The Role of Matching Grants as a Commitment Device in the Federation Model with a Repeated Soft Budget Setting[1]

Nobuo Akai[†] Motohiro Sato[2]

Version 10/16/2015

Abstract

This paper reexamines the optimality of the soft budget constraint against the hard budget constraint in federations. By extending Besfamille and Lockwood (2008) that raises the case that the soft budget is ex ante favorable, we consider the commitment problem in the repeated-game setting. Policy instruments by the federal government are enriched to include a matching grant scheme. We then establish that the matching grant scheme works as a commitment device and improves social welfare relative to when ex ante policy option of the federal government is limited to either the hard or the soft budget.

Key Words: soft budget, time inconsistency

JEL: H71, H72, H73, H77

[1] We also thank Chikara Yamaguchi and Noriyuki Yanagawa for their helpful comments. An early version of this paper was presented at Annual Meetings of Japanese Economic Association in 2011 and Public Choice Society and Association of Public Economic Theory in 2012. The authors gratefully acknowledge the financial support of a Grant-in-Aid for Scientific Research from the Japan Society for the Promotion of Science (#23330105 and #20243022, respectively).

[†]Correspondence to: Osaka School of International Public Policy, Osaka University. 1-31 Machikaneyama, Toyonaka, Osaka, 560-0043 Japan Tel:+81-(0)6-6850-5624(Dial-In), E-Mail akai@osipp.osaka-u.ac.jp

[2] Hitotsubashi University

1. Introduction

There is a large literature on the soft budget constraint (SBC) problem, pioneered by Kornai (1986). Dewatripont and Maskin (1995) formulate the problem as the case of time inconsistency in the context of a principal and agent relationship. To be specific, due to a lack of commitment, a lender, acting as the principal, bails out a costly project started by a borrower, acting as the agent. Such a bailout might be inefficient from the ex ante perspective when the project is not yet initiated, but turn out to be efficient ex post once the initial cost is sunk. In anticipation of the ex post rescue, the agent undertakes the ex ante inefficient project. In this setting, although a hard budget constraint (HBC) is ex ante efficient, the principal cannot commit to it and, therefore, the SBC prevails. .

Wildasin (1998) applies the SBC to the context of intergovernmental relations. The central government as the principal ex post bails out the local government, acting as the agent, which undersupplies an interregional spillover-generating public service. The bailout is motivated to enhance ex post efficiency, but it exacerbates ex ante inefficiency given that the local government acts in a strategic manner to manipulate ex post transfers in its favor. This study has been extended by Akai and Sato (2008).

The above studies presume that the HBC is ex ante preferable to the SBC. There may be circumstances, however, when the SBC constitutes an optimal contract from the second-best perspective. Indeed, Kornai et al.'s (2003) survey addresses such a case, where the government deliberately utilizes a SBC as a device to affect the incentives of its agent, such as local government. In this regard, the SBC may ex ante dominate the HBC. In this spirit, Besfamille and Lockwood (2008) consider that the central government ex ante imposes either a HBC or a SBC on the local government. The project requires a certain effort at the local level. Without it, completion of the project will require additional costs. In the case of the HBC, there is no transfer to finance the extra cost of the project, so it must be terminated. In the case of the SBC, on the other hand, the central government grants a transfer ex post to complete the project. Besfamille and Lockwood (2008) suppose that the central government is ex ante capable of committing to either a HBC or a SBC and derive when and then gives the circumstance that the SBC can be the ex ante second-best.

The distinct feature of their model is that the policy option of the central government is limited to that it either gives no grant or fully bears the additional cost required to complete the project. The analysis presumes that commitment to the policy designed ex ante is possible. No ex post discretion is allowed to change from the HBC to the SBC. Moreover, the game between the governments is one-shot game, without consideration for future welfare.

The current paper, on the other hand, extends Besfamille and Lockwood (2008) in several directions. It enriches policy instruments of the central government to include matching grants. In addition, we incorporate both the commitment problem and the reputation effect. The commitment problem arises when the central government is allowed to pursue the ex post optimum by changing its policy ex post, whereas the reputation effect can be considered by extending the model to the repeated-game setting. Related to it, Crivelli and Staal (2012) account for additional cost of bailing out. The reputation effect in our model may constitute such a cost.

In addition, we account for a matching grant scheme to subsidize part of the additional cost ex post whereas remaining is self financed by the regional government. We then establish that the matching grant scheme works as a commitment device and improves social welfare relative to the case where ex ante option of the federal government is limited to either the hard or the soft budget. It is because the additional benefit of bail out becomes relatively smaller with the matching grant, which makes commitment possible.

The present paper does not intend to replicate certain practices regarding intergovernmental relations. Rather, we aim to provide a theoretical framework to synthesize the different views on the HBC versus the SBC.

The rest of the paper is organized as follows. Section 2 describes the basic setting of the model. Social and regional gains from HBCs and SBCs are discussed in Section 3. In Section 4, we discuss the commitment problem in the case of a one-shot game. Section 5 examines the case of a repeated game. In Section 6, the model is extended, and the role of matching grants as a commitment device is clarified. Section 7 concludes the paper.

2. Model setting:

The present model considers an economy with a representative regional government (RG) and a federal government (FG). In order to focus on the problem in vertical fiscal relations, the model abstracts from horizontal or interregional interaction but addresses vertical fiscal relations. Later we briefly address the case of multiple regions.

There arises a non-cooperative game between the RG and the FG. The game is static but is assumed to be repeated infinitely, with a public project being undertaken and completed in each period. Both governments maybe concerned about the future periods that gives rise to reputation effect. (An infinite discount rate implies that government is myopic.)

To be specific, in each period, one project is initiated ex ante that generates a benefit ex post. Assume that the initial investment cost is $c_1 > 0$ incurred by the RG. There is uncertainty with regard to the project outcome. If it turns out to be successful with probability of $p > 0$, the project will be completed, generating benefit of $b_g > 0$. With probability of $1 - p$, an additional cost represented by $c_2 > 0$ must be spent to complete it, which is referred to as 'bad project'. The benefit in this case is denoted by b_b. Otherwise, the project is terminated no benefit being accrued to the economy. Ex ante, the effort at the regional level can enhance the probability of success; for simplicity, we assume that the effort level directly corresponds to p.[3] The cost for such an effort is assumed to be $\Delta(p) > 0$ with $\Delta(0) = 0$, $\Delta(p) > 0$, $\Delta'(p)\big|_{p \neq 0} > 0$, $\Delta''(p) > 0$, $\Delta'(0) = 0$ and $\Delta'(1) = \infty$[4]. The last two conditions assure interior solution of p

[3] This implicitly assume that the project fails with no effort, p=0. The result holds even in the case where the probability p is positive without any efforts.

[4] In order to interpret our analysis easily, it would be assumed that $U > 0$ in the case of

Fiscal regimes

In the basic setting, we assume that only the FG can incur the extra expense $c_2 > 0$ and then consider two regimes of vertical fiscal relation, namely the HBC and the SBC. Later, we introduce matching grants scheme between the FG and RG sharing the extra expense. In the HBC, any ex post rescue is ruled out, which implies that the bad project must be terminated. Such rescue is allowed in the SBC in contrast and thus the bad project is continued.

Ex ante, the FG chooses either regime in every period. The FG may not be able to commit to an ex ante selected regime, however. In this paper, we discuss the disparity between ex ante and ex post regimes that leads to the commitment problem, in the sense that SBC occurs ex post, even if HBC is socially desirable ex ante. Then, the ex ante inefficiency is exacerbated.

Although it is the FG's mandate to continue or terminate the bad project ex post, the decision to start the project ex ante can be assigned to either the FG or the RG. We refer to the former situation as a centralized authority, whereas the latter situation is noted as a decentralized authority. The ex ante decentralized authority may be plausible in a situation where the FG has no information about the project cost at all, but the RG has such knowledge. It also includes the circumstance that the FG rely on reports on c_1 from the RG to determine whether or not to initiate the project. However, such report may be manipulated in favor of the regional interest.

Timing

To summarize, we can describe the structure of the game in each period as the following steps. We call ***Ex ante*** from Stage 0 to Stage 2 and ***Ex Post*** after Stage 3.

Ex Ante Stage

- (Stage 0) FB determines regime of either HBC or SBC.
- (Stage 1) The decision to launch the project is made by the RG in the decentralized authority and by the FG in the centralized one.
- (Stage 2) If the project starts, the RG makes its effort. The game ends otherwise.

Ex Post Stage

- (Stage 3) If the project is successful, it is completed and creates the benefit $b_g > 0$. In the case that the project turns out to be bad,
  - Under the HBC regime, it is terminated and generates no benefit.
  - Under the SBC regime, the FG rescues the RG by financing $c_2 > 0$ and the project is completed yielding the benefit of $b_b > 0$

Regional welfare

We suppose that both the FG and the RG are benevolent, in the sense that they pursue

---

$\Delta(1)$.

the welfare of their jurisdictions. The present model can be easily extended to a political economy setting as Goodspeed (2002), but the essence would not be altered.

We consider a conflict of interest between the two governments, with the FG maximizing social interest, whereas the RG pursues the regional interest, as is standard in the literature. The difference between them is that the RG does not account for the additional financing cost required to complete the bad project, whereas the FG does. Of course, such cost must be financed by raising a central tax, which will ultimately be borne by the residents of the RG. To highlight the inefficiency of the SBC, we assume that the RG does not take into account such feed back as it does not see through the central budget.

In the SBC regime, we assume that no additional cost is incurred at the regional level as $c_2 > 0$ is fully financed by FG.[5] Then, from the ex ante perspective, the regional welfare per period becomes:

$$U = pb_g + (1-p)b_b - c_1 - \Delta(p).$$

Turn to the HBC regime. Given that the bad project is terminated with a probability of $1-p$, the expected regional welfare is:

$$U = pb_g - c_1 - \Delta(p).$$

We suppose that the RG cannot self-finance the cost of the project. The presumption behind this is that ex post local taxation is quite costly, as is assumed in Besfamille and Lockwood (2008).

Now, we establish the effort level chosen by the RG at Stage 2 in each regime. In the SBC regime, the effort level is derived from the first-order condition $\Delta'(p) = b_g - b_b$, which gives $p = p_S^*$. Similarly, the HBC induces the regional effort at $\Delta'(p) = b_g$, yielding $p = p_H^*$. The convexity of the cost function $\Delta(p)$ gives the following lemma.

<Lemma 1> The effort level under the SBC is lower than that under the HBC; namely, $p_S^* < p_H^*$.

The incentive for the effort making deteriorates under the SBC relative to the HBC, This is due to the insurance effect of the bailout ex post, which is often referred to as the ex ante moral hazard.

Denote respectively by $u_S(p_S^*)$ and $u_H(p_H^*)$ the maximized objectives of the RG under the SBC and the HBC regimes. Comparing them establishes that from the regional standpoint, the SBC is always more desirable than the HBC.

<Lemma 2> $\underline{u_S(p_S^*) > u_H(p_H^*)}$

where

$$u_S(p_S^*) \equiv p_S^* b_g + (1-p_S^*)b_b - c_1 - \Delta(p_S^*)$$

[5] In order to capture the effect created by the ignorance of the bailout cost, we assume that the regionals are myopic, namely the RG does not consider the bailout cost.

$$u_H(p_H^*) \equiv p_H^* b_g - c_1 - \Delta(p_H^*)$$

Proof:

$$u_H(p_H^*) \equiv p_H^* b_g - c_1 - \Delta(p_H^*) < p_H^* b_g + (1-p_H^*) b_b - c_1 - \Delta(p_H^*) < u_S(p_S^*)$$

The last inequality is due to the fact that p_s^* maximizes $u_S(p)$. Q.E.D.

3. Social welfare

In the present section, we compare ex ante (expected) social welfare in the two regimes as the benchmark for examining whether commitment works. The social welfare differs from the regional one in the SBC regime since the additional cost c_2 [6]is accounted for:.

$$v_S(c_2) \equiv p_S^* b_g + (1-p_S^*)(b_b - c_2) - c_1 - \Delta(p_S^*)$$

Then two welfare functions coincide in the HBC regime on the other hand;

$$v_H \equiv p_H^* b_g - c_1 - \Delta(p_H^*) = u_H(p_H^*)$$

As noted above, FG ex ante pursues the social welfare in selecting the regimes.

At this point, we would like to distinct two dimensions of the distortion (from the ex ante perspective) namely the choices of regime and effort level. To begin with, note that the social welfare maximizing effort under the SBC regime needs to fulfill

$$\Delta'(p) = b_g - b_b + c_2$$

Define the corresponding effort by p_S^{**}. It is immediate to see that $p_S^* < p_S^{**} < p_H^*$. This implies that effort level chosen by RG is too small relative to the social optimum given the SBC. It is because RG does not account for ex post cost c_2 incurred by FG.

In our context, the first best is not achievable due to the distortion on effort level under SBC and the regime decision must incorporate such under-provision of the effort that lowers $v_S(c_2)$ and in turn raises scope of HBC regime as the second best outcome. As addressed later, lack of commitment generates further ex ante inefficiency distorting ex post regime choice.

Turn to regime choice. From the ex ante perspective, the SBC is efficient when $v_S(c_2) > v_H$, whereas the HBC is favored otherwise. Let c_2^* denote the critical cost such that $v_S(c_2^*) = v_H$. Given that $v_S(0) > v_H$ and $v_S'(c_2) < 0$, c_2^* is unique and positive. Moreover, we can establish the following lemma.

<<u>Lemma 3></u> $\underline{0 < c_2^* < b_b}$

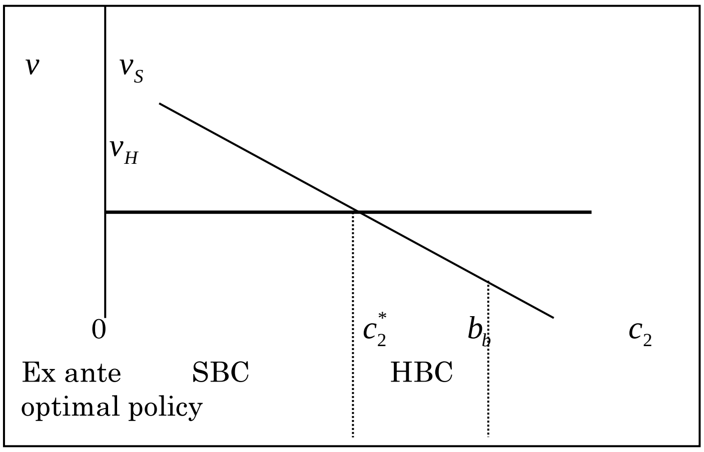
The proof is in the Appendix. As depicted in Figure 1. the SBC is ex ante efficient for

---

[6] We focus on the case where $b_b > c_2$ because the bailout does not occur in this case so any regimes are efficient.

$0 < c_2 < c_2^*$, whereas HBC is ex ante optimal when $c_2^* < c_2$. In slightly different setting, this confirms Besfamille and Lockwood (2008) that addresses the case that the SBC becomes more favorable than the HBC.

Figure 1 Social welfare ex ante under HBC and SBC



4. The case of a one-shot game

For clarification, we begin with a one-shot game. This corresponds to the case where the FG is not concerned about the future or its time discount rate is infinite. In addition, we allow FG to alter the regime ex post. Then FG may not commit to the HBC regime even if it is ex ante optimal. This is the commitment problem in our context.

To see it we start from the ex post comparison of the benefits between the HBC and SBC given that the project is bad. It is then straightforward to see that the SBC is favored when $b_b > c_2$ i.e., the benefit of the project exceeds the additional expense. The initial investment cost does not appear since it has been sunk. From Lemma 3, $0 < c_2^* < b_b$. This in turn implies that FG's decision switches from the HBC to the SBC ex post for $c_2^* < c_2 < b_b$. Therefore we have:

<Proposition 1>

The commitment problem arises for $c_2^* < c_2 < b_b$ because that the efficient policy, HBC, selected ex ante at Stage 0 becomes inefficient ex post at Stage 3 (see Figure 1).

In the above, it has been taken as given that the project is ex ante initiated. To be precise, the ex ante optimum then contains two dimensions of policy decisions. One is whether the SBC or the HBC is favored and another is whether the project should be started. The latter relies upon the initial cost c_1 as well as c_2. We can then state the following lemma.

<Lemma 4>

From an ex ante viewpoint, the project should be initiated if and only if $MAX\left[v_S(c_2), v_H\right] \geq 0$.

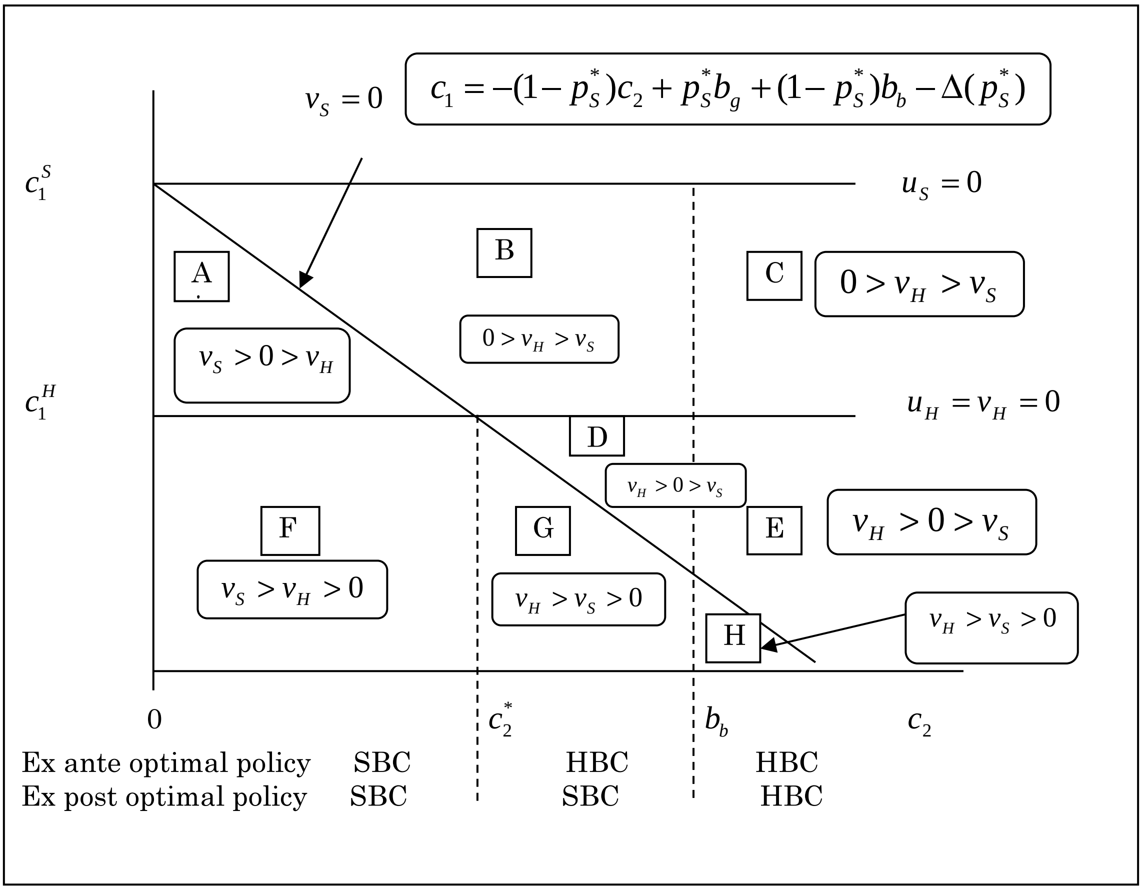
The SBC regime is desirable from an ex ante viewpoint when (i) $v_S(c_2) > v_H$ and (ii) $v_S(c_2) > 0$, whereas the HBC regime is desirable if (i) $v_S(c_2) < v_H$ and (ii) $v_H > 0$. Without commitment, however, ex ante favored HBC may not be implemented ex post.

At this point, we define c_1^S, such that $v_S(c_1^S, c_2) = 0$; i.e.,

$$c_1^S = p_S^* b_g + (1 - p_S^*)(b_b - c_2) - \Delta(p_S^*) \quad \text{with} \quad \partial c_1^S / \partial c_2 = -(1 - p_S^*) < 0$$

and c_1^H refers to $v_H = 0$ or $c_1^H = p_H^* b_g - \Delta(p_H^*)$. Using these relations, we can depict the ex ante optimal regime as in Figure 2 taking c_1 and c_2 as vertical and horizontal axes.

Figure 2 Regional welfare, social welfare and the optimal policy in each regime



Consider the centralized authority for starting the project. Then, in cases A, D, E, F, G and H in Figure 2, the project will be initiated by the FG if it can commit. Among these situations, it is optimal to undertake the project in the HBC regime in areas D, E, G and H and in the SBC regime in areas A and F. However, without commitment, the HBC regime cannot be achieved in D and G creating ex ante inefficiency. It is particularly interesting to focus on D, where, despite the fact that the project is socially desirable, generating a positive expected social welfare outcome in the HBC regime, the FG decides not to commence the project because it foresees an ex post bailout occurring due to its lack of commitment, which makes the social welfare negative.

This is reminiscent of Kornai, Maskin and Roland (2003), that addresses "naturally, the S-organization would never have wished to see the enterprise set up in the first place had it known that this trouble would occur." Such consideration in turn prevents innovative investment and hampers long run development.

Turn to the decentralized authority. it further exacerbates the ex ante inefficiency as the socially undesirable project is undertaken and rescued ex post in the bad state, if $v_S < 0$ but $b_b > c_2$. To be specific, B and D correspond to such a case. In anticipation that the SBC prevails ex post without commitment, the RG will choose to undertake the project as $u_S > 0$. This is referred to as the dynamic inconsistency in that the project that should not have been initiated ex ante starts and becomes ex post optimal to complete it.

At this point, we compare the above results with Besfamille and Lockwood (2008), who note that dynamic inconsistency arises when (i) $b_b < c_1 + c_2$ and (ii) $b_b > c_2$. Their definition is different from ours, i.e., (i) $v_S < 0$ and (ii) $b_b > c_2$ that refers to B and D. These areas can contain $b_b \geq c_1 + c_2$ that is above the line of $c_1 = -c_2 + b_b$ starting at the point $(c_1, c_2) = (0, b_b)$ in Figure 2. Ex ante, it is not certain whether the project is good or bad and thus the ex ante net benefit should be defined in the expected term as in the present model instead of realized one as $b_b - (c_1 + c_2)$.

5.The case of repeated game

Now, we introduce the repeated-game setting. As noted earlier, the game is static and each period is divided into several stages of decision making within each period. To clarify terms, note that "ex ante" refers to the beginning of each period (Stage 0) and "ex post" denotes Stage 3. Given that the ex post decision regarding the bailout is exclusive to the FG and that the RG's current effort does not affect its future gain, the discount rate of the RG is irrelevant to the game outcome. We thus focus on the FG's discount rate on the future gain.

In this context, the result may change as the FG is concerned about its reputation or future gains from sustaining the HBC regime. To be precise, we consider a trigger strategy whereby the RG chooses to maintain a high effort as long as the FG has committed to the HBC regime until the previous period. Once the FG deviates from this regime to pursue an ex post optimum by the rescue for the bad project, the RG subsequently sets its own effort at a low level at p_s^*. The game then reduces to a one-shot game as the FG finds the SBC regime ex post optimal, given the RG's strategy. We do not allow for renegotiation, which would raise the cost of ex post deviation from the HBC regime. This is an extension of the reputation game pioneered by Barro and Gordon (1983) in relation to intergovernmental relations.

In the following, we focus on the case where $c_2^* < c_2 < b_b$, which leads to the commitment problem in the one-shot game; i.e., the HBC is ex ante favored but is not optimal from the ex post perspective. If $0 < c_2 \le c_2^* (< b_b)$, the SBC is ex ante optimal and time consistent irrespective of the discount factor.

<u>Incentive for commitment</u>

We compare the long-term benefit from the HBC with the short-run (ex post) gain from deviating to the SBC. Insofar as the former exceeds the latter, the FG can commit to the HBC regime. Discount factor about welfare in the next period is denoted by $\beta < 1$. (The discount rate is equal to $1/\beta - 1$.) Then, the present value of the net gain from the HBC relative to the SBC is given by:

$$\Omega(\beta, c_2) = \left(\beta + \beta^2 + \beta^3 + \ldots\ldots\right)\left(v_H - v_S(c_2)\right) = \frac{\beta}{1-\beta}\left(v_H - v_S(c_2)\right).$$

It is increasing with β and c_2. On the other hand, given that the project turns out to be bad and requires an additional cost, c_2, the bailout policy creates the gain $\Pi(c_2) = b_b - c_2$. Then, the HBC can be sustained and commitment becomes possible ex post if and only if:

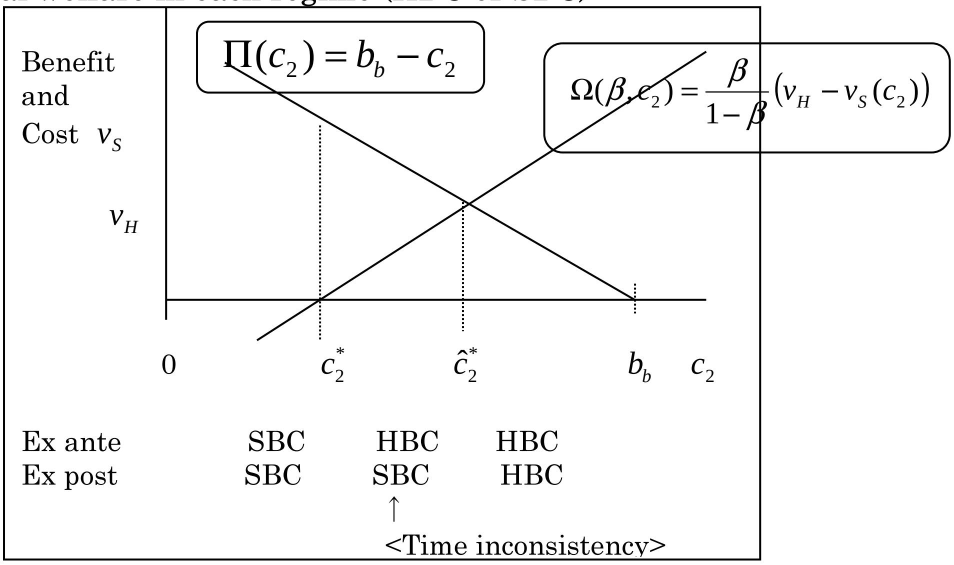
$$\Omega(\beta, c_2) \ge \Pi(c_2) \text{; that is, } \frac{\beta}{1-\beta}\left(v_H - v_S(c_2)\right) \ge b_b - c_2 .$$

Define the cost binding the above inequality by $\hat{c}_2^*$. $\hat{c}_2^*$ relies upon β. FG can fully commit to the HBC ex post if it approaches to unity ($\beta \to 1$), whereas the reputation effect does not work when $\beta = 0$. Now FG can commit to the HBC regime if $c_2 > \hat{c}_2^*$.

With a finite discount factor, the scope of the ex post deviation is diminished to $c_2^* < c_2 < \hat{c}_2^*$. Recall that it arises for $c_2^* < c_2 < b_b$ in one shot game that corresponds to the case of $\beta = 0$.

The intuition is obvious. With $\beta > 0$, ex post deviation from the HBC leads to the welfare loss in the future periods, so raising cost of ex post bailout in the current period. The repeated-game setting works as a commitment device, which is known as reputation effect.

Figure 3 Social welfare in each regime (HBC or SBC)



Now we have the following proposition.

<Proposition 2>

In the repeated game setting with the trigger strategy, FG can commit to the ex ante favored HBC if $c_2 > \hat{c}_2^*$ that is decreasing in β with $c_2^* < \hat{c}_2^*$.

The reputation effect can generate additional gain when ex ante decision of initiating the project is concerned. Recall area D in Figure 2. There, the project is ex ante socially desirable only under the HBC regime. In one shot game, FG chooses not to start the project because it cannot commit to HBC and social welfare turns to be negative ($v_S < 0$). With the reputation effect, on the other hand, the area of D is reduced whereas E is expanded because commitment now becomes effective as well for $\hat{c}_2 < c_2 < b_b$. [7]Therefore, the project can be undertaken yielding the social benefit.

[7] The border for commitment now becomes $\hat{c}_2$ in the repeated game setting lower than b_b in one shot game setting.

Multiple regions

So far, we have examined the interaction between the FG and a representative RG. The model can easily be extended to the case of multiple regions. We denote by N the total number of identical regions. $\overline{p}$ is the average probability or the share of the regions in which the project succeeds. This implies that there are $(1-\overline{p})N$ regions that require additional funds to complete the project. We suppose that N is large enough so that the individual regions take $\overline{p}$ as given. The total future gain from commitment amounts to $N\Omega(\beta,c_2)$, whereas the corresponding gain from deviating from the commitment and bailing out the regions is given by $(1-\overline{p})N\Pi(c_2)$. Therefore, the condition under which the FG prefers to commit becomes:

$$N\Omega(\beta,c_2) \geq N(1-\overline{p})\Pi(c_2) \quad \text{or} \quad \frac{\beta}{1-\beta}(v_H - v_S(c_2)) \geq (1-\overline{p})(b_b - c_2).$$

Let $1-\hat{p}$ be the share of the regions with bad projects that binds the above inequality . Then, we can establish the following proposition:

<Proposition 3>

Suppose that $p_S^* < \hat{p} < p_H^*$. Then, we have multiple equilibria, whereby either (i) all regions choose p_H^* and the FG commits to the HBC; or (ii) all regions select p_S^* and the FG cannot commit.

Proof:

Consider that all regions expect p_H^*. Given that share of the regions with bad projects remains at p_H^*, so that

$$\frac{\beta}{1-\beta}(v_H - v_S(c_2)) > (1-p_H^*)(b_b - c_2),$$

the FG has no motive to deviate from the HBC regime. Then, the regions do not anticipate a bailout ex post and exert a high effort, leading to p_H^*. On the contrary, suppose that all regions expect p_S^*. In anticipation of an ex post rescue, the regions make a low effort, resulting in $(1-p_S^*)N$ regions with bad projects. Then, the commitment condition is violated. Q.E.D.

6. Matching grants as a commitment device

This section introduces the matching grants scheme that share c_2 between RG and FG. This scheme corresponds to retrospective cost sharing arrangement that covers cost overrun as well as initial expenses. The new regime stands between the HBC and the SBC and is shown to expand the scope of commitment whereas enhancing ex ante social welfare.

Refinancing by the RG

In the preceding models, the ex post cost to continue the bad project was not incurred by the RG. In the following we allow the RG to collect the extra funds to cover c_2. In practice, however, such collection cost is more expensive at the regional level than at the federal one, reflecting the scale of the economy and the tax-induced mobility of resources across regions. To capture this, we set the cost for refinancing ex post by the RG relative to the FG as $\gamma \geq 1$. To be specific, we assume $\gamma c_2 > b_b$ implying that the RG cannot alone finance a bad project.

Matching grants

We now introduce a subsidy scheme under which the FG may partially share the ex post cost to continue the bad project at the rate of m. It may be regarded that the matching grant encompasses the two polar regimes, the SBC and the HBC. The SBC regime corresponds to $m = 1$, whereas the HBC is designated $m = 0$.

In the following, we focus on the case where m selected ex ante satisfies $b_b > mc_2 + (1-m)\gamma c_2$ or $m > \underline{m} = (\gamma - b_b / c_2)/(\gamma - 1)$, which guarantees that the continuing project with the matching grants, m, is socially desirable ex post. It is straightforward that the FG has an incentive to raise the matching grant rate up to unity (i.e., $m = 1$) ex post (which corresponds to the SBC) because the financing cost by the FG is cheaper. Furthermore, in order to make our story interesting, we focus on the case that FG cannot commit to the HBC regime $(m = 0)$ in the repeated game setting, i.e., $c_2^* < c_2 < \hat{c}_2^*$. Then, we consider the level of m that induces the FG's commitment and enhances welfare.

Regional welfare

Given the matching rate, m, regional welfare becomes:

$$U = pb_g + (1-p)(b_b - (1-m)\gamma c_2) - c_1 - \Delta(p).$$

$m > \underline{m}$ ensures that the RG can self-finance the cost of continuing the project and completes it ex post when there is a bad situation ex ante. Then, the effort made in Stage 2 satisfies:

$$\Delta'(p_m) = b_g - b_b + (1-m)\gamma c_2.$$

This gives the effort level at Stage 2 as $p_m^* \equiv p_m(m, c_2, \gamma)$.

Now, we have the following lemma for the effect on the effort level.

<Lemma 5>

$p_m^* \equiv p_m(m, c_2, \gamma)$ is decreasing in m, whereas it is enhanced with c_2 and γ.[8] The intuition is that the increase of m induces moral hazard, whereas the increases of c_2 and γ enhance the effort ex ante.

The proof is in the Appendix.

Expected social welfare

Given the matching rate, m, social welfare becomes:

$$v_m(m, c_2, \gamma) \equiv p_m^* b_g + (1 - p_m^*)[b_b - mc_2 - (1-m)\gamma c_2] - c_1 - \Delta(p_m^*),$$

with:

$$\frac{\partial}{\partial m} v_m \equiv (1 - p_m^*)(\gamma - 1)c_2 + mc_2 \frac{dp_m^*}{dm},$$

$$\frac{\partial}{\partial c_2} v_m \equiv -(1 - p_m^*)(m + (1-m)\gamma) + mc_2 \frac{dp_m^*}{dc_2},$$

and

$$\frac{\partial}{\partial \gamma} v_m \equiv -(1 - p_m^*)(1-m)c_2 + mc_2 \frac{dp_m^*}{d\gamma}.$$

The signs of these derivatives are not deterministic because the direct effect and the indirect one through p_m^* work in opposite directions. In the following, we postulate that $v_m(m, c_2, \gamma)$ is concave in m. The next lemma gives such a case.

<Lemma 6>

Suppose that $\Delta(p) = p^2 / 2\alpha$ and $\gamma < 2$, then $v_m(m, c_2, \gamma)$ becomes concave in the matching rate m.

The proof is in the Appendix.

Similarly to Section 5, we examine whether the commitment works by comparing the benefit of commitment and the benefit of the SBC below. Q.E.D.

Condition of commitment

Consider the repeated-game setting. Then, under the matching grant scheme, m, the present value of the net gain from the HBC relative to the SBC is given by:

8 $\frac{\partial p_m^*(m, c_2, \gamma)}{\partial m} < 0, \frac{\partial p_m^*(m, c_2, \gamma)}{\partial c_2} \geq 0 \ \frac{\partial p_m^*(m, c_2, \gamma)}{\partial \gamma} \geq 0$. The equality holds when $m = 1$.

$$\Omega(\beta,m,c_2,\gamma)\equiv(\beta+\beta^2+\beta^3+......)(v_m(m,c_2,\gamma)-v_S(c_2))=\frac{\beta}{1-\beta}(v_m(m,c_2,\gamma)-v_S(c_2)).$$

It can immediately be seen that $\Omega(\beta,m,c_2,\gamma)$ is decreasing in γ and increasing in β.

On the other hand, given that the project turns out to be bad and requires an additional cost, c_2, the bailout yields the following gain:

$$\Pi(m,c_2)\equiv(b_b-c_2)-(b_b-mc_2-(1-m)\gamma c_2)=(\gamma-1)(1-m)c_2,$$

given that $m>\underline{m}$. The net gain from the deviation $\Pi(m,c_2)$ is increasing in γ and c_2, decreasing in m and is independent of β. It takes different form from the case without matching grants in the last section since the welfare from the commitment is different.

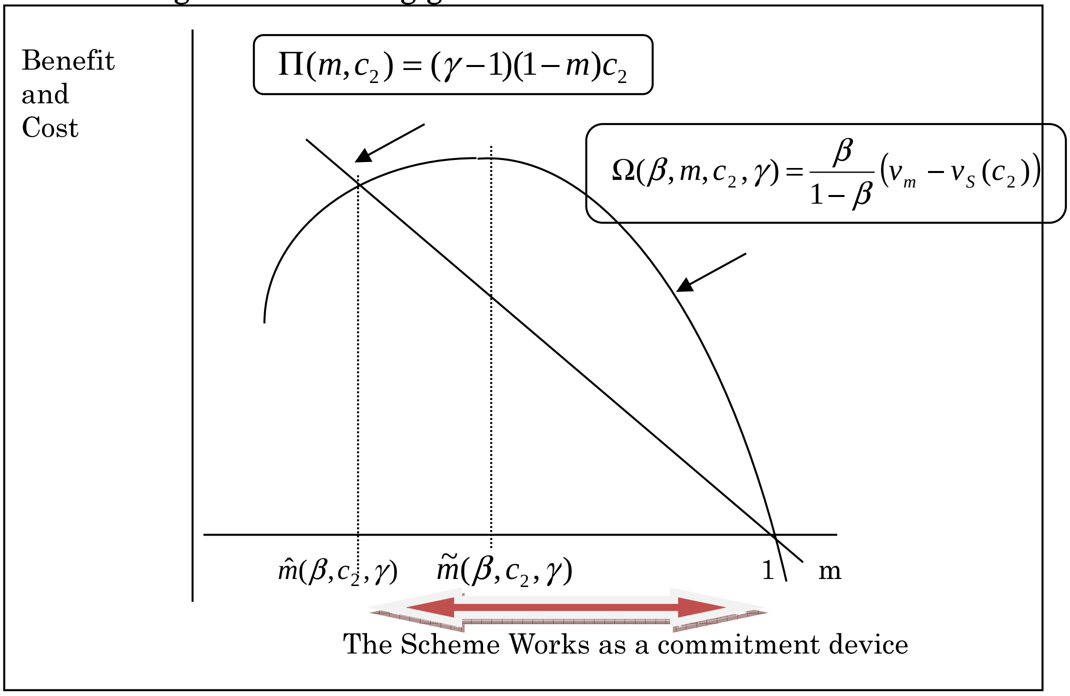
Then, the matching grant scheme can be sustained and committed to ex post if and only if the following commitment condition is fulfilled:

$$\Omega(\beta,m,c_2,\gamma)\geq\Pi(m,c_2)\text{; that is, }\frac{\beta}{1-\beta}(v_m(m,c_2,\gamma)-v_S(c_2))\geq(\gamma-1)(1-m)c_2.$$

Figure 4 depicts the relationship between the long-run (ex ante) gain from the commitment and the short-run (ex post) gain from the deviation to the SBC. We use $\hat{m}(\beta,c_2,\gamma)$ to denote the matching rate yielding the above with equality. Recall that we let $c_2^*<c_2<\hat{c}_2^*$ so that the FG cannot commit to HBC regime that implies $\hat{m}>0$. This in turn implies that the matching grants serve to improve the ability of the FG to commit.

<Proposition 4>: The government can commit to the ex ante policy at the level of m when $\hat{m}(\beta,c_2,\gamma)\leq m\leq 1$.

Figure 4 Matching grants as a commitment device



We now turn to the comparative statics to see how the threshold of the matching grant rate is related to γ, c_2 and β. As $\hat{m}$ is lowered, the scope of commitment expands. We can establish the following:

<**Lemma 7**> $\hat{m}(\beta,c_2,\gamma)$ is increasing with γ and decreasing with β.

The proof is in the Appendix.

We can confirm these results in Figure 4. First, consider the effect of γ. As the increase of γ pushes the level of $\Pi(m,c_2)$ up and the level of $\Omega(\beta,m,c_2,\gamma)$ down, $\hat{m}(\beta,c_2,\gamma)$ increases. The intuition is as follows. The increase of γ results in self- financing by the RG becoming costly and makes the incentive for rescue high. Therefore, commitment to the ex ante policy requires a higher-level matching grant ex ante, meaning that the boundary for commitment, $\hat{m}(\beta,c_2,\gamma)$, increases. Next, consider the effect of β. The decrease of β lowers the level of $\Omega(\beta,m,c_2,\gamma)$, given m. It can be seen immediately that $\hat{m}(\beta,c_2,\gamma)$ converges to unity as β goes to zero; namely, the FG is perfectly myopic. This implies that, as long as the FG is myopic, any $\hat{m}(\beta,c_2,\gamma)$ that is less than one cannot be committed to and the SBC arises because there is no benefit from the HBC.

<u>Welfare Implications</u>

Now let us turn to welfare effect of the matching rate. We remove the assumption of $c_2^* < c_2 < \hat{c}_2^*$ so that we account for the case that FG can commit to the HBC. Even so, matching grant may be desirable from social welfare standpoint. In ex ante selecting m, FG maximizes the social welfare subject to the commitment condition, or it solves the following:

$$\begin{array}{ll} Max_{\{m\}} & v_m(m, c_2, \gamma) \\ \text{subject to} & \Omega(\beta, m, c_2, \gamma) \geq \Pi(m, c_2) \end{array}$$

Denote by $\tilde{m} = \tilde{m}(c_2, \gamma)$ the unconstrained solution to the above maximization that is given by:

$$(1 - p_m(\tilde{m},\ c_2, \gamma))(\gamma - 1) + \tilde{m}\frac{\partial p_S(\tilde{m},\ c_2, \gamma)}{\partial m} = 0 \quad .$$

Since $\gamma > 1$, it is immediate to see that $\tilde{m} > 0$. Noting that HBC regime corresponds to $m = 0$. $\tilde{m} > 0$ means that setting matching rate m enhances social welfare. $\tilde{m}$ is the ex ante optimal matching rate and FG can commit to it if $\tilde{m} > \hat{m}(\beta, c_2, \gamma)$. Otherwise, FG ex ante sets the rate at $\hat{m}$ that just binds the commitment condition. Define

$$m^*(\beta, c_2, \gamma) = Max[\tilde{m}(c_2, \gamma), \hat{m}(\beta, c_2, \gamma)]$$

Then we can show:

<u><Proposition 5>: F</u>G can choose matching rate m ex ante to enhance social welfare compared to the SBC and HBC regimes whereas assuring commitment or

$$v_m(m^*(\beta, c_2, \gamma), c_2, \gamma) \geq Max[v_H, v_S(c_2)]$$

with strict inequality insofar as $m^*(\beta, c_2, \gamma) < 1$.

The proposition is intuitive given that matching grants improves social welfare given that it covers the SBC (m=1) and HBC (m=0) regimes as special cases. The present model addresses that the SBC may become ex ante optimal as in Besfamille and Lockwood (2008) because ex ante policy option feasible to FG is limited to the two extreme regimes. Moreover, as we have established in Section 5, there is the circumstance that FG cannot commit to the HBC that is ex ante desirable since the regime is too costly to sustain from the ex post viewpoint. The matching grants that enriches FG's option serves to resolve these dilemmas.

7. Conclusion

The present paper reexamines the argument of Besfamille and Lockwood (2008) that the SBC regime may ex ante dominate the HBC. They suppose that the FG can commit to an ex ante policy decision and its policy instrument is limited to either no refinancing or $m = 0$ (referred to as the HBC regime) or full refinancing or $m = 1$ (corresponding to the SBC).

First, we showed that, in the one-shot game, the commitment problem arises depending on c_2 and b_b (where $c_2^* < c_2 < b_b$ shown in Figure 1). There exists an interesting case where the project is socially desirable in the HBC regime, but not in the SBC regime (where $v_H > 0 > v_S$, shown by area D in Figure 2). In this case, the allocation of the authority for starting the project to either FG or RG becomes important.

Moreover, when considering the situation where the game is repeated, The scope of commitment is enhanced because the FG cares more about the future period welfare, as seen in Figure 3. We confirm that the repeated-game setting works as a commitment device.

We allow the FG to establish an ex post cost sharing (subsidy) in a more flexible way, by choosing a matching grant scheme In this setting, the matching grants scheme works as a commitment device and social welfare improves. The results imply that an additional policy instrument, apart from an all-or-nothing bailout policy, is useful.

Appendix

<u>Proof of Lemma 3</u>

Suppose that $c_2^* > b_b$. Then, we have $p_H^* b_g - c_1 - \Delta(p_H^*) < p_S^* b_g - c_1 - \Delta(p_S^*)$, which contradicts that p_H^* is the optimal value to maximize $pb_g - c_1 - \Delta(p)$.
Q.E.D.

<u>Proof of Lemma 4</u>

Suppose that $0 \geq c_2^*$. Then, we have:

$$\begin{aligned} v_S(c_2^*) &\equiv p_S^* b_g + (1-p_S^*)(b_b - c_2^*) - c_1 - \Delta(p_S^*) \geq p_S^* b_g + (1-p_S^*) b_b - c_1 - \Delta(p_S^*) \\ &> p_H^* b_g + (1-p_H^*) b_b - c_1 - \Delta(p_H^*) > p_H^* b_g - c_1 - \Delta(p_H^*) = v_H \end{aligned}.$$

This contradicts the definition of c_2^* such that $v(c_2^*) = v_H$. Q.E.D.

<u>Proof of Lemma 5</u>

Noting that $\Delta''(p_S^m) > 0$, we have:

$$\frac{\partial p_S(m,c_2,\gamma)}{\partial m} = \frac{-\gamma c_2}{\Delta''(p_S^m)} < 0, \frac{\partial p_S(m,c_2,\gamma)}{\partial c_2} = \frac{(1-m)\gamma}{\Delta''(p_S^m)} \geq 0, \frac{\partial p_S(m,c_2,\gamma)}{\partial \gamma} = \frac{(1-m)c_2}{\Delta''(p_S^m)} \geq 0.$$

The denominator is positive ($\Omega_m(\beta,m,c_2,\gamma) > \Pi_m(m,c_2)$) at $m = \hat{m}$ under the assumption that $v_m(m,c_2,\gamma)$ is concave. We also have $\Omega_\gamma < 0$ and $\Omega_\beta > 0$. Q.E.D.

<u>Proof of Lemma 6</u>

With $\Delta(p) = p^2/2\alpha$, $p_m^* = \alpha\left(b_g - b_b + (1-m)\gamma c_2\right)$ or

$$\frac{d}{dm} p_m^* = -\alpha\gamma c_2,$$

which in turn gives:

$$\frac{\partial}{\partial m} v_m \equiv (1-p_m^*)(\gamma-1)c_2 - m\alpha\gamma(c_2)^2.$$

Therefore,

$$\frac{\partial^2}{\partial m^2} v_m \equiv -\left(\frac{d}{dm} p_m^*\right)(\gamma-1)c_2 - \alpha\gamma(c_2)^2 = \alpha\gamma c_2(\gamma-1)c_2 - \alpha\gamma(c_2)^2 = \alpha\gamma(\gamma-2)(c_2)^2 \quad <0.$$

<u>Proof of Lemma 7</u>

$$\frac{\partial \hat{m}(\beta,c_2,\gamma)}{\partial \gamma} = -\frac{\Omega_\gamma(\beta,\hat{m},c_2,\gamma)}{\Omega_m(\beta,m,c_2,\gamma) - \Pi_m(m,c_2)} > 0,$$

$$\frac{\partial \hat{m}(\beta, c_2, \gamma)}{\partial \beta} = -\frac{\Omega_\beta(\beta, \hat{m}, c_2, \gamma)}{\Omega_m(\beta, m, c_2, \gamma) - \Pi_m(m, c_2)} < 0 .$$

References

Akai, N. and Sato, M., "Too Big or Too Small? A Synthetic View of the Commitment Problem of Interregional Transfers," Journal of Urban Economics, 64(3), 2008, 551–559.

Barro, R. and Gordon, D., "A Positive Theory of Monetary Policy in a Natural Rate Model," Journal of Political Economy, 91(5), 1983, 89–610.

Besfamille, M. and Lockwood, B., "Bailouts in Federations: Is a Hard Budget Constraint Always Best?," International Economic Review, 49, 2008, 577–593.

Crivelli, E. and K. Staal, Size and Soft Budget Constraint, International tax and public finance, forthcoming.(Published online: 2012)

Dewatripont, M. and Maskin, E., "Credit and Efficiency in Centralized and Decentralized Economics," Review of Economic Studies 62, 1995, 541–555.

Goodspeed, Timothy J. (2002), "Bailouts in a Federation," International Tax and Public Finance, Vol.9, No.4 ,August, pp.409-421.

Kornai, J., "The Soft Budget Constraint," Kyklos, 39(1), 1986, 3–30.

Kornai, J., Maskin, E. and Roland, G., "Understanding the Soft Budget Constraint," Journal of Economic Literature, 41(4), 2003, 1095–1136.

Wildasin, D.E., "Externalities and Bailouts: Hard and Soft Budget Constraints in Intergovernmental Fiscal Relations," Policy Research Working Paper Series 1843, 1997, The World Bank.