Brazilian Superior Court of Justice to select its members. Each proposer votes for three candidates from a set with six candidates, and if there are three candidates with more votes than half of the total number of voters, they will form the list. It is used in the election of the members of the Brazilian Superior Court of Justice.

Example 3

Let $A = \{a, b, c, d, e\}$, and let $N = \{1, \ldots, 11\}$. Each proposer votes for one candidate and the list has the names of the three most voted candidates, with a tie breaking rule when needed: $b \succ a \succ e \succ d \succ c$.

Preference Profile

1 proposers type 1	7 proposers type 2	3 proposers type 3	Chooser
b	a	c	b
a	c	a	c
e	d	e	a
d	b	d	e
c	e	b	d

A strategy profile that sustains c as a strong Nash equilibrium outcome:
The seven type 2 proposers cast four votes for a and three votes for e. Type 1 proposer casts a vote for b, while the three type 3 proposers cast three votes for c. Thus, the selected list is $\{a, c, e\}$ and c is the winning candidate.

A strategy profile that sustains a as a strong Nash equilibrium outcome:
The seven type 2 proposers cast three votes for a, two votes for e, one vote for b and one vote for d.
Type 1 proposer casts a vote for b, while the three type 3 proposers cast two votes for d and one for e.
So, a,d and e will have three votes each, while b only two. Thus, the selected list is $\{a, d, e\}$ and a is the winning candidate.
The readers can check that there is no coalition of voters that has incentive in deviating.

Parameters of Screening Rules:

q_1 is the minimum q such that for every candidate there exists an action profile such that any coalition with size higher or equal to q can impose the inclusion of this candidate among the k chosen candidates.

q_k is the minimum q such that for every set with k candidates any coalition, with size higher or equal to q; can impose the choice of this set.

Remark: Notice that these definitions apply to any screening rule which is anonymous and has full range, though we shall only use it here for the case of v-votes screening rules for k names.

Parameters of v- votes screening rules for k names:

$$q_1 = \left\lceil \frac{vn}{(k+v)} \right\rceil + I\left(\frac{vn}{k+v} = \left\lceil \frac{vn}{k+v} \right\rceil \right)$$

$$q_k = \left\lceil \frac{kn}{(k+n)} \right\rceil + I\left(\left\lfloor \frac{v\left\lceil \frac{kn}{k+v} \right\rceil}{k} \right\rfloor \leq n - \left\lceil \frac{kn}{k+v} \right\rceil \right)$$

Parameters of Screening Rules:

$q_1(x)$ is the minimum q such that there exists an action profile such that any coalition with size higher or equal to q can impose the inclusion of candidate x among the k chosen candidates.

$q_k(X)$ is the minimum q such that any coalition with size higher or equal to q; can impose the choice of X.

Remark: if x is one of the k-top candidates according to the tie breaking criterion then $q1(x)$ = q1 or $q1(x)$ = q1-1.
if the set X is formed by the k-top candidates according to the tie breaking criterion then qk(X) = qk or qk(X) = qk-1.

We face several difficulties when trying to characterize equilibria:

-single peakedness is not sufficient condition for existence ;

We'll analyze a specific but significant case, where the choosers are polarized in two groups with opposite preferences.

Necessary Conditions for a Strong Nash Equilibrium Outcome

Proposition: Consider any v-votes screening rules for k names, if candidate *x* is a strong Nash equilibrium outcome of the Constrained Chooser Game then it satisfies the following three conditions :

1. It is a chooser's (a-k+1)-top candidate.

2. If *y* is a chooser's (a-k+1)-top candidate then $\{i \in N | y \succ_i x\} < q_k(Y)$ for any $Y \in A_k$ such that y is the chooser best candidate in Y .

3. If *y* is the chooser's 1-top candidate then $\{i \in N | y \succ_i x\} < q_1(y)$.

Example 5: The example below shows that the set of necessary conditions established by the previous proposition are not sufficient conditions and the equilibrium may not exist.

Let $\mathbf{A} = \{a, b, c, d\}$ and let $\mathbf{N} = \{1, 2, 3\}$. Suppose that each proposer votes for one candidate and the two most voted candidates form the list, with the following tie breaking rule when needed: $a \succ c \succ b \succ d$.
The preferences of the chooser and the committee members are as follows:

Preference Profile

Proposer 1	Proposer 2	Proposer 3	Chooser
c	b	b	a
b	c	a	c
d	d	c	b
a	a	d	d

We have that $q_1(a) = q_1(c) = 1$, $q_1(x) = 2$ for any $x \in A \backslash \{a, c\}$ and $q_k(X) = 2$ for any $X \in A_k$ such that $a \in X$ and $q_k(Y) = 3$ for any $Y \in A_k$ such that $a \notin Y$. Inspecting the preference profile above, we have that:

1. Condition 1: $\{a, b, c\}$.
2. Condition 2: $\{b, c\}$.
3. Condition 3: $\{b\}$

So, only candidate b satisfies the three necessary conditions stated in Proposition 2.
However, b is not an equilibrium outcome. He is not an equilibrium outcome, since Proposer 1 always have incentive in preventing the election of b by casting vote in c.
Notice also that the proposers preference profile satisfies single peakedness, so this example teaches us that this property does not guarantee existence of an equilibrium. If we had considered a 2-votes screening rules for two names, candidate b would be the unique strong Nash equilibrium outcome of the game.The table below presents the set of strong Nash equilibrium for different values of v.

Set of strong Nash equilibrium outcomes

k=2	$v = 1$	ϕ
k=2	$v = 2$	$\{b\}$

Sufficient Conditions for a Strong Nash Equilibrium Outcome

Proposition 3: Consider any v-rule for k names and let x be one of the chooser´s (A-k+1)-top candidates and X□A_k such that x is the chooser best candidate in X, if $q_k(X)$ proposers rank x highest then x is the unique strong Nash equilibrium outcome of the Constrained Chooser Game..

Proposition 4: Consider any v-rule for k names, if a candidate x is a $n-\lfloor nv/2k \rfloor+1$-Condorcet winner over the set of chooser's (a-k+1)-top candidates then it is the unique strong Nash equilibrium outcome of the Constrained Chooser Game.

Proposition 5: Consider any v- rule for k names, let x be the chooser´s 1-top candidate. If x is also a q□(x)-Condorcet winner over the set of chooser's (A-k+1)-top candidates then it is the unique strong Nash equilibrium outcome of the Constrained Chooser Game..

On the number of votes that each proposer can cast.
What is good for the chooser?

Proposition 6: If the chooser 1-top-candidate is a strong Nash equilibrium outcome of the Constrained Chooser Game under v´-rule for k names then it is also a strong Nash equilibrium outcome of the Constrained Chooser Game under any v- rule for k names whenever v<v´ provided that both screening rules have the same tie breaking criteria.

Example 2: Chooser is better off under a small parameter v

Let $\mathbf{A} = \{a, b, c, d, e\}$ and let $\mathbf{N} = \{1, 2, 3\}$. Suppose that each proposer votes for one candidate and the three most voted candidates form the list , with a tie breaking rule when needed: $b \succ a \succ e \succ d \succ c$. .The preferences of the chooser and the committee members are as follows:

Preference Profile

Proposer 1	Proposer 2	Proposer 3	Chooser
e	e	e	a
d	d	d	b
c	c	b	c
a	a	a	d
b	b	c	e

Notice that , we have that $q_1(x) = 1$ for any $x \in A$ and $q_k(X) = 3$ for any $X \in A_k$.

Only candidate a that satisfies all three conditions.

The following strategy profile sustains a as a strong Nash equilibrium outcome:

Proposer 1 votes for a, Proposer 2 votes for d and Proposer 3 votes for b.

Notice that , in this example, the chooser is weakly worst off as v increases.

Set of strong Nash equilibrium outcomes

k=3 $v = 1$ $q_1 = 1$ $q_k = 3$ $\{a\}$

k=3 $v = 2$ $q_1 = 2$ $q_k = 3$ $\{a\}$

k=3 $v = 3$ $q_1 = 2$ $q_k = 2$ $\{c\}$

Example 7: Chooser is better off under a large parameter v

Let $A = \{a, b, c, d\}$, and let $N = \{1, 2, 3\}$. Each proposer votes for one candidate and the list has the names of the two most voted candidates, with a tie breaking rule when needed: $c \succ d \succ b \succ a$.

Preference Profile

Proposer 1	*Proposer 2*	*Proposer 3*	*Chooser*
b	a	a	b
d	c	c	a
c	d	d	c
a	b	b	d

As can be verified, c is the unique equilibrium outcome of the Constrained Chooser Game under the 1-vote screening rule for 2 names.
Here there is a intuition for this result: notice that candidate a cannot be a strong equilibrium outcome of the Constrained Chooser Game, because as long as proposer 1 votes for b, proposers 2 and 3 cannot get a to be the outcome, even if they can force a to be in the list. Short of that, proposers 2 and 3 coordinate their actions so that one of them votes for c and the other for d. If 1 persists in voting for b, this creates a tie between the three candidates that is solved in favor of c and d, out of which the chooser selects c. If 1 votes for c instead, the same outcome ensues. And all other actions by any combination for agents would lead some of them to outcomes that would be worse than c for some of them. Hence, c is the unique strong Nash equilibrium of the Constrained Chooser Game under our proposed rule.
Now let us change the screening rule for 2 names. Suppose that the proposers use 2-votes screening rule for 2 names. Now, a is the strong Nash equilibrium outcome of the Constrained Chooser Game.
This shows that here the chooser is better off under 2-vote screening rule for 2 names than under the 1-vote screening rule for 2 names.

Set of strong Nash equilibrium outcomes

k=2 $v = 1$ $\{c\}$

k=2 $v = 2$ $\{a\}$

The Polarized Proposers Model

The Polarized Proposer Model will allow us to:

- Guarantee existence and uniqueness of an equilibrium outcome.

- Obtain a result characterizing a whole family of rules, the v voting rules, which include the majoritarian ones as a special case.

- Discuss the role of the relevant parameters determining the power distribution among players.

- Perform explicit probability calculations for expected rankings.

The Polarized Proposers Model

1. (Assumption 1). There are two groups of proposers, denoted by G1 and G2, such that G2 = N\G1.

2. (Assumption 2). All the proposers in G1 share the same preferences over the set of candidates.

3. (Assumption 3). All the proposers in G2 share the same preferences over the set of candidates and it is the reverse of the preference profile of the proposers in G1.

4. (Assumption 4). The tie breaking rule coincides with at least one of the agents´ preferences over the set of candidates.

Without loss of generality, we denote by G1 the largest of the two groups, and we let m be the number of members it contains (m>n/2).

Proposition 4 (**Equilibrium Outcome Characterization**):
Consider the polarized proposers model with an odd number of proposers. Under any v-votes screening rule for k names the strong Nash equilibrium outcome of the Constrained Chooser Game is unique and always exists.
In addition:

1) Suppose that the tie breaking criterion coincides with the chooser´s preferences over the set of candidates or with the minoritarian group ´s preferences over the set of candidates.
• If $\mathbf{m \geq q_k}$ then the equilibrium outcome is the best alternative of individuals in the majoritarian group out of chooser´ s (a -k + 1)-top alternatives.
• If $\mathbf{q_k > m}$ then the equilibrium outcome is the chooser´ s top alternative.

2) Suppose that the tie breaking criterion coincides with the majoritarian group´s preferences over the set of candidates.
• If $\mathbf{m \geq q_k}$ then the strong Nash equilibrium outcome is the best alternative of individuals in the majoritarian group out of chooser´ s (a-k+1)-top alternatives;
• If $\mathbf{q_k > m \geq q_1 > n-m}$ then the strong Nash equilibrium outcome is the chooser´s best alternative out of the majoritarian group´s k-top candidates;
•If $\mathbf{q_k > m > n-m \geq q_1}$ then the equilibrium outcome is the chooser´s top alternative.

Compatative Statics

Corollary 1: Consider the Polarized Proposers Model and odd number of proposers. The chooser cannot be worst off under v´-rule for k names than under v- rule for k names whenever v>v´.

Corollary 2: Consider the Polarized Proposers Model, odd number of proposers and v-rule of k names. The chooser cannot be worst off under a more polarized set of proposers (small m) than under a less polarized set of proposers (big m).

Corollary 3: Consider the Polarized Proposers Model and odd number of proposers. The chooser cannot be worst off under v- rule for k´ names than under v- rule for k´´ names whenever k´´>k´.

Expected Rankings of the Equilibrium Outcome (N=1)

- All preference profiles are considered equally likely at the time of the choice of k.

- The agents will know the preference profile at time of the vote.

- The expected ranking of the equilibrium outcome according to proposer´s preference is equal to:

$$E(r_1 \mid k) = \frac{a+1}{a-k+2}$$

- The expected ranking of the equilibrium outcome according to chooser´s preference is equal to:

$$E(r_2 \mid k) = \frac{a-k+2}{2}$$

Results: one proposer

Proposition: There exist at most two values of k (utilitarian k_u) that minimize the sum of agents´ expected ranking of the equilibrium outcome.

Proposition: There exist at most two values of k (the egalitarian k_e) that minimize absolute difference of agents´ expected ranking of the equilibrium outcome.

Proposition: The utilitarian k is equal to the egalitarian k and they are not lower than ((a+1)/2).

$$k_u = k_e = \left\lfloor a + \frac{5}{2} - \sqrt{2a + \frac{9}{4}} \right\rfloor$$

Table 1: Expected Ranking Of Equlibrium Outcomes

Expected Ranking Of Equilibrium Outcome

	Number of Candidates															
k	3		4		5		6		7		8		9		10	
1	1	3	1	3,5	1	4	1	4,5	1	5	1	5,5	1	6	1	6,5
		2		2,5		3		3,5		4		4,5		5		5,5
2	1,333	2,833	1,25	3,25	1,2	3,7	1,167	4,167	1,143	4,643	1,125	5,125	1,111	5,611	1,1	6,1
		1,5		2		2,5		3		3,5		4		4,5		5
3	2	3	1,667	3,167	1,5	3,5	1,4	3,9	1,333	4,333	1,286	4,786	1,25	5,25	1,222	5,722
		1		1,5		2		2,5		3		3,5		4		4,5
4			2,5	3,5	2	3,5	1,75	3,75	1,6	4,1	1,5	4,5	1,429	4,929	1,375	5,375
				1		1,5		2		2,5		3		3,5		4
5					3	4	2,333	3,833	2	4	1,8	4,3	1,667	4,667	1,571	5,071
						1		1,5		2		2,5		3		3,5
6							3,5	4,5	2,667	4,167	2,25	4,25	2	4,5	1,833	4,833
								1		1,5		2		2,5		3
7									4	5	3	4,5	2,5	4,5	2,2	4,7
										1		1,5		2		2,5
8											4,5	5,5	3,333	4,833	2,75	4,75
												1		1,5		2
9													5	6	3,667	5,167
														1		1,5
10															5,5	6,5
																1

Heterogeneous Preferences

Let denote by $d_i : A \to \mathbb{R}$ the agent i´s disultility function.
Suppose that $d_i(x) = 1 + (r_i(x) - 1)\theta_i$ where $r_i(x) = a - \#\{y \in A \backslash \{x\} | x \succ_i y\}$ and $\theta_i > 0$.

Notice that if x is the agent i´s best candidate then $d_i(x) = 1$ and if it is the agent i´s worst candidate $d_i(x) = 1 + (a-1)\theta_i$.

The agent 1´s expected disultility is $E(d_1 | k, a) = (1 - \theta_1) + \theta_1 \frac{a+1}{a-k+2}$.
The agent 2´s expected disultility is $E(d_2 | k, a) = \frac{\theta_2 (a-k)+2}{2}$

Let $\beta = \frac{\theta_2}{\theta_1}$

$$k_u = \begin{cases} \left\lfloor a+\frac{5}{2}-\frac{1}{\sqrt[2]{\beta}}\sqrt[2]{2a+2+\frac{\beta}{4}} \right\rfloor \text{ if } a+\frac{5}{2}-\frac{1}{\sqrt[2]{\beta}}\sqrt[2]{2a+2+\frac{\beta}{4}} \geq 1 \\ 1 \text{ if } a+\frac{5}{2}-\frac{1}{\sqrt[2]{\beta}}\sqrt[2]{2a+2+\frac{\beta}{4}} < 1 \end{cases}$$

k_e is equal to $\lfloor a + 1 + \frac{1}{\beta} - \frac{1}{\sqrt[2]{\beta}} \sqrt{2a + \beta + \frac{1}{\beta}} \rfloor$ or $\lceil a + 1 + \frac{1}{\beta} - \frac{1}{\sqrt[2]{\beta}} \sqrt{2a + \beta + \frac{1}{\beta}} \rceil$.

An Alternative Story: Expected Rankings of the Outcome Under Complete Ignorance

- The agents will not know the other players´ preferences at time of the vote. All preference profiles are considered equally likely at the time of the choice of k.

The case of one proposer and one chooser:

- The expected ranking of the equilibrium outcome according to proposer´s preference is:

$$E(r_1 \mid k) = \frac{k+1}{2}$$

- The expected ranking of the equilibrium outcome according to chooser´s preference is:

$$E(r_2 \mid k) = \frac{a+1}{k+1}$$

Table 2: Expected Ranking Of Equlibrium Outcomes Under Complete Ignorance

Expected Ranking Of Outcome: Under Completely Ignorance Scenario

		Number of Candidates															
		3		4		5		6		7		8		9		10	
	1	1	3	1	3,5	1	4	1	4,5	1	5	1	5,5	1	6	1	6,5
			2		2,5		3		3,5		4		4,5		5		5,5
	2	1,5	2,833	1,5	3,167	1,5	3,5	1,5	3,833	1,5	4,167	1,5	4,5	1,5	4,833	1,5	5,167
			1,333		1,667		2		2,333		2,667		3		3,333		3,667
	3	2	3	2	3,25	2	3,5	2	3,75	2	4	2	4,25	2	4,5	2	4,75
			1		1,25		1,5		1,75		2		2,25		2,5		2,75
	4			2,5	3,5	2,5	3,7	2,5	3,9	2,5	4,1	2,5	4,3	2,5	4,5	2,5	4,7
					1		1,2		1,4		1,6		1,8		2		2,2
	5					3	4	3	4,167	3	4,333	3	4,5	3	4,667	3	4,833
k							1		1,167		1,333		1,5		1,667		1,833
	6							3,5	4,5	3,5	4,643	3,5	4,786	3,5	4,929	3,5	5,071
									1		1,143		1,286		1,429		1,571
	7									4	5	4	5,125	4	5,25	4	5,375
											1		1,125		1,25		1,375
	8											4,5	5,5	4,5	5,611	4,5	5,722
													1		1,111		1,222
	9													5	6	5	6,1
															1		1,1
	10															5,5	6,5
																	1

Example: Suppose that a=1000.

In the complete information scenario:

The optimal k is equal to 957 ($\lfloor a+(5/2)-(2a+(9/4))^{1/2} \rfloor=957$). Under k=957, the expected ranking of the equilibrium outcome is 22.244 ($(a+1)/(a-k+2)=22.244$) according to the proposer ´s preferences and 22.5 ($(a-k+2)/2)=22.5$) according to the chooser´s preferences.

In the complete ignorance scenario:

The optimal k is equal to 44. Under k=44, the expected equilibrium outcome is 22.5 ($(k+1)/2 =22.5$) according to the proposers preferences and 22.244 ($(a+1)/(k+1)=22.244$) according to the chooser´s preferences.

Polarized Proposers Model
Calculating the expected rankings.

(Assumption 5). The preferences of the proposers in G1 and the chooser are independently and uniformly distributed over the domain of strict preferences.

Proposition 7
Consider the Polarized Proposers Model and odd number of proposers. Under any v-votes screening rule for k names the expected ranking of the equilibrium outcome of the Constrained Chooser Game according to players preferences are equal to:

1) Suppose that the tie breaking criterion coincides with the chooser´s preferences over the set of candidates or with the minoritarian group ´s preferences over the set of candidates.

Scenarios	*Majority Group*	*Minority Group*	*Chooser*
$m \geq q_k$	$\frac{(a+1)}{a-k+2}$	$\frac{(a+1)(a-k+1)}{a-k+2}$	$(\frac{a-k+2}{2})$
$q_k > m$	$\frac{(a+1)}{2}$	$\frac{(a+1)}{2}$	1

2) Suppose that the tie breaking criterion coincides with the majoritarian group´s preferences over the set of candidates.

Scenarios	*Majority Group*	*Minority Group*	*Chooser*
$m \geq q_k > n-m$	$\frac{(a+1)}{a-k+2}$	$\frac{(a+1)(a-k+1)}{a-k+2}$	$(\frac{a-k+2}{2})$
$q_k > m \geq q_1 > n-m$	$\frac{k+1}{2}$	$\frac{2a-k+1}{2}$	$\frac{a+1}{k+1}$
$q_k > m > n-m \geq q_1$	$\frac{(a+1)}{2}$	$\frac{(a+1)}{2}$	1

Example: Here we compute the expected rankings of the strong Nash equilibrium outcome according to the agents´ distributions of preferences. Suppose n=33 (number of proposers), a=6 (number of candidates), m=21 (size of the majority).

1) Suppose that the tie breaking criterion coincides with the chooser´s preferences over the set of candidates or with the minoritarian group ´s preferences over the set of candidates.

Screening Rules	q_1	q_k	*Condition*	$E(r_m)$	$E(r_{n-m})$	$E(r_c)$
$v = 1$ *and* $k = 3$	9	26	$q_k > m$	3.5	3.5	1
$v = 1$ *and* $k = 5$	6	29	$q_k > m$	3.5	3.5	1
$v = 3$ *and* $k = 3$	17	17	$m \geq q_k$	1.4	5.6	2.5
$v = 3$ *and* $k = 5$	13	22	$q_k > m$	3.5	3.5	1

2) Suppose that the tie breaking criterion coincides with the majoritarian group´s preferences over the set of candidates.

Screening Rules	q_1	q_k	*Condition*	$E(r_m)$	$E(r_{n-m})$	$E(r_c)$
$v = 1$ *and* $k = 3$	9	26	$q_k > m > n - m \geq q_1$	3.5	3.5	1
$v = 1$ *and* $k = 5$	6	29	$q_k > m > n - m \geq q_1$	3.5	3.5	1
$v = 3$ *and* $k = 3$	17	17	$m \geq q_k$	1.4	5.6	2.5
$v = 3$ *and* $k = 5$	13	22	$q_k > m \geq q_1 > n - m$	3	4	1.17

Some unsolved problems

- What if the chooser is more active? The Unconstrained Chooser Game.
- A reference to the expert opinion literature.
- The dynamics of appointment rules.
- Approaching Arrow and Condorcet.