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The spenders-hoarders theory of capital accumulation,
wealth distribution and fiscal policy

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**The spenders-hoarders theory of capital accumulation,
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Abstract

This paper proposes a simple OLG model which is consistent with the essential facts about consumer behavior, capital accumulation and wealth distribution, and yields some new and surprising conclusions about fiscal policy. By considering a society in which individuals are distinguished according to two characteristics, altruism and wealth preference, we show that those who in the long run hold the bulk of private capital are not so much motivated by dynastic altruism as by preference for wealth. Two types of social segmentation can result with different wealth distribution. To a large extent our results seem to fit reality better than those obtained with standard optimal growth models in which dynastic altruism (or rate of impatience) is the only source of heterogeneity: overaccumulation can appear, public debt and unfunded pensions are not neutral, estate taxation can improve the welfare of the top wealthy.

Keywords: altruism, preference for wealth, capital accumulation, wealth distribution, Ricardian equivalence.

JEL classification: D64, H55, H63

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“The love of wealth is therefore to be traced, as either a principal or accessory motive, at the bottom of all that the Americans do.”

Alexis de Tocqueville 1841.

1 Introduction

The model of optimal capital accumulation with infinitely lived agents developed by Ramsey (1928) is one of the most popular models in macroeconomics. As infinitely lived agents are now often reinterpreted as dynasties of altruists (see Barro, 1974), it forms the core of many models of economic growth and it is extensively used for analyzing the effects of government policy.

In Barro-Ramsey framework, it is well known that all the capital would end up in the hands of the most patient (or alternatively, the most altruistic) households in a competitive equilibrium.¹ Before Ramsey (1928), as noted by Boyd (2000), Rae (1834) and Fisher (1930) had already suggested that the most patient households accumulate all of the capital.

This result seems to be robust since it holds as soon as there exists some agents with infinite horizon (infinitely lived agents or unconstrained altruistic agents). Indeed, in societies with at least an agent *à la* Barro-Ramsey, the steady state is always² Pareto optimal and the long-run capital stock, driven by the degree of patience (or altruism) of the most patient (or altruistic) agents, is only held by these agents.

The main goal of this paper is to question these well established results in optimal growth theory by constructing a simple and realistic overlapping generations model which is consistent with the essential facts about consumer behavior, capital accumulation and wealth distribution, and to yield some new and surprising conclusions about fiscal policy. Then, our paper can be viewed as an alternative modeling of dynamic wealth distribution that is generally dealt with using calibrated versions of stochastic growth models³ or using theoretical models with imperfect credit market⁴.

¹This property has already been conjectured by Ramsey (1928) and it has been formally proved by Rader (1971), Becker (1980) and Becker and Foias (1987) for the case with borrowing constraints, and by Bewley (1982) for the case without borrowing constraints.

²See Muller and Woodford (1988), Nourry and Venditti (2001) or Thibault (2005).

³An elegant way to generate non-degenerate wealth distribution consists of introducing some idiosyncratic uninsurable risks in a growth model and calibrate it to fit data. Calibrated economies with Barro-Ramsey households have been for example studied by Aiyagari (1994), Castaneda, Diaz-Giménez and Rios-Rull (1998, 2003) and Quadrini (2000) when agents have identical preferences or by Krusell and Smith (1998) when agents differ regarding to time discount rate.

⁴These standard theoretical models on wealth accumulation and on wealth distribution (for example, Banerjee and Newman, 1991, Galor and Zeira, 1993, Aghion and Bolton, 1997, Piketty, 1997, or Matsuyama, 2000, 2006) generally have three main ingredients: imperfect capital market, exogenous prices and warm-glow altruism. This form of altruism (joy of giving) is the most tractable, but it implies, contrary to the data, that the wealth held by an individual always has an inherited component.

1.1. *Why do we need a new model ?*

Recently, Mankiw (2000) presented three pieces of evidence suggesting that we need a new macroeconomic model of fiscal policy. Indeed, the two canonical macrodynamic models – namely the Barro-Ramsey model with infinite horizon and the standard OLG model (due to Samuelson, 1958, and Diamond, 1965) with finite horizon – are inconsistent with the empirical finding that consumption tracks current income and with the numerous households with near zero wealth. In addition, the Diamond-Samuelson model is inconsistent with the great importance of bequests in aggregate wealth accumulation. Then according to Mankiw (2000, p. 121): “A new model of fiscal policy needs a particular sort of heterogeneity. It should include both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption not only from year to year but also from generation to generation. That is, we need a model in which some consumers plan ahead for themselves and their descendants, while others live paycheck to paycheck.”

From these observations, macroeconomists have focused on a new distinction to segment society in a dual way, that between spenders and savers, which echoes that introduced some time ago by Ramsey (1928) between people with high and low impatience and more recently that between altruists and non altruists.⁵ The gist of these later distinctions is that savers, patient agents or altruistic households, end up accumulating wealth for the sake of transmission to their children whereas spenders, impatient agents or non altruistic households, don't save at all and if they do so, they do it for their own future consumption.

The question one can raise at this point is whether such a simple representation of society bears any resemblance to reality. In others words, is the degree of altruism the key parameter of wealth accumulation? It is undoubtedly an important parameter as without bequest motive dynastic wealth is not sustainable. Whatever the huge wealth of Warren Buffet or Bill Gates their decision not to leave any sizeable bequest to their heirs does not favor wealth accumulation on behalf of their descents. Admittedly it is highly rare for a family fortune to last more than three generations, so much so that there's a well known adage: “Shirt sleeves to shirt sleeves in three generations”.⁶

However, even if individuals differ in many respects including altruism, we observe at the same time in real life societies that those who control capital accumulation are

⁵Among the models with “savers-spenders” such as presented by Mankiw (2000), let us mention Michel and Pestieau (1998, 1999, 2005) who analyze the effects of alternative fiscal policies (debt, PAYG, estate taxation) in settings where the only heterogeneity is the degree of altruism with fixed or endogenous labor supply or where people vary according to altruism and productivity. Using the same dichotomy “savers-spenders”, Smetters (1999) analyses the robustness of the Ricardian equivalence; Nourry and Venditti (2001) study the stability and the determination of the long-run equilibrium, Laitner (2001) try to explain secular changes in wealth inequality and inheritance in the US and UK data and Thibault (2005) tries to explain the emergence of rentiers (i.e., of capitalists).

⁶Degan and Thibault (2007) analyze the dynamics of accumulation within a dynasty and explain the emergence, the persistence and/or the disappearance of “rentiers” and “dilapidators” over time.

not particularly altruistic. For instance, Arrondel and Laferrère (1998) who distinguish very wealthy and just wealthy in France, show that for the former altruism plays a much smaller role than for the latter. More recently De Nardi (2004) and Reiter (2004) developed general equilibrium models calibrated on US and/or Sweden data. They show that altruism cannot explain the top tail of the wealth distribution. To do it, they use growth models based on Max Weber’s theory of “the spirit of capitalism” and a mathematical model of Kurz (1968): capitalists accumulate wealth for the sake of wealth. To cite Weber (1958, p. 53): “Man is dominated by making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal relationship, so irrational from a naive point of view, is evidently a leading principle of capitalism.”

This view has been shared by many other contemporary and past economists including A. Smith, J.S. Mill, J. Schumpeter or J.M. Keynes⁷ and has been used by many authors who have tried to explain growth and/or savings (see for instance Bakshi and Chen, 1996, Gong and Zou, 2002, Zou, 1994, 1995, Carroll, 2000, De Nardi 2004, Reiter 2004 or Galor and Moav, 2006). To summarize, as argued by Carroll (2000): “the saving behavior of the (American) richest households cannot be explained by models in which the only purpose of wealth accumulation is to finance future consumption, either their own or that of heirs.” Then, to explain such a behavior one has to assume that some consumers regard accumulation as a end in itself or as channel leading to power which is equivalent to assume that wealth is intrinsically desirable, what we call here “preference for wealth”.

In a nutshell it seems that those who hold the bulk of private wealth are not so much motivated by dynastic altruism as by a preference for wealth. In other words, the key source of heterogeneity would not only be impatience or altruism but preference for holding wealth. In this paper, we look at this issue by considering a society in which individuals or rather dynasties are distinguished according to these two characteristics, altruism and wealth preference.

1.2. An overview of our main results.

Let us describe the main results that we obtain in this paper by considering an OLG economy with individuals differing in altruism and in preference for wealth. Those with preference for wealth are labeled “hoarders-altruists”; even though their degree of altruism can be nil. Those without such preference are either weak or strong altruists (see Table 1). To summarize, our society can be view as a society which mixes agents *à la Diamond*, agents *à la Barro*, hoarders with positive intergenerational bequest motive and hoarders without intergenerational bequest motive. Depending on the degree of altruism and the preference for wealth of these individuals, two kinds of equilibrium and segmentation can emerge. In the first type, denoted Equilibrium I,

⁷See Zou (1992, 1994) for a review of the history of economic thought and more references on this topic.

only the hoarders-altruists leave a positive bequest. In the second type of equilibrium, denoted Equilibrium II, only the weak altruists don't bequeath.

	Individual preferences		Wealth transmission	
Types of agents	Degree of Altruism γ	Wealth Preference δ	Equilibrium I (two-class)	Equilibrium II (three-class)
<i>Weak altruists</i>	weak (or nil)	nil	nil (spenders)	nil (spenders)
<i>Strong altruists</i>	strong	nil	nil (spenders)	positive (savers)
<i>Hoarders altruists</i>	positive (or nil)	positive	positive (hoarders)	positive (hoarders)

Table 1: Social segmentation

Regarding capital accumulation, our results differ following the two types of equilibrium. When the degree of altruism of the strong altruists is low, the stock of capital can then exceed its Golden Rule (hereafter GR) level. In other words, equilibrium I can be dynamically inefficient (marginal productivity of capital exceeds the rate of population growth) even though agents motivated by a dynastic altruism leave positive bequests. The result of “savers-spenders” models that it suffices to have one altruistic agent leaving bequests for the economy to be dynamically inefficient does not resist to the arrival of hoarders-altruists. When the altruism of strong altruists is sufficiently high, then the stock of capital is ruled by a Modified Golden Rule (hereafter MGR) based on their degree of altruism. We have then equilibrium II wherein the result of “savers-spenders” models is verified: the stock of capital is in the long run ruled by the most altruistic individuals.

Our findings are particularly interesting with regard to wealth distribution. The bequeathing dynasties hold a strong stand as to the way aggregate wealth is shared. In the two-class equilibrium, the hoarders impose their view and in the three class equilibrium, both hoarders and savers impose their view. Thus in equilibrium II the strong altruists who determine the MGR equilibrium are not the only ones to hold wealth. The hoarders also hold some wealth.⁸ In this case wealth distribution is not reduced to a single point, but reflects what can be observed in reality. We show that it is possible that few hoarders with a low degree of altruism hold more capital than a

⁸We are not the first to exhibit theoretical variants of the Barro-Ramsey model in which the long-run distribution of wealth can be non-degenerate. For example, Epstein and Hynes (1983) or Lucas and Stokey (1984) show that there may exist stationary equilibria in which all households own positive amounts of capital when preferences are described by recursive utility. Sarte (1997) establishes that progressive taxation as another reason for the existence of stationary equilibria with a non-degenerate wealth distribution. A non-degenerate wealth distribution is also obtained by Dutta and Michel (1998) in a setting with imperfect altruism and linear price, by Boyd (2000) in an endogenous growth framework with learning-by-doing or by Falk and Stark (2001) with other form of altruism. Recently, Sorger (2002) shows, in the case where a government levies a progressive income tax, there exist infinitely many stationary equilibria in which all households own positive capital stocks. However, contrary to these papers, we are the first to obtain a distribution of wealth non-degenerate in a simple framework with logarithmic (and not recursive) utility function and without taxation and in which the standard MGR capital stock is definable and holds.

large number of altruists with strong altruism.

With the relation between wealth holding and altruism being more complex than in earlier models, we obtain new results concerning the incidence of fiscal policy. We first summarize the main effects of PAYG pensions and of public debt. In equilibrium I these policies reduce capital accumulation and can either increase or decrease the welfare of both spenders and hoarders. In Equilibrium II, these policies don't have any effect on the stationary capital stock. However, as it appears on Table 2, they increase wealth holding by the savers and decrease wealth holding by both spenders and hoarders.

	Spenders	Savers	Hoarders
Share of capital held	<i>Decreased</i>	<i>Increased</i>	<i>Decreased</i>
Welfare	<i>Decreased</i>	<i>Increased</i>	<i>Decreased</i>

Table 2: Effects of a PAYG pension system or a public debt at the MGR equilibrium.

In the traditional “savers-spenders” models à la Mankiw, those fiscal policies seem to be regressive as they were favoring the savers at the expenses of the spenders. Introducing hoarders leads to a different finding. Now there is a transfer of resource from hoarders to the other individuals. Note also that a PAYG pension system or public borrowing have an opposite effect of unconstrained altruists whether or not there exist a preference for wealth, even a very light one. These two policies benefit the savers and hurt both spenders and hoarders.

As to estate taxation our results differ also from those obtained in the “saver-spender” setting. As it appears on Table 3, in the MGR equilibrium, estate taxation reduces capital accumulation. It increases the share of capital held by spenders and hoarders and decreases the share held by savers. Consequently, if one wants to hurt the savers, estate taxation is good and if one wants to favor them, either PAYG pension or public debt become the appropriate instrument.

	Spenders	Savers	Hoarders
Share of capital held	<i>Increased</i>	<i>Decreased</i>	<i>Increased</i>
Welfare	<i>Decreased</i>	<i>Decreased</i>	<i>Increased if δ^{HO} sufficiently large Decreased if δ^{HO} sufficiently low</i>

Table 3: Effects of the estate taxation at the MGR equilibrium.

Estate taxation is clearly a questionable instrument of redistribution: it hurts the wealthy, but favors the top wealthy. Moreover, estate taxation worsens the welfare of both the spenders and the savers but increases (decreases) the one of the hoarders if the degree of altruism of the savers is sufficiently high (low). We found here a new reason to deal with estate taxation with caution. If the objective of such a tax is to fight top wealth holding, we show that society might be better off using another tool.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 is devoted to the long-run capital accumulation and the long run wealth distribution. Section 4 studies the incidence of public debt, social security and estate taxation. A final section concludes. Proofs are gathered in Appendices.

2 The model

Consider a perfectly competitive economy which extends over infinite discrete time periods. The economy consists of $N \geq 1$ dynasties denoted by $h \in \{1, \dots, N\}$. In each period t , the size of each dynasty h is denoted by N_t^h and grows at rate n . Total population size is N_t . We denote by p_t^h the positive relative size of each dynasty h . It is time invariant. Hence, we have $N_t^h/N_t = p_t^h = p^h$, $\sum_{h=1}^N p^h = 1$ and $N_{t+1}/N_t = N_{t+1}^h/N_t^h = 1 + n$.

Individuals of a dynasty h are identical within as well as across generations and live for two periods. All dynasties are made of altruistic individuals. We adopt Barro (1974)'s definition of altruism: parents care about their children welfare by including their children's utility in their own utility function and possibly leaving them a bequest. When young, altruists of dynasty h , born at time t , receive a bequest x_t^h , work during their first period (inelastic labor supply), receive the market wage w_t , consume c_t^h and save s_t^h . When old, they consume d_{t+1}^h a part of the proceeds of their savings and bequeath the remainder $(1+n)x_{t+1}^h$ to their $(1+n)$ children. Agents perfectly foresee the interest factor R_{t+1} . Bequest is restricted to be non-negative, which is an important assumption. We denote by V_t^h the utility of an altruist of dynasty h :

$$V_t^h(x_t^h) = \max_{c_t^h, s_t^h, d_{t+1}^h, x_{t+1}^h} \ln c_t^h + \beta \ln d_{t+1}^h + \delta^h \ln x_{t+1}^h + \gamma^h V_{t+1}^h(x_{t+1}^h)$$

$$s.t. \quad w_t + x_t^h = c_t^h + s_t^h \tag{1}$$

$$R_{t+1} s_t^h = d_{t+1}^h + (1+n)x_{t+1}^h \tag{2}$$

$$x_{t+1}^h \geq 0$$

where $V_{t+1}^h(x_{t+1}^h)$ denotes the utility of a representative child who inherits x_{t+1}^h . Parameter $\delta^h \geq 0$ measures the preference for wealth, $\gamma^h \in [0, 1)$ is the intergenerational degree of altruism of the dynasty h and $\beta \in (0, 1]$ is the factor of time preference.

Importantly, contrary to Barro (1974), our log-linear life-cycle utility is not restricted to depend only on life-cycle consumption. Indeed, the agent enjoys accumulating wealth for itself when $\delta^h > 0$. The reasons why wealth directly enters in the utility function have been previously explained in our introduction. Such a specification is old. For example, before⁹ the well-known article of Kurz (1968), Yaari (1964) focuses "on the notion that consumer preferences depend not only on the rate of consumption but also on terminal wealth (or bequests). This notion is, of course, not new. Marshall (1920, p. 228), for instance, refers to family affections as *the chief motive of saving*".

⁹A theoretical discussion on this issue can also be found in the Tobin's unpublished dissertation (1947).

In our economy, the heterogeneity comes from the two parameters triggering saving (besides old age consumption): the preference for wealth δ^h and the degree of altruism γ^h . Then, each dynasty h can be characterized by a pair $(\delta^h, \gamma^h) \in \mathbb{R}_+ \times [0, 1)$. From each pair (δ^h, γ^h) we can define the key parameter $\bar{\gamma}^h$ as follows:

$$\bar{\gamma}^h = \frac{\gamma^h(1 + \beta) + \delta^h}{1 + \beta + \delta^h} \geq \gamma^h.$$

This parameter represents a “modified degree of altruism” which is larger or equal to γ^h . Indeed, when $\delta^h = 0$ we have $\bar{\gamma}^h = \gamma^h$ whereas $\bar{\gamma}^h > \gamma^h$ as soon as $\delta^h > 0$. This modified degree of altruism modified by the wealth preference allows us to segment, without loss of generality, the $N > 0$ dynasties as follows:

We first have M dynasties ($M > 0$) which have no preference for wealth (i.e., $\delta^h = 0$) and are labeled from $h = 1$ to M . We assume that $\gamma^M \in (0, 1)$ and (if $M > 1$) $\gamma^h \in [0, \gamma^M)$ for $h \in \{0, \dots, M - 1\}$. Therefore, M is the most altruistic dynasty among the dynasties which don't have any preference for accumulating wealth. By convention, dynasties 1 to $M - 1$ are considered as dynasties of weak altruists (WA) whereas the dynasty M is a dynasty of strong altruists (SA).

We then have $N - M$ dynasties ($N > M$) which have some preference for wealth (i.e., $\delta^h > 0$) and are indexed $h = M + 1, \dots, N$. We assume that $\gamma^h \in [0, 1)$ and (if $N > M + 1$) $\bar{\gamma}^h \in [0, \bar{\gamma}^N)$ for $h \in \{M, \dots, N - 1\}$. Therefore, N is the dynasty with the higher modified degree of altruism among the dynasties with preference for wealth. In our terminology these $N - M$ dynasties are dynasties of hoarders-altruists (HA) whereas in the specific case where $\gamma^h = 0$, individuals are only hoarders and not hoarders-altruists.

Maximizing $V_t^h(x_t^h)$ subject to (1) and (2) gives the following first order conditions:

$$\forall h \in \{0, N\} \quad d_{t+1}^h = \beta R_{t+1} c_t^h \quad (3)$$

$$\forall h \leq M \quad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\gamma^h}{c_{t+1}^h} \leq 0 \quad (= \text{if } x_{t+1}^h > 0) \quad (4)$$

$$\forall h > M \quad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\delta^h}{x_{t+1}^h} + \frac{\gamma^h}{c_{t+1}^h} = 0 \quad (5)$$

Unlike the M dynasties without wealth preference, the optimal bequests of the $N - M$ dynasties with wealth preference are necessarily positive at all date, and hence, in the long run.

We can also write the saving of a given dynasty h . Indeed, we have:

$$s_t^h = \frac{1}{1 + \beta} \left[\beta(w_t + x_t^h) + \phi(R_{t+1})x_{t+1}^h \right] \quad (6)$$

where $\phi(R) = (1+n)/R$ can be interpreted as a dynastic discount factor. Thus, *ceteris paribus*, the higher the inheritance x_t^h and earning w_t , the higher is saving. Saving increases also with intended bequest x_{t+1}^h .

Let us now turn to the production side. Production technology is represented by a Cobb-Douglas function with two inputs, capital K_t and labor L_t i.e., $Y_t = F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0, 1)$ and $A > 0$. Homogeneity of degree one allows us to write output per worker as a function of the capital/labor ratio per worker, $Y_t/L_t = F(k_t, 1) = f(k_t) = Ak_t^\alpha$ with $k_t = K_t/L_t$, the capital/labor ratio.

Markets are perfectly competitive. Assuming, without loss of generality, that capital fully depreciates after one period, each factor is paid its marginal product:

$$w_t = f(k_t) - k_t f'(k_t) = A(1 - \alpha)k_t^\alpha \quad \text{and} \quad R_t = f'(k_t) = A\alpha k_t^{\alpha-1} \quad (7)$$

In each period, the labor market clears, i.e., $L_t = N_t$ and the capital stock at time $t + 1$ is financed by the savings of the young generation born in t . Hence we have $K_{t+1} = N_t s_t$ with $s_t = \sum_{h=1}^{h=N} p^h s_t^h$. Therefore, in intensive form:

$$(1 + n)k_{t+1} = \sum_{h=1}^{h=N} p^h s_t^h \quad (8)$$

We finish this section by three remarks. First, our approach comprises a wide class of OLG models.¹⁰ Second, we do not make any assumption on the sign of $\bar{\gamma}^N - \gamma^M$. Finally, note that, according to Weil (1987) or Thibault (2000), the Barro's (1974) model with our Cobb-Douglas specification exhibits positive bequest if and only if $\gamma^M > \varepsilon \equiv \beta(1 - \alpha)/[\alpha(1 + \beta)]$. Even if, for the sake of generality, we allow for $\varepsilon > 1$, we have to keep in mind that $\varepsilon > 1$ corresponds to the case where the Barro's (1974) model has constrained altruists.

3 Capital accumulation and wealth distribution

In this section, we restrict our analysis to the steady states. We first study the long run behavior of hoarders-altruists. We have seen that their bequests x_t^h are positive at each date t . Then, according to equations (3) and (5) we obtain in the steady-state denoted by subscript \star :

$$\forall h > M \quad c_\star^h = \frac{\phi(R_\star) - \gamma^h}{\delta^h} x_\star^h \quad \text{and} \quad d_\star^h = \beta R_\star c_\star^h \quad (9)$$

¹⁰When $N = M = 1$ and $\gamma^1 \leq \varepsilon \equiv \beta(1 - \alpha)/[\alpha(1 + \beta)]$ we obtain the Diamond's (1965) model (or equivalently the Barro's (1974) model with constrained altruists). When $N = M = 1$ and $\gamma^1 > \varepsilon$ we have Barro's (1974) model with positive bequests. When $N = M > 1$ and $\gamma^M > \varepsilon$, we obtain the Barro's (1974) model with heterogenous dynasties studied by Vidal (1996). When $N = M = 2$, $\gamma^1 = 0$ and $\gamma^2 > \varepsilon$ our economy is similar to that studied by Michel and Pestieau (1998), Mankiw (2000) or Nourry and Venditti (2001).

Merging (1), (2) and (9) we obtain the (positive) level of stationary bequest of a dynasty h of hoarders-altruists:

$$\forall h > M \quad x_{\star}^h = \frac{\bar{\delta}^h w_{\star}}{\phi(R_{\star}) - \bar{\gamma}^h} \quad (10)$$

where $\bar{\delta}^h \equiv \delta^h / (1 + \beta + \delta^h)$. This term is the relative weight of wealth in the life-cycle utility $\ln c_t^h + \beta \ln d_{t+1}^h + \delta^h \ln x_{t+1}^h$. In the specific case of hoarders dynasty h without intergenerational bequest motive (i.e., $\gamma^h = 0$), we can note that $\bar{\gamma}^h$ corresponds to $\bar{\delta}^h$.

Since, the bequest of hoarders-altruists is positive, the steady-state $\phi(R_{\star})$ necessarily satisfies $\bar{\gamma}^N < \phi(R_{\star})$ and, according to (6) and (10), stationary saving $s_{\star}^h = x_{\star}^h + w_{\star} - c_{\star}^h$ of a dynasty h of hoarders-altruists is given by:

$$\forall h > M \quad s_{\star}^h = \frac{1}{1 + \beta} \left(\beta + \frac{\bar{\delta}^h [\beta + \phi(R_{\star})]}{\phi(R_{\star}) - \bar{\gamma}^h} \right) w_{\star} \quad (11)$$

In the specific case of a hoarders dynasty h without intergenerational bequest motive (i.e., $\gamma^h = 0$), we can note that (11) gives $(1 + \beta)(\phi(R_{\star}) - \bar{\delta}^h)s_{\star}^h = (\beta + \bar{\delta}^h)\phi(R_{\star})w_{\star}$.

We now turn to the dynasties with no wealth preference. According to equations (3) and (4), the long-term behavior of each of them must satisfy:

$$\forall h \leq M \quad \gamma^h \leq \phi(R_{\star}) \quad (= \text{if } x_{\star}^h > 0) \quad (12)$$

Hence, among these M dynasties, only the strong altruists, i.e., the dynasty M with the highest degree of altruism, has the possibility to leave a bequest. Indeed, if there exists a dynasty $m \in \{1, \dots, M - 1\}$ such that $x_{\star}^m > 0$ then equation (12) is not satisfied for dynasties h where $h \in \{m + 1, \dots, M\}$. Then, the weak altruists are constrained altruists in the long run and their saving s_{\star}^h is such that:

$$\forall h \leq M - 1 \quad s_{\star}^h = \frac{\beta}{1 + \beta} w_{\star} \quad (13)$$

Things are more complicated for the behavior of strong altruists. However, according to (12), if x_{\star}^M is positive then the steady state capital stock k_{\star} is equal to that of the MGR capital stock (i.e., $k_{\star} = f'^{-1}[(1 + n)/\gamma^M]$) and we have:

$$k = \left[\frac{\alpha A \gamma^M}{1 + n} \right]^{\frac{1}{1-\alpha}} \equiv k_{\star}^M, \quad R = \frac{1 + n}{\gamma^M} \equiv R_{\star}^M, \quad w = A(1 - \alpha) \left[\frac{\alpha A \gamma^M}{1 + n} \right]^{\frac{\alpha}{1-\alpha}} \equiv w_{\star}^M$$

For the altruists without wealth preference we follow Mankiw (2000) by labeling as spenders the dynasties of constrained altruists and as savers the dynasties of unconstrained altruists. Since the hoarders-altruists wish to accumulate capital for its own sake we label them as hoarders. Then, to sum up our economy (see Table 1), we have shown that the weak altruists are always spenders and the hoarders-altruists are always hoarders. The case of the strong altruists is ambiguous: they are spenders when $x_{\star}^M = 0$ and savers when x_{\star}^M is positive. As a consequence, the bequest motive of the

strong altruists determines the long run wealth distribution of the society.

Knowing the saving function of each dynasty allows us to characterize the long run capital accumulation in our economy. Indeed, using (6), (11) and (13) we can rewrite the equation of capital accumulation (8) according to whether or not the strong altruists leave a bequest.

When $x_\star^M = 0$, according to (11), (13), and using the fact that $(1+n)k/w = \alpha\phi(R)/(1-\alpha)$, the equilibrium condition (8) is equivalent to:

$$F(\phi(R_\star)) \equiv \frac{\beta(\phi(R_\star) - \varepsilon)}{\varepsilon(\phi(R_\star) + \beta)} - \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{\phi(R_\star) - \bar{\gamma}^h} = 0 \quad (14)$$

This equation determines the capital stock of the economy when the strong altruists are spenders.

When x_\star^M is positive, we have $\phi(R^M) = \gamma^M$ and according to (6), (11) and (13), the equilibrium condition (8) gives:

$$x_\star^M = \left[\frac{\beta(\gamma^M - \varepsilon)}{\varepsilon(\gamma^M + \beta)} - \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{\gamma^M - \bar{\gamma}^h} \right] \frac{w_\star^M}{p^M} = F(\gamma^M) \times \frac{w_\star^M}{p^M} \quad (15)$$

This equation determines the bequest left by the strong altruists in the steady-state. Accordingly, the strong altruists are savers or spenders according to whether or not $F(\gamma^M)$ is positive.

From equations (14) and (15) we can now study both the existence of the steady state and the long run capital accumulation of our economy.

Proposition 1 - THE LONG RUN CAPITAL ACCUMULATION.

a - When strong altruists are insufficiently altruist, the steady state is a spenders-hoarders equilibrium where the stationary capital stock k_\star is the solution of (14).

This capital stock increases with the proportion, the degree of altruism and the wealth preference of the hoarders but is independent of the degree of altruism of the spenders.

The stationary capital stock k_\star is in¹¹ under-accumulation, at the GR or in over-accumulation of capital if respectively $\sum_{h=M+1}^N p^h \bar{\delta}^h / (1 - \gamma^h)$ is smaller than, equal to or larger than $\beta(1 - \varepsilon) / \varepsilon$.

b - When strong altruists are sufficiently altruist, the steady state is a savers-spenders-hoarders equilibrium where the stationary capital stock is equal to that of the MGR capital stock k_\star^M .

¹¹Over (under)-accumulation of capital occurs when k_\star is greater (lower) than the golden rule (GR) capital stock $k^G = f'^{-1}(1+n)$. Using Cass (1972), k_\star is said dynamically (in)efficient if and only if k_\star is in (over)under-accumulation of capital.

This capital stock increases with the degree of altruism of the savers but is independent of the proportion of each dynasty, and of the degree of altruism of the spenders and the hoarders.

The stationary capital stock k_^M is below the GR level.*

PROOF – See Appendix A. \square

In Appendix A, we exhibit a threshold value $\tilde{\gamma}$ (satisfying $\tilde{\gamma} \geq \bar{\gamma}^N$) of the degree of altruism of strong altruism below which the strong altruists are insufficiently altruist and, consequently, spenders and above which they are sufficiently altruist and, consequently, savers.

The uniqueness of the stationary equilibrium is standard given the Cobb-Douglas specification. It is however interesting to observe that even in this very simple setting we end up with an endogenously segmented society.

The present paper generalizes the results of the “savers-spenders” models in several respects. It presents a more realistic setting. Also, the introduction of agents with two different characteristics (altruism and preference for wealth) allows for testing the robustness of those models. Indeed, the mere introduction of at least one dynasty of hoarders sizably affects the equilibrium and the resulting stratification.

Two patterns are now possible in the long term. In the first, we have a two-class society with spenders and hoarders that has never been studied in the literature. The spenders belong both to the weak and strong altruists whereas the hoarders belong to hoarders-altruists. The second pattern generalizes the “savers-spenders” models by introducing a third class: the hoarders. We now have a society of savers, spenders and hoarders since in this pattern the strong altruists are savers.

We observe that the three-class result does not hold anymore when the hoarders have a modified degree of altruism, $\bar{\gamma}^h$, that is high enough relatively to the others. Indeed, it suffices that $\bar{\gamma}^N > \gamma^M$ for not having savers, in other words, for having the strong altruists bequeathing nothing. This does not mean that $\gamma^N > \gamma^M$. A low factor of altruism γ^N is compatible with a high $\bar{\gamma}^N$. The degree of altruism γ^M of strong altruists is nevertheless fundamental to determine which kind of equilibrium we end up with.

If it is to bequeath, the strong altruists needs to have a degree of altruism sufficiently high, i.e., such that $\gamma^M > \bar{\gamma}^N$. Note that the value of $\tilde{\gamma}$ is independent of the degree of altruism and of the proportion of the spenders. On the contrary this threshold value $\tilde{\gamma}$ depends on the proportion and the degree of altruism of the $N - M$ dynasties of the hoarders. All things being equal the higher the proportion, the degree of altruism and the wealth preference of the hoarders, the higher is the threshold $\tilde{\gamma}$ and the lower is the likelihood to have a three-class equilibrium that vindicates the results of Michel and Pestieau (1998) and Mankiw (2000).

Importantly, the macrodynamic properties of the equilibrium spenders-hoarders are very different from those of the equilibrium savers-spenders-hoarders. When strong altruists are savers, the economy is at the MGR steady state which depends on the degree of altruism γ^M of savers, but not on their proportion. Note that this result is similar to those of Kaldorian models (see, for instance, Kaldor, 1956, Pasinetti, 1962, or Britto, 1972) but it is obtained in an endogenous way. As well-known since Ramsey (1928) and Becker (1980), the most patients (or altruists) impose their view on the long-run capital accumulation, whatever their size. Then, our savers-spenders-hoarders equilibrium seems to be equivalent to the equilibrium obtained in the “savers-spenders” models with heterogenous agents with no wealth preference. It is however noteworthy that unlike the “savers-spenders” models and contrary to Ramsey’s intuition,¹² equilibrium II with MGR has not two categories of individuals but three: savers, spenders and hoarders. Equilibrium I has two categories of agents, but it is not consistent with the MGR.

At the microeconomics level the introduction of altruists-hoarders has an important implication. Even though the dynasty of strong altruists imposes its view on capital accumulation, it is not the only one to bequeath and hold wealth. The hoarders are also bequeathing even though they can have a negligible degree of altruism. When instead we have a spenders-hoarders equilibrium, the stock of capital is not any more determined by a single dynasty. Now the stock of capital depends on both the degree of altruism and on the preference for wealth of the $N - M$ dynasties of hoarders and on the proportion of each of them.

The fact that the logic of capital accumulation is totally different in the two equilibriums is reinforced by our results on the dynamic efficiency of the two steady-states. These results also emphasize the influence of wealth preference. Even though all agents have some altruism for their children, the competitive equilibrium can be dynamically inefficient when some agents have some preference for wealth per se. Indeed at the spenders-hoarders equilibrium, the higher the proportion, the degree of altruism and the wealth preference of a dynasty of hoarders, the higher is the possibility of over-accumulation. Thus the main result of Barro (1974) that dynastic altruism implies dynamic efficiency does not resist to the introduction of agents having some preference for wealth not just as a means of saving for retirement or for bequests but just for itself. Intuitively it is clear that with such a taste for wealth the stock of capital is likely to be higher than when there is no such a taste.

Things are different in the three-class equilibrium. The stock of capital being consistent with the MGR, the economy is dynamically efficient irrespectively of the presence of hoarders. To sum up, depending on the fundamentals, the introduction of hoarders may have an impact or not on the accumulation of capital. Note that the introduction of hoarders is not necessarily negative. Even when it increases the capital stock, it can be efficient as long as the GR level is not overtaken.

¹²Even though optimal growth theorists and Ramsey in particular are not concerned by intragenerational issue, he notes in his 1928’s seminal paper that if agents differ in their time preference society will be split in two classes: “the thrifty enjoying bliss and the improvident at the subsistence level”.

To illustrate numerically the possibility of a dynamically inefficient equilibrium even if all agents are altruistic, we now consider an economy in which the population consists of a proportion p_1 of weak altruists with degree of altruism $\gamma_1 < \varepsilon$, a proportion p_2 of strong altruists with degree of altruism $\gamma_2 > \varepsilon$ and a proportion p_3 of hoarders with degree of altruism γ_3 . We adopt the following scenario: $\gamma_2 = 0.9$, $\gamma_3 = 0.89$, $\beta = 0.5$, $\alpha = 1/3$, $A = 1$ and $n = 0$. Clearly hoarders is very altruistic. On Figure 1, we see how the steady-state stock of capital changes with δ and increasing values of p_3 . The GR level of capital is given by the horizontal dotted line. The MGR is given by the horizontal segment of the continuous line. When the proportion of hoarders is very low (1/140) the economy follows the MGR as long as $\delta \leq 0.25$. When $\delta > 0.25$ we have a two-class equilibrium but the level of capital stock stays dynamically efficient. In this particular case, increasing wealth preference can be viewed as socially desirable. When the proportion of hoarders is a bit higher, we have the three-class equilibrium for low values of δ ($\delta = 0.1$ for $p_3 = 13/160$). When δ increases, we move to the two-class equilibrium but also to dynamically inefficient capital stocks. For $p_3 = 1/3$ the economy turns dynamically inefficient for $\delta = 0.1$. To summarize, Figure 1 provides an example showing how Barro (1974)'s intuition that dynastic altruism leads to dynamic efficiency is not robust to a slight introduction of preference for wealth.

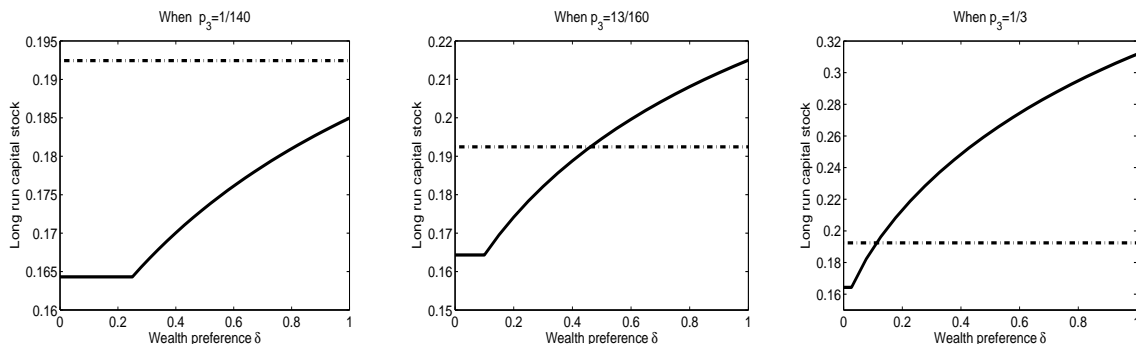


Figure 1: Capital accumulation and dynamic efficiency

We now turn to the way capital is held by the different agents. So doing we obtain a better grasp of the long run distribution of wealth in the two alternative equilibria. One of the motivations of this paper was to show that top wealth is not necessarily held by the strong altruists, which was the conclusion as soon as there exists some savers (infinitely lived agents or unconstrained altruistic agents).

Comparing to the “savers-spenders” literature, our model seems to be most relevant to study the long term distribution of wealth. In the earlier models relying on the single characteristic of either patience or altruism, the equilibrium wealth distribution was reduced to two points: positive wealth for the most patient or altruist, and zero wealth for the others. By introducing some preference for wealth and thus the category of hoarders we now have a more complex and realistic distribution of wealth. We now have $N - M + 1$ or $N - M$ types of wealth-holders. Indeed, it is straightforward that the dynasties which held long run wealth are the dynasties of bequeathers. We could get the share of wealth held by each of the N dynasties. However to keep the analysis

simple, we now focus on the share of each of the three types of dynasties: savers, spenders, hoarders. With this simple presentation, we now study the comparative statics of this wealth-holding.

Proposition 2 - THE LONG RUN WEALTH DISTRIBUTION.

a – At the spenders-hoarders equilibrium, the higher the proportion, the degree of altruism and the preference for wealth of hoarders, the higher is the fraction μ^{HO} of the stationary capital k_ held by the hoarders and the lower is the fraction μ^{SP} held by the spenders.*

b – At the savers-spenders-hoarders equilibrium, the higher the degree of altruism of the savers, the higher is the fraction ν^{SA} of the stationary capital k_^M held by the savers and the lower are ν^{HO} and ν^{SP} the fractions respectively held by the hoarders and the spenders. The higher is the proportion, the degree of altruism and the preference for wealth of a dynasty of hoarders, the higher is ν^{HO} .*

PROOF – See Appendix B. \square

Whatever the equilibrium regime, the spenders hold some capital that is related to saving for retirement.¹³ The distribution of wealth in the equilibrium with spenders and hoarders depends on the stock of capital, which is not the case of the three-class equilibrium. The larger the capital stock, the higher is the share held by the hoarders. Wealth sharing in the two-class equilibrium is more or less imposed by the behavior of hoarders. Both the overall stock of capital and the share they hold increase with their degree of altruism and their taste for wealth. Conversely, the share of wealth held by the spenders is independent of their own characteristics. Intuitively, one could say that the hoarders get the additional capital accumulation they generate ($k_* - k_*^M$).

In the three-class equilibrium, things are clearly different. Sharing now depends on the degree of altruism of both savers and hoarders. The more altruistic the savers, the higher is the capital stock k_*^M , and the higher is their share. When the degree of altruism of the savers increases, ν^{HO}/ν^{SP} decreases and, thus, even if the shares of both savers and hoarders diminish, the hoarders loose more that the spenders. When the factor of altruism of a dynasty of hoarders increases, then the share and the amount of capital held by the hoarders increase at the only expenses of savers. In other words, variation is the degree of altruism of hoarders have not impact on the share of capital held by the spenders.

Our findings are summarized in table 4.

¹³This fact implies that, even if the spenders save, we can consider that they do not hold wealth. Indeed, in a successive generations model, that is, a model without retirement period, the spenders do not hold any wealth.

	Variations of capital sharing				
	EQUILIBRIUM I (two-class)		EQUILIBRIUM II (three-class)		
	Spenders μ^{SP}	Hoarders μ^{HO}	Spenders ν^{SP}	Savers ν^{SA}	Hoarders ν^{HO}
Increase of Parameter					
Altruism Degree γ of WA	<i>Unchanged</i>	<i>Unchanged</i>	<i>Unchanged</i>	<i>Unchanged</i>	<i>Unchanged</i>
Altruism Degree γ^M of SA	<i>Unchanged</i>	<i>Unchanged</i>	<i>Decreased</i>	<i>Increased</i>	<i>Decreased</i>
Altruism Degree γ of HA	<i>Decreased</i>	<i>Increased</i>	<i>Unchanged</i>	<i>Decreased</i>	<i>Increased</i>
Wealth Preference δ of HA	<i>Decreased</i>	<i>Increased</i>	<i>Unchanged</i>	<i>Decreased</i>	<i>Increased</i>

Table 4: Changes in Wealth Distribution according to individual parameters.

To sum up, the dynasties that bequeath are in a strong position in the way capital is shared. In the two-class equilibrium, the hoarders impose their view and in the three-class equilibrium, both hoarders and savers impose theirs. It is important to see the key role played by the preference for wealth. Whatever the final equilibrium the share of capital held by the hoarders increases with their preference for wealth.

To illustrate numerically the relevance of our model we consider the three-type setting used previously for illustrate the possibility of dynamic inefficiency. We now introduce a scenario to illustrate the fact that the relation between wealth holding and altruism is not as simple as thought previously. Figure 2 represents the case where: $\gamma_2 = 0.8$, $\gamma_3 = 0.5$, $\delta = 0.5$, $\beta = 0.5$, $\alpha = 1/3$, $A = 1$ and $n = 0$.

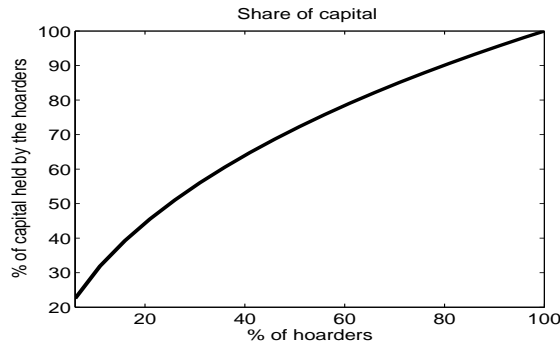


Figure 2: Share of capital held by the hoarders (Scenario 2)

As soon as $p_3 > 0.06$, we have a two-class equilibrium. We see that as p_3 increases beyond this threshold value the share of capital held by the hoarders increase from a share 0.225 ($p_3 = 0.06$) to 1 ($p_3 = 1$). Consequently, even though hoarders are less altruistic than the strong altruists, they end up holding relatively more wealth. In the example depicted Figure 2, the hoarders hold almost 25% of total wealth even though they are poorly altruistic and only represent 5% of total population.

4 Alternative fiscal policies

We now turn to alternative fiscal policies such as PAYG pension system, public debt and estate taxation. We want to see how the results obtained in optimal growth models with spenders and/or savers change or can be extended with the introduction of the hoarders.

4.1 Pay-as-you-go pensions

Our PAYG pension system consists of a payroll levy, τ_t , paid in period t by workers and a pension benefit, θ_t , paid to the retirees of generation $t-1$ so that $\theta_t = (1+n)\tau_t$. Then, the budget constraints (1) and (2) become:

$$w_t + x_t^h - \tau_t = c_t^h + s_t^h \quad \text{and} \quad R_{t+1}s_t^h + \theta_{t+1} = d_{t+1}^h + (1+n)x_{t+1}^h \quad (16)$$

We begin our study of the incidence of PAYG pension system in our economy with a fraction of hoarders by focusing both on the savings and the bequests behavior of each types of individuals.

Obviously, the spenders do not leave bequest. Then, after calculus, their savings and their bequests satisfy:

$$\forall h < M \quad s_\star^h(\tau) = \frac{\beta w_\star(\tau) - [\beta + \phi_\star(\tau)]\tau}{1 + \beta} \quad \text{and} \quad x_\star^h(\tau) = 0 \quad (17)$$

Concerning the hoarders, they always bequeath and the equilibrium conditions (9) always holds. Using $\tilde{\omega}_\star(\tau) = w_\star(\tau) - [1 - \phi_\star(\tau)]\tau$ the part of the life-cycle income $\Omega_\star^h(\tau)$ independent of h , their savings and their bequests is such that:

$$\forall h > M \quad s_\star^h(\tau) = \frac{\zeta_\star^h(\tau)w_\star(\tau)}{1 + \beta} - \frac{\tau[\beta + \phi_\star(\tau)]\chi_\star^h(\tau)}{1 + \beta} \quad \text{and} \quad x_\star^h(\tau) = \frac{\bar{\delta}^h \tilde{\omega}_\star(\tau)}{\phi_\star(\tau) - \bar{\gamma}^h} \quad (18)$$

where:

$$\zeta_\star^h(\cdot) = \beta + \frac{\bar{\delta}^h[\beta + \phi_\star(\cdot)]}{\phi_\star(\cdot) - \bar{\gamma}^h} \quad \text{and} \quad \chi_\star^h(\cdot) = 1 + \frac{\bar{\delta}^h[1 - \phi_\star(\cdot)]}{\phi_\star(\cdot) - \bar{\gamma}^h}$$

Finally, the strong altruists are savers if $x_\star^M(\tau)$ is positive and are spenders if $x_\star^M(\tau)$ is nil. Then, we now distinguish these two cases to determine how the PAYG pension system affect the capital accumulation.

When strong altruists are spenders, using both (17) and (18), we can rewrite (8) to obtain at the equilibrium:

$$F[\phi_\star(\tau)] = -\frac{\tau}{w_\star(\tau)}\vartheta[\phi_\star(\tau)] \quad \text{with} \quad \vartheta[\phi_\star(\cdot)] = 1 + \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h [1 - \phi_\star(\cdot)]}{\phi_\star(\cdot) - \bar{\gamma}^h} \quad (19)$$

Importantly, even if $\phi_\star(\cdot)$ can be greater than one, the function $\vartheta[\phi_\star(\cdot)]$ is always positive. Indeed, $\vartheta[\phi_\star(\cdot)]$ is a decreasing function of $\phi_\star(\cdot)$ such that $\lim_{\phi_\star(\cdot) \rightarrow +\infty} \vartheta[\phi_\star(\cdot)] =$

$1 - \sum_{h=1}^{+\infty} p^h \bar{\delta}^h > 0$. Note that, when $\tau = 0$, we recoup (14).

When strong altruists are savers, the economy is at the MGR. Indeed, (12) holds because the first order conditions (3) and (4) are not modified by the PAYG pension system. Then, the equality (8) allows us to obtain, after some tedious computations, the bequests of savers:

$$x_{\star}^M(\tau) = F(\gamma^M) \times \frac{w^M}{p^M} + \frac{\tau}{p^M} \times \vartheta(\gamma^M) \quad (20)$$

Note that, when $\tau = 0$, we recoup (15).

From equations (19) and (20) we can now study the impact of a PAYG pension system both on the long run capital accumulation of the economy, the redistribution across the dynasties and the welfare of each dynasties.

Proposition 3 THE EFFECTS OF A PAYG PENSION SYSTEM.

a – At the spenders-hoarders equilibrium, the PAYG pension system reduces the accumulation of capital. The decrease in savings of the spenders is not compensated by the hoarders. Indeed, concerning both the bequest and the savings of the hoarders, the impact of the PAYG pension system is ambiguous.

The introduction of the PAYG pension system can improve or worsen both the welfare of the spenders and the one of the hoarders.

b – At the savers-spenders-hoarders equilibrium, the PAYG pension system has no effects on the accumulation of capital. The decrease in the saving both of the spenders and the hoarders is compensated by an increase of the savings of the savers. If the bequest of the spenders remains nil, the introduction of the PAYG pension system reduces the one of the hoarders but increases the one of the savers. This introduction increases the share of capital held by the savers but decreases the shares of capital held respectively by the spenders and the hoarders.

The introduction of the PAYG pension system improves the welfare of the savers but it worsens the welfare both of the spenders and the hoarders.

PROOF – See Appendix C. \square

According to (20), the equilibrium is a three-class one if and only if $\tau > \tau^M \equiv -w^M F(\gamma^M)/\vartheta(\gamma^M)$. If without PAYG pension we have a three-class equilibrium (case where $F(\gamma^M) > 0$), this type of equilibrium remains so with PAYG regardless of the size of τ . On the contrary, if the economy is in a two-class equilibrium without PAYG pensions (case where $F(\gamma^M) < 0$) there exists a payroll tax rate τ^M above which introducing a PAYG pension scheme will imply a three-class equilibrium. Then, the introduction of a PAYG pension system can imply a transition from a spenders-hoarders equilibrium to a spenders-savers-hoarders equilibrium. The opposite shift is not possible. The intuition underlying this possible transition is that in the two-class

equilibrium introducing a PAYG pension leads to capital decumulation in the long run. This decline can lead to the capital level consistent with the MGR, for which we get a three-class equilibrium.

In the two class equilibrium, the higher τ , the less do spenders save. The first effect induces a drop in the capital stock. Furthermore this effect can be offset or reinforced depending on the influence of the PAYG pension system on the level of bequests and saving by the hoarders. Concerning the bequests of the hoarders, we have (see Appendix C):

$$\forall h > M \quad \frac{\partial x_{\star}^h(k_{\star}(\tau), \tau)}{\partial \tau} = \frac{\partial x_{\star}^h(k_{\star}(\tau), \tau)}{\partial k_{\star}(\tau)} \frac{\partial k_{\star}(\tau)}{\partial \tau} - \frac{\bar{\delta}^h [1 - \phi_{\star}(\tau)]}{\phi_{\star}(\tau) - \bar{\gamma}^h}$$

When α and τ are sufficiently low we show that the first term of RHS is positive. Then, if the equilibrium value $k_{\star}(\tau)$ is low enough, hoarder's bequests increase with τ . Intuitively, the introduction of the PAYG pension system leads to an increase in the second period income of the hoarders. This wealth gain in the second period induces the hoarders to increase their bequests. Note however that when the equilibrium is dynamically inefficient (i.e., $\phi[k_{\star}(\tau)] < 1$), the second term of the RHS is negative and the effect of the PAYG pension system on the hoarder's bequests is ambiguous. This effect can even be positive as the first term of the RHS can also be negative when τ and α are sufficiently high. Intuitively bequests by the hoarders can decrease as the PAYG pension system has a depressive effect on first-period income. Thus there exists a wealth loss in the first period that depresses bequests by the hoarders.

Concerning the saving of the hoarders we establish that it is reduced by the PAYG pension system when their bequests are themselves reduced. Conversely, note that the fact that the PAYG pension system increases the bequest of the hoarders is not sufficient to imply an increase in the saving of the hoarders. Intuitively, it exists two antagonist effects when τ increases. First, according to (6) an increase of the bequest $x_{\star}^h = x_t^h = x_{t+1}^h$ imply an increase of the saving of the hoarders when the prices w and R are given. However, it also exists when τ increases a "general equilibrium effect" which leads to reduce the capital stock and, consequently, w_{\star} and $\phi(R_{\star})$. According to (6), when $x_{\star}^h = x_t^h = x_{t+1}^h$ is given this effect implies a decrease of the saving of the hoarders.

To sum up PAYG pension systems always generate a drop in saving by spenders. This decline can be accompanied by a drop in the saving of the hoarders thus reinforcing the decrease in the stationary capital stock. On the contrary, a PAYG scheme can also lead to an increase in the saving of hoarders. Yet this possible increase is too weak to compensate for the decline in the saving of spenders. In other words, PAYG pension systems are not macroeconomically neutral when the economy is in a two-class equilibrium at the outset.

Things are different when society is segmented in three classes at the start. In that case, the PAYG pension system is neutral given that the stationary stock of capital is consistent with the MGR capital stock. This neutrality result generalizes those ob-

tained by Barro (1974), Michel and Pestieau (1998) and Mankiw (2000). In that case, neutrality property obtained in the “savers-spenders” literature resists to the introduction of hoarders-altruists.

Even if the PAYG pension system has no effect on the stationary stock of capital, it modifies the long run wealth distribution because saving by the three classes is affected by the fiscal policy. That of the spenders and the hoarders decreases and that of the savers increases. Saving by the hoarders and the spenders does not equally react to a change in the payroll tax because bequests by the savers and the hoarders don’t move in the same way. PAYG pension systems increase bequests by the savers while reducing those by the hoarders. Intuitively, the hoarders have a bequest motive that is related to wealth accumulation. Their bequests are thus a constant proportion of income $w_\star^M - (1 - \gamma^M)\tau$, which decreases with τ . Things are different for the savers as their bequests increase with τ , given that $x_\star^M(\tau) = x_\star^M(0) + \tau\vartheta(\gamma^M)/p^M$. Following equation (8), the sum of all the bequests (i.e., $\sum_{h=1}^N x_\star^h$) is constant in the savers-spenders-hoarders equilibrium. Thus as τ is raised, increased bequests by the savers fully compensate the drop in bequests by the hoarders.

The fact that, contrary to the capital stock, wealth distribution is modified is already a result obtained by Michel and Pestieau (1998) or Mankiw (2000): they show that a PAYG pension system implies wealth redistribution from the spenders to the savers. Introducing hoarders extends those results: the fraction of capital they hold $\nu^{HO}(\tau)$ decreases with τ . There is thus some redistribution from hoarders to savers. Then, a PAYG pension system, which seemed unfair in “savers-spenders” models, is in fact a fiscal policy which hurts the top wealthy. The direction of redistribution between spenders and hoarders is ambiguous; it depends on the proportion of hoarders; it has to be high enough so that $\nu^{HO}(\tau) - \nu^{SP}(\tau)$ increases with τ .

The effect of a PAYG pension system on individual welfare¹⁴ depends on the type of equilibrium we are concerned with. In the two-class case, both k_\star and R_\star vary with τ . Thus, the welfare of the spenders can be written as:

$$V_\star^{SP}(\tau) = (1 + \beta) \ln \tilde{\omega}_\star(\tau) + \beta \ln R_\star(\tau) + cst$$

As $R_\star(\tau)$ increases with τ but $w_\star(\tau)$ decreases, the variations of $V_\star^{SP}(\tau)$ can be ambiguous. For low values of τ , $\tilde{\omega}_\star(\tau)$ and $R_\star(\tau)$ vary in opposite directions. As $w'_\star(\tau)$ and $\phi'_\star(\tau)$ are both negative, $\tilde{\omega}_\star(\tau)$ is a decreasing function if we are in underaccumulation. On the contrary, if we are in overaccumulation, $\tilde{\omega}_\star(\tau)$ increases with τ and the PAYG pension system may increase the welfare of the spenders. Moreover, the welfare of the hoarders is given by:

$$V_\star^{HO}(\tau) = V_\star^{SP}(\tau) + \delta^h \ln \tilde{\omega}_\star(\tau) + (1 + \beta) \ln \{ \phi_\star(\tau) - \bar{\gamma}^h + \bar{\delta}^h [1 - \phi_\star(\tau)] \} \\ - (1 + \beta + \delta^h) \ln (\phi_\star(\tau) - \bar{\gamma}^h) + cst$$

¹⁴Given that $d_\star^h = \beta R_\star c_\star^h$ and $\Omega_\star^h = c_\star^h + d_\star^h / R_\star$, the long run welfare of a dynasty h can be rewritten as $V_\star^h = (1 + \beta) \ln \Omega_\star^h + \beta \ln R_\star + \delta^h \ln x_\star^h + cst$ where $\Omega_\star^h(\tau) = \tilde{\omega}_\star(\tau) + [1 - \phi_\star(\tau)] x_\star^h(\tau)$.

The relation between $V_{\star}^{HO}(\tau)$ and τ is also ambiguous. However, when the preference for wealth is weak enough and if there is a lot of overaccumulation, a PAYG pension system increases the welfare of the hoarders. Henceforth, in overaccumulation, a PAYG pension system may be Pareto improving. After all, this is not a surprising result.

In the case of a three-class equilibrium the welfare incidence is easier as the stock of capital k_{\star}^M , and thus R_{\star}^M , don't depend on τ . Thus, both the welfare of the spenders and the hoarders can be rewritten as:

$$V_{\star}^{SP}(\tau) = (1+\beta) \ln[w_{\star}^M - (1-\gamma^M)\tau] + cst \quad \text{and} \quad V_{\star}^{HO}(\tau) = (1+\beta+\delta^h) \ln[w_{\star}^M - (1-\gamma^M)\tau] + cst$$

Hence, one clearly sees that τ has a depressive effect on the welfare of these two types of dynasties. Turning to the savers, we have:

$$V_{\star}^{SA}(\tau) = \ln[w_{\star}^M + (1-\gamma^M)(x_{\star}^M(\tau) - \tau)] + cst$$

Since $x_{\star}^M(\tau) - \tau - x_{\star}^M(0) = \tau[\vartheta(\gamma^M)/p^M - 1] > 0$, one clearly sees that τ has a positive effect on the welfare of the savers.

We find here an extension of the key result of Michel and Pestieau (1998) and Mankiw (2000) according to whom a PAYG scheme improves the welfare of the savers while decreasing the welfare of the spenders. Their result resists to the introduction of hoarders but their are gloss over by the fact introducing a PAYG pension system increases the welfare of the savers but decreases the one of the hoarders; even if the hoarders has an infinitesimal preference for wealth and consequently if the hoarders is quasi similar to the savers.

4.2 Public debt

We now turn to the standard question of whether or not debt policy can be steady state welfare improving. In each period, the government faces the budget constraint $B_t = (1+r_t)B_{t-1} - L_t T_t$, where B_t is the total level of debt in t and T_t is a lump-sum tax paid by the working generation. We assume that the debt was used at time 0 to the benefits of the retirees. There is no other government spending. We write $b_t = B_{t-1}/N_t$ and assume that $b_t = b$ is constant. This yields $T_t = (r_t - n)b$.

With this public debt scheme, only two equations are changed. The first period budget constraint (1) is now:

$$w_t + x_t^h - T_t = c_t^h + s_t^h \quad (21)$$

and the relation linking capital and savings (8) becomes:

$$(1+n)(k_{t+1} + b) = \sum_{h=1}^{h=N} p^h s_t^h \quad (22)$$

We begin our study of the incidence of public debt in our economy with a fraction of hoarders by focusing focus both on the savings and the bequests behavior of each

types of individuals. Obviously, the spenders do not leave bequest. Then, after calculus and using the first period income net of tax b , $\tilde{\omega}_*(b) = w_*(b) - [R_*(b) - (1+n)]b$, their savings and their bequests satisfy:

$$\forall h < M \quad s_*^h(b) = \frac{\beta}{1+\beta} \tilde{\omega}_*(b) \quad \text{and} \quad x_*^h(b) = 0 \quad (23)$$

Concerning the hoarders, they always bequeath and the equilibrium conditions (9) always holds. Thus, after some computations, their bequests and their bequest is such that:

$$\forall h > M \quad s_*^h(b) = \frac{\zeta_*^h(b) \tilde{\omega}_*(b)}{1+\beta} \quad \text{and} \quad x_*^h(b) = \frac{\bar{\delta}^h \tilde{\omega}_*(b)}{\phi_*(b) - \bar{\gamma}^h} \quad (24)$$

where $\zeta_*^h(\cdot)$ is defined in equation (18).

Finally, the strong altruists are savers if $x_*^M(b)$ is positive and are spenders if $x_*^M(b)$ is nil. We now distinguish these two cases to see how the public debt affects the capital accumulation.

When strong altruists are spenders, using the equilibrium condition (22) we obtain after some calculus:

$$F[\phi_*(b)] = -\frac{(1+n)b}{\phi_*(b)w_*(b)} \vartheta[\phi_*(b)] \quad (25)$$

where $\vartheta[\phi_*(\cdot)]$ is defined in equation (19). Note that, when $b = 0$, we recoup (14).

When strong altruists are savers, the economy is at the MGR capital stock k_*^M . Indeed, (12) holds because the first order conditions (3) and (4) are not modified by the public debt scheme. Then, the equation of capital accumulation (22) allows us to obtain, after some tedious computations, the bequests of the savers:

$$x_*^M(b) = F(\gamma^M) \times \frac{w_*^M}{p^M} + \frac{(1+n)b}{\gamma^M p^M} \times \vartheta(\gamma^M) \quad (26)$$

Note that, when $b = 0$, we recoup (15).

From equations (25) and (26) we can now study the impact of the public debt both on the long run capital accumulation of the economy, the redistribution across the dynasties and the welfare of each dynasties.

Proposition 4 THE EFFECTS OF A PUBLIC DEBT.

a – At the spenders-hoarders equilibrium, the public debt always reduces the accumulation of capital. When the stationary capital stock without debt is in under accumulation of capital, then both the savings of the spenders and the bequests of the hoarders are reduced by the public debt. However, it is possible that both the savings of the spenders and the bequests of the hoarders are augmented when the stationary capital stock without debt is in over accumulation of capital. Concerning the savings of the hoarders, the impact of the public debt is ambiguous. Importantly, if the economy

without debt is dynamically inefficient, it exists a constant public debt policy b^G which restore the dynamic efficiency by leading the economy at the GR equilibrium.

The introduction of the public debt can improve or worsen both the welfare of the spenders and the one of the hoarders.

b – At the savers-spenders-hoarders equilibrium, the public debt has no effects on the capital accumulation. The decrease in the saving both of the spenders and the hoarders is compensated by an increase of the savings of the savers. If the bequest of the spenders remains nil, the introduction of the public debt reduces the one of the hoarders but increases the one of the savers. This introduction increases the share of wealth (capital plus bonds) held by the savers but decreases the shares of wealth held respectively by the spenders and the hoarders.

The introduction of the public debt improves the welfare of the savers but it worsens the welfare both of the spenders and the hoarders.

PROOF – See Appendix D. \square

As in the case of a PAYG pension system, the public debt can make the economy shift from a two-class equilibrium to a three-class equilibrium, the reverse being impossible. The reason is simple: a three-class equilibrium occurs if and only if $b > b^M \equiv -F(\gamma^M)w^M\gamma^M/[(1+n)\vartheta(\gamma^M)]$. Then, if without debt the equilibrium is one with three classes (case where $F(\gamma^M) > 0$), it remains so with debt and whatever the level of the debt. On the contrary if, without debt, the equilibrium is one with two classes (case where $F(\gamma^M) < 0$), then there exists a level of debt b^M above which public borrowing leads to a three-class equilibrium. This change of regime is made possible because in a two-class equilibrium public borrowing reduces steady-state capital accumulation. This reduction can be such that the steady-state capital stock converges to the MGR, which implies a three-class equilibrium.

At the two-class equilibrium, contrary to the PAYG pension system case, the saving of the spenders is not always reduced by public borrowing because it increases with $\tilde{\omega}_*(b)$. Then, as capital stock is reduced (i.e. $k_*(0) > k_*(b)$), saving by the spenders is always reduced by the public debt when $R_*(b) > 1+n$, i.e. when $k_*(b)$ is below the GR level of capital. However, when $k_*(b)$ is above the GR level saving by the spenders is reduced (resp: increased) by public borrowing if $W_*(b)$ is larger (resp: lower) than one where $W_*(b) = w_*(0)/\tilde{\omega}_*(b)$. According to (24), the bequest of the hoarders is always reduced by the public debt when $k_*(b)$ (and consequently $k_*(0)$) is in under-accumulation of capital. However, when $W_*(b) < 1$ (in this case, the capital stock $k_*(0)$ is necessarily in over-accumulation) the bequest of the hoarders is reduced (resp: augmented) by the public debt if $W_*(b)$ is larger (resp: lower) than $\zeta_*^h(b) = (\phi_*(0) - \bar{\gamma}^h)/(\phi_*(b) - \bar{\gamma}^h) > 1$. Since $\zeta_*^h(b) > 0$, we have $\zeta_*^h(b) > \zeta_*^h(0)$. Then, the saving of the hoarders is augmented by the public debt when $W_*(b) < 1$. When $W_*(b) > 1$ this saving is augmented (resp: reduced) according to $\zeta_*^h(b)$ is larger (resp:) than $\zeta_*^h(0)W_*(b)$.

To sum up the saving behavior that follows public borrowing in the two-class equilibrium, we distinguish among three cases. When $W_*(b) > \zeta_*^h(b)/\zeta_*^h(0) > 1$, public

borrowing reduces saving by both the spenders and the hoarders. When $\zeta_\star^h(b)/\zeta_\star^h(0) > W_\star(b) > 1$, saving by the spenders decreases and that by the hoarders increases following the introduction of the public debt. When $W_\star(b) < 1$, both savings increase. Note that this latter case is only operative if the equilibrium without debt is below the GR. This case is surprising and impossible with PAYG pension systems. The reason is that aggregate private saving is not, contrary to the PAYG pension system case, equal to just the stock of capital, but to the stock of capital plus bonds. Thus in case (iii), $k_\star(b) + b$ increases with respect to b even if $k_\star(b)$ decreases.

From an initial situation of overaccumulation, we show that there exists a level of debt $b^G \in (0, b^M)$ that leads to the GR capital stock. We thus find for this heterogeneous society the result obtained by Diamond (1965) for a society consisting only of spenders: public debt can lead to a Pareto optimal growth path. In such a society consisting only of spenders, if at the outset the economy is in under accumulation, the public debt is welfare worsening in the steady-state. With heterogeneous agents, this negative effect is mitigated because when b reaches b^M the economy switches to a three-class equilibrium in which the stock of capital corresponds to the MGR and is invariant to the public debt.

Public debt has no macroeconomic effect on the three-class equilibrium. This neutrality result at the aggregate level strengthens the intuition that just one saver is enough to obtain Ricardian equivalence. Michel and Pestieau (1998) and Mankiw (2000) show that this result keeps holding with the introduction of spenders. Here we show that it resists to the further introduction of hoarders.

As in the case of PAYG pension systems, if the public debt is neutral in aggregate terms, it modifies wealth distribution because the saving of our three classes of individuals change. Saving by the spenders and the hoarders is reduced and that by the savers is increased. Saving by the hoarders and the spenders does not react equally to public borrowing because of bequests. Public debt increases bequests of the savers while reducing bequests of the hoarders. Hoarders leave a bequest that is a proportion of the income $\tilde{\omega}_\star^M(b) = w_\star^M - (1+n)(1/\gamma^M - 1)b$ which declines as b increases. Things are different for the savers; their bequests increase with b as $x_\star^M(b) = x_\star^M(0) + (1+n)b\vartheta(\gamma^M)/(\gamma^M p^M)$. However, the sum of all the bequests is an increasing function of b in the savers-spenders-hoarders equilibrium. Hence, in contrast with the PAYG pension system case, the savers necessarily increase their bequests by an amount higher than what is necessary to compensate the decrease of bequests by the hoarders when b increases.

The fact that, contrary to the capital stock, wealth distribution is modified is already a result already obtained for PAYG pension systems. The share of capital held by the savers increases with b whereas the share held both by the spenders and the hoarders decreases. The direction of redistribution between the hoarders and the savers is ambiguous; it depends on the proportion of hoarders in society. Consequently, if the government wants to hurt the hoarders (i.e., the top wealthy) and to favor the savers, it can use a PAYG pension or a public debt.

Concerning individual welfare¹⁵, as we now show, the distinction between two types of equilibrium is going to provide for the effect of the public debt results quite similar to those obtained for the effect of a PAYG pension system. At the spenders-hoarders equilibrium, we have for the welfare of the spenders:

$$V_{\star}^{SP}(b) = (1 + \beta) \ln \tilde{\omega}_{\star}(b) + \beta \ln R_{\star}(b) + cst$$

As $R_{\star}(b)$ increases and $w_{\star}(b)$ decreases with b , the variations of $V_{\star}^{SP}(b)$ can be ambiguous. For low variations of b , $\tilde{\omega}_{\star}(b)$ and $R_{\star}(b)$ vary in opposite directions. As $w'_{\star}(b) - R'_{\star}(b)$ is negative, remark that $\tilde{\omega}_{\star}(b)$ decreases with b when $\phi_{\star}(b) < 1$. On the contrary, $\tilde{\omega}_{\star}(b)$ increases with b in case of overaccumulation and thus the public debt improves the welfare of the spenders. The welfare of the hoarders is given by:

$$\begin{aligned} V_{\star}^{HO}(b) &= V_{\star}^{SP}(b) + \delta^h \ln \tilde{\omega}_{\star}(b) + (1 + \beta) \ln[\phi_{\star}(b) - \bar{\gamma}^h + \bar{\delta}^h(1 - \phi_{\star}(b))] \\ &\quad - (1 + \beta + \delta^h) \ln(\phi_{\star}(b) - \bar{\gamma}^h) + cst \end{aligned}$$

The relation between $V_{\star}^{HO}(b)$ and b are also ambiguous. However with weak preference for wealth and important overaccumulation introducing public debt increases the welfare of the hoarders. In that case, public debt is Pareto improving.

Moving to the three-class equilibrium, one writes the welfare of spenders as:

$$V_{\star}^{SP}(b) = (1 + \beta) \ln[w_{\star}^M - (1/\gamma^M - 1)(1 + n)b] + cst$$

As to the hoarders, their welfare is:

$$V_{\star}^{HO}(b) = (1 + \beta + \delta^h) \ln[w_{\star}^M - (1/\gamma^M - 1)(1 + n)b] + cst$$

Thus, the higher public debt, the lower the welfare of both the spenders and the hoarders is. Turning to the savers, we have:

$$V_{\star}^{SA}(b) = \ln[w_{\star}^M + (1 - \gamma^M)(x_{\star}^M(b) - \frac{b(1+n)}{\gamma^M})] + cst$$

Since $x_{\star}^M(b) - b(1+n)/\gamma^M - x_{\star}^M(0) = b(1+n)[\vartheta(\gamma^M)/p^M - 1]/\gamma^M > 0$ and $\vartheta(\gamma^M) > 1$, introducing public borrowing increases the welfare of the savers.

To sum up, we have the same type of results as for the PAYG pension. Unconstrained altruists benefit from national debt whereas both the hoarders and the spenders welfare decreases.

¹⁵Given that $d_{\star}^h = \beta R_{\star} c_{\star}^h$ and $\Omega_{\star}^h = c_{\star}^h + d_{\star}^h/R_{\star}$, the long run welfare of a dynasty h can be rewritten as $V_{\star}^h = (1 + \beta) \ln \Omega_{\star}^h + \beta \ln R_{\star} + \delta^h \ln x_{\star}^h + cst$ where $\Omega_{\star}^h(b) = \tilde{\omega}_{\star}(b) + [1 - \phi_{\star}(b)]x_{\star}^h(b)$.

4.3 Estate taxation

We now turn to a third instrument that is natural in a setting where bequests play such an important role: inheritance or estate taxation. Again, we focus on the steady state solution. The tax scheme is simple: an estate tax of fixed rate $\kappa \in [0, 1)$ that is redistributed in each period t in a lump-sum way in an amount θ_t , the same for all. Hence, the revenue constraint is simply equivalent to:

$$\theta_t = \kappa \sum_{h=1}^N p^h x_t^h$$

Then, the first budget constraint (1) of an agent of dynasty h becomes:

$$w_t + (1 - \kappa)x_t^h + \theta_t = c_t^h + s_t^h \quad (27)$$

Moreover, given θ_t , the optimal condition for saving (3) is unchanged but that for bequests both of the savers and the hoarders (4) and (5) are now distorted:

$$\forall h \leq M \quad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\gamma^h(1-\kappa)}{c_{t+1}^h} \leq 0 \quad (= \text{if } x_{t+1}^h > 0) \quad (28)$$

$$\forall h > M \quad -\frac{(1+n)\beta}{d_{t+1}^h} + \frac{\delta^h}{x_{t+1}^h} + \frac{\gamma^h(1-\kappa)}{c_{t+1}^h} = 0 \quad (29)$$

We now want to see what is the incidence of estate taxation in our economy with a fraction of hoarders. In particular, we focus both on the savings and the bequests behavior of each types of individuals. Obviously, the spenders do not leave bequest. Then, after calculus and using the notation $\tilde{\omega}_*(\kappa) = w_*(\kappa) + \theta_*(\kappa)$, their savings and their bequests satisfy:

$$\forall h < M \quad s_*^h(\kappa) = \frac{\beta}{1+\beta} \tilde{\omega}_*(\kappa) \quad \text{and} \quad x_*^h(\kappa) = 0 \quad (30)$$

Concerning the hoarders, they always bequeath and (9) always holds. Thus, after computations, their savings and their bequests are such that:

$$\forall h > M \quad s_*^h(\kappa) = \frac{\hat{\zeta}_*^h(\kappa) \tilde{\omega}_*(\kappa)}{1+\beta} \quad \text{and} \quad x_*^h(\kappa) = \frac{\bar{\delta}^h \tilde{\omega}_*(\kappa)}{\phi_*(\kappa) - \bar{\gamma}^h(1-\kappa)} \quad (31)$$

where $\hat{\zeta}_*^h(\kappa) = \bar{\delta}^h[\beta(1-\kappa) + \phi_*(\kappa)]/[\phi_*(\kappa) - \bar{\gamma}^h(1-\kappa)] + \beta$. Note that, larger is the redistribution θ_* , larger is the bequest of the hoarders whereas larger is the tax rate κ , lower are the bequests of the hoarders.

Finally, the strong altruists are savers if $x_*^M(\kappa)$ is positive and are spenders if $x_*^M(\kappa)$ is nil. We now distinguish these two cases to see how the estate taxation affects the capital accumulation.

When the strong altruists are spenders, we can use the equilibrium condition $(1 + n)k_*(\kappa) = \sum_{h=1}^M p^h s_*^h(\kappa)$ to obtain after some calculus:

$$\Lambda(\phi_*(\kappa), \kappa) \equiv \frac{\beta(\phi_*(\kappa) - \varepsilon)}{\varepsilon(\phi_*(\kappa) + \beta) + \kappa\beta(\phi_*(\kappa) - \varepsilon)} - \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{\phi_*(\kappa) - \bar{\gamma}^h(1 - \kappa)} = 0 \quad (32)$$

When $\kappa = 0$, we recoup (14) since $\Lambda(\phi_*(0), 0) = F(\phi_*(0))$.

When the strong altruists are savers, according to (28) we have $\phi_*(\kappa) = \gamma^M(1 - \kappa)$. Then, contrary to the public debt scheme or the PAYG pension system, an estate tax modifies the stock of capital of the savers-spenders-hoarders equilibrium. Indeed, the MGR capital stock is affected by the estate taxation since we have in long run:

$$k_*^M(\kappa) = \left[\frac{\alpha A \gamma^M (1 - \kappa)}{1 + n} \right]^{\frac{1}{1-\alpha}}$$

Then, larger is the tax rate κ , lower is the long run capital stock $k_*^M(\kappa)$. We can remark that this capital stock does not depend on the proportion of the savers, the spenders or the hoarders. In the no tax case, this capital stock is obviously the one of the MGR. Using the fact that $\phi_*(\kappa) = \gamma^M(1 - \kappa)$, equation (8) allows us to obtain, after some tedious computations, the bequests of the savers:

$$x_*^M(\kappa) = \left\{ \frac{\beta}{\varepsilon} \left[\frac{\gamma^M(1 - \kappa) - \varepsilon}{\gamma^M + \beta + \kappa\gamma^M} \right] - \left[\frac{\gamma^M + \beta + \kappa\beta\gamma^M/\varepsilon}{\gamma^M + \beta + \kappa\gamma^M} \right] \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{\gamma^M - \bar{\gamma}^h} \right\} \frac{w_*^M(\kappa)}{p^M} \quad (33)$$

where $w_*^M(\kappa) = A(1 - \alpha)k_*^M(\kappa)^\alpha$.

From equations (32) and (33) we can now study the impact of an estate taxation both on the existence of the steady states, the long run capital accumulation and the redistribution across the dynasties. Given the complexity of the problem at hand, we make two simplifications. First we take a tax reform viewpoint by focusing on an infinitesimal change in the tax rate at a zero level. Second we assume that $N = M + 1$; in other words, there is only one dynasty of hoarders.

Proposition 5 THE EFFECTS OF THE ESTATE TAXATION.

(a) *At the spenders-hoarders equilibrium, with only one dynasty of hoarders, the estate tax reduces the capital accumulation. It also depresses the bequest of the hoarders. Its impact on savings is ambiguous. However, when the introduction of estate taxation increases (decreases) the savings of the hoarders (spenders), it also increases (decreases) the savings of the spenders (hoarders).*

The introduction of the estate taxation can improve or worsen the welfare of both the spenders and the hoarders. Note that estate taxation improves (worsens) the welfare of the hoarders (spenders), it improves (worsens) also that of the spenders (hoarders).

(b) *At the savers-spenders-hoarders equilibrium, estate taxation reduces the capital accumulation. It reduces the savings of all the agents. It also depresses the bequest of the savers but increases (decreases) that of the hoarders if the degree of altruism of the savers is sufficiently high (low). Moreover, the introduction of estate taxation decreases the share of wealth held by the savers but increases that held by the spenders and the hoarders.*

The introduction of estate taxation worsens the welfare both of the spenders and the savers but increases (decreases) that of the hoarders if the degree of altruism of the savers or their preference for wealth are sufficiently high (low).

PROOF – See Appendix E. \square

According to (33) we have a three-class equilibrium if and only if $\kappa < \kappa^M \equiv \varepsilon(\gamma^M + \beta)\Lambda(\gamma^M, 0)/[1 + \sum_{h=M+1}^N p^h \delta^h / (\gamma^M - \bar{\gamma}^h)]$. Thus if, in the absence of estate tax, we have a two-class equilibrium $F(\gamma^M) = \Lambda(\gamma^M, 0) < 0$, we keep this type of equilibrium with estate taxation. On the contrary, if we have a savers-spenders-hoarders equilibrium without estate taxation $F(\gamma^M) = \Lambda(\gamma^M, 0) > 0$, there is a positive level of taxation κ^M above which the equilibrium becomes a spenders-hoarders equilibrium. Hence, introducing an estate tax can lead to go from a three-class equilibrium to a two-class one; the other way around is not possible. Estate taxation has thus the opposite effect relative to public debt and PAYG pension.

What is the intuition of such a switch of regime? Public debt or PAYG pension system induces the strong altruists to increase their bequests and hence reinforces the portion of savers and may lead to a switch of regime from two- to three-classes. Estate taxation discourages bequeathing by the strong altruists and may lead to the disappearance of savers. Consequently, the only possible switch is that from three- to two-classes.

We now analyze the behavior of different agents in the two types of equilibrium. In the spenders-hoarders equilibrium, according to (30), saving by the spenders depends on $\tilde{\omega}_*(\kappa)$. Then, according to Appendix E, there exists two opposite effects. The first one follows from the decrease of capital accumulation triggered by estate taxation; the second one is redistributive and follows from the lump-sum transfer financed by estate taxation. Concerning saving by the hoarders, beyond the two effects just mentioned, there is a third one due to changes in $\hat{\zeta}_*^N(\kappa)$. This third effect is negative. Hence if estate taxation increases saving by the hoarders, then it also increases saving by the spenders. Conversely, if estate taxation depresses saving by the spenders, it also depresses saving by the hoarders. Turning to the level of bequests by the hoarders (one single dynasty) it can be expressed as $x_*^N(\kappa) = \bar{\delta}^N w_*(\kappa) / \{\phi_*(\kappa) - \bar{\gamma}^N + \kappa(\bar{\gamma}^N - p^N \bar{\delta}^N)\}$. As the numerator decreases and the denominator increases with κ , the estate tax has a depressive effect on the bequests of the hoarders.

In the three-class equilibrium, estate taxation has quite different effects. First of all, let us remember that it has a depressive incidence on capital accumulation. This is a result that is consistent with that obtained by Mankiw (2000) and Michel and Pestieau (1998). As in the two-class equilibrium, saving varies with disposable income

$\tilde{\omega}_*(\kappa)$. However, $\tilde{\omega}_*(\kappa)$ decreases with κ when the rate of estate taxation is low and $\hat{\zeta}_*^h(\kappa) = \beta + \bar{\delta}^h(\beta + \gamma^M)/(\gamma^M - \bar{\gamma}^h)$ does not depend on κ . Thus, according to (30) and (31), savings of both spenders and hoarders decrease when κ decreases, contrary to what is happening in the two-class equilibrium.

As to the influence of κ on saving by the savers, consider first the effect of κ on their bequests described by (33). The higher the estate tax rate, the lower the bequests by the savers are. Since $(1 + \beta)s^M(\kappa) = \beta\tilde{\omega}_*(\kappa) + (1 - \kappa)(\gamma^M + \beta)x_*^M(\kappa)$, this negative effect of taxation on bequests has an impact on saving. The first term of the RHS decreases with κ (for low κ) and the second term always decreases with κ . Consequently, estate taxation depresses saving by the savers.

To sum up, for low tax rates, saving by the three types of agents decreases and capital accumulation goes down unambiguously. This is in contrast with the neutral effect of either public debt or unfunded pensions.

Turning to the bequests of the hoarders, according to (31), there exists two opposite effects when κ increases and calculations of Appendix E lead to:

$$\forall h > M \quad \left. \frac{\partial x_*^h(\kappa)}{\partial \kappa} \right|_{\kappa=0} \geq 0 \quad \text{if and only if} \quad \gamma^M \geq \frac{\alpha\beta}{\alpha\beta + 1 - \alpha}.$$

Thus contrary to the bequests of the savers, the bequests of the hoarders don't necessarily decrease as a result of estate taxation. The sign of the variation of hoarders' bequests depend on savers' characteristics (degree of altruism) and not on their own characteristics (degree of altruism and preference for wealth). This is typically a general equilibrium result. The key economic variable is disposable income of the hoarders in the second period: $R_*(\kappa)s_*^h(\kappa)$. We know that for the hoarders, $s_*^h(\kappa)$ decreases for low value of κ and that $R_*(\kappa) = (1 + n)/[\gamma^M(1 - \kappa)]$ increases with κ . We show that for low κ , $x_*^h(0)$ and $R'_*(0)s_*^h(0) + R_*(0)s_*^{h'}(0)$ have the same sign. The lower saving by the hoarders is more than compensated by the increase of the interest factor. This increase is particularly important when the degree of altruism of the savers is high enough.

This result is surprising in several respects. First, it does not concern the savers whose bequests always decrease. It applies to the hoarders even if they have a very low degree of altruism. Altruism and preference for wealth determine the level of bequests of the hoarders, but not how these bequests react to estate taxation. Note that all dynasties of hoarders behave identically as to an increase or a decrease of their bequests. This is quite different from what we observe in the two-class equilibrium where bequests are always negatively influenced by estate taxation.

Estate taxation affects also wealth distribution in the three-class equilibrium. It increases the share of capital held by the spenders and the hoarders at the expense of the share held by the savers. If the government wants to hurt the savers, it can introduce estate taxation. Estate taxation is clearly a questionable instrument of redistribution: it hurts the wealthy, but favors the top wealthy.

We now turn to the incidence of estate taxation on welfare¹⁶ in the two kinds of equilibrium. Starting with the two-class equilibrium, the welfare of the spenders is:

$$V_{\star}^{SP}(\kappa) = (1 + \beta) \ln \tilde{\omega}_{\star}(\kappa) + \beta \ln R_{\star}(\kappa) + cst$$

As $R_{\star}(\kappa)$ increases with κ and the sign of $\tilde{\omega}'_{\star}(\kappa)$ is ambiguous, the reaction of welfare of the spenders when κ varies is also ambiguous even for low values of κ . As to the welfare of the hoarders, we have:

$$V_{\star}^{HO}(\kappa) = V_{\star}^{SP}(\kappa) + (1 + \beta) \ln \hat{\zeta}_{\star}(\kappa) + \delta^N \ln x_{\star}^{HO}(\kappa) + cst$$

The effect of estate taxation here is also ambiguous even though we know that both $\hat{\zeta}_{\star}(\kappa)$ and $x_{\star}^{HO}(\kappa)$ decreases with κ when κ is small.

In the three-class equilibrium, the welfare of the spenders is:

$$V_{\star}^{SP}(\kappa) = (1 + \beta) \ln \tilde{\omega}_{\star}(\kappa) + \beta \ln R_{\star}(\kappa) + cst$$

where $\tilde{\omega}_{\star}(\kappa)$ decreases with κ whereas $R_{\star}(\kappa)$ increases with low κ . According to Appendix E, this last effect is always dominated by the first effect at the capital stock $k_{\star}^M(\kappa)$. Then, a (low) increase of (low) κ worsens the welfare of the spenders. Turning to the savers, we have:

$$V_{\star}^{SA}(\kappa) = (1 + \beta) \ln \Omega_{\star}^{SA}(\kappa) + \beta \ln R_{\star}(\kappa) + cst$$

and we can show that a (low) increase of (low) κ worsens the welfare of the savers. Note that the decrease of the welfare of the savers is larger than the decrease of the welfare of the spenders. We find here one of the main results of Michel and Pestieau (1998): estate taxation worsens the welfare both of the spenders and the savers. Do we find the same result for the hoarders? From (31) their welfare can be written as:

$$V_{\star}^{HO}(\kappa) = (1 + \beta + \delta^h) \ln \tilde{\omega}_{\star}^h(\kappa) + \beta \ln R_{\star}(\kappa) - \delta^h \ln(1 - \kappa) + cst$$

and we can show that:

$$\left. \frac{\partial V_{\star}^{HO}(\kappa)}{\partial \kappa} \right|_{\kappa=0} \geq 0 \quad \text{if and only if} \quad \gamma^M \geq \frac{\beta[1 + \alpha(\beta + \delta^h)]}{\beta[1 + \alpha(\beta + \delta^h)] + \delta^h(1 - \alpha)}$$

Thus if the altruism of the savers is sufficiently strong, estate taxation can have a positive effect on the hoarders, namely on bequeathers who expectedly should be penalized. This surprising result can be explained by the general equilibrium mechanism described previously that leads to increased bequests by the hoarders. However, note that the degree of altruism of the savers that is needed to increase the welfare of the hoarders is higher than that needed to increase their bequests. These increases occur when the reduction of saving by the hoarders is more than compensated by the increase in interest rate. We have seen that an increase in bequests by the hoarders does not

¹⁶Given that $d_{\star}^h = \beta R_{\star} c_{\star}^h$ and $\Omega_{\star}^h = c_{\star}^h + d_{\star}^h / R_{\star}$, the long run welfare of a dynasty h can be rewritten as $V_{\star}^h = (1 + \beta) \ln \Omega_{\star}^h + \beta \ln R_{\star} + \delta^h \ln x_{\star}^h + cst$ where $\Omega_{\star}^h(\kappa) = \tilde{\omega}_{\star}(\kappa) + [1 - \kappa - \phi_{\star}(\kappa)]x_{\star}^h(\kappa)$.

depend on their altruism nor on their preference for wealth. The increase in utility depends (positively) on δ^h .

To sum up, we have found a new reason to deal with estate taxation with caution. If the objective of such a tax is to fight top wealth holding, we have shown that society might be better off without it.

5 Conclusion

Traditional macroeconomic models rest on the assumption that agents are either altruistic or not and look at the shape of wealth distribution and at the effect of alternative fiscal policies on both capital accumulation and wealth distribution. Though very insightful these models fail to reflect some real life features, particularly the fact that wealth is not predominantly held by altruistic agents. Empirical studies point out to the fact that wealth accumulation, specially top wealth accumulation, is not motivated by the presence of children but by some type of preference for wealth or for the power and the prestige that wealth conveys. To incorporate this relevant and important fact, this paper considers agents who are characterized not only by some degree of altruism, but also by some preference for wealth. It appears that with this double heterogeneity results obtained with the sole difference in altruism don't hold true (see Table 1, 2, 3 and 4). One does not obtain necessarily the MGR; in fact, overaccumulation can occur. Also, we don't have necessarily the neutrality of either public borrowing or unfunded pensions. We also show that estate tax depresses both the capital accumulation and the bequest of the altruists with no preference for wealth but not necessarily the one of the altruists with preference for wealth.

Above all our paper helps to understand why in reality top wealth is not held by the most altruistic individuals. Introducing preference for wealth allows for distinguishing two types of equilibrium, with and without the MGR prevailing. In the equilibrium with MGR, the most altruists impose their own rate of time preference, but they don't necessarily hold the bulk of wealth. Thus, we explain the top tail of the distribution of estates by not only altruism, as usually done, but mainly by some dynastic taste for wealth. Our finding is in the line of recent papers of Carroll (2000), De Nardi (2004) or Reiter (2004) who explain the top tail of wealth distribution in the US and/or Sweden economy by a capitalist spirit motive according to which capital provides utility services directly, but not just through consumption. This is what we do here with our preference for wealth. Then, our paper proposed an alternative modeling of dynamic wealth distribution that is generally dealt with using calibrated versions of stochastic growth models or using theoretical models with imperfect credit market.

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Appendix

APPENDIX A – PROOF OF PROPOSITION 1.

Step 1: Existence and uniqueness of the steady state.

Since $\bar{\gamma}^N < \phi(R_*)$ and according to (12) and (14), we have a spenders-hoarders society if and only if there exists $\phi(R_*) > \max\{\bar{\gamma}^N, \gamma^M\}$ such that $F(\phi(R_*)) = 0$. As we have:

$$F'(\phi(R)) = \frac{\beta(\beta + \varepsilon)}{\varepsilon(\phi(R) + \beta)^2} + \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{(\phi(R) - \bar{\gamma}^h)^2} > 0$$

the function $F(\phi(R))$ increases from $-\infty$ to β/ε when $\phi(R)$ increases from $\bar{\gamma}^N$ to $+\infty$. Then, there exists a unique solution $\phi(R_*) > \bar{\gamma}^N$ of the equation $F(\phi(R_*)) = 0$. Consequently, we have a spenders-hoarders society if and only if $\phi(R_*) > \gamma^M$ or equivalently if and only if $F(\gamma^M) \leq 0$. When $\bar{\gamma}^N > \gamma^M$ we have $F(\gamma^M) \leq 0$. Conversely, when $\bar{\gamma}^N < \gamma^M$, $F(\gamma^M) \leq 0$ if and only if $\sum_{h=M+1}^N [p^h \bar{\delta}^h / (\gamma^M - \bar{\gamma}^h)] \geq \beta(\gamma^M - \varepsilon) / [\varepsilon(\beta + \gamma^M)]$.

According to (15), we have a savers-spenders-hoarders society if and only if $x_*^M > 0$ i.e., if and only if $F(\gamma^M) > 0$. Using the definition of $F(\cdot)$ we obtain that $F(\gamma^M) > 0$ if and only if $\bar{\gamma}^N < \gamma^M$ and $\sum_{h=M+1}^N [p^h \bar{\delta}^h / (\gamma^M - \bar{\gamma}^h)] > \beta(\gamma^M - \varepsilon) / [\varepsilon(\beta + \gamma^M)]$.

Finally, the existence of equilibrium is always guaranteed by the fact that we have necessarily $F(\gamma^M) \leq 0$ (Equilibrium I) or $F(\gamma^M) > 0$ (Equilibrium II). The uniqueness of the steady state comes from the fact that the conditions $F(\gamma^M) \leq 0$ and $F(\gamma^M) > 0$ cannot be satisfied at the same time.

Step 2: Steady state and degree of altruism of strong altruists.

According to Step 1, the steady state is a savers-spenders-hoarders equilibrium if and only if $\gamma^M > \bar{\gamma}^N$ and $F(\gamma^M) > 0$. As

$$F'(\gamma) = \frac{\beta(\beta + \varepsilon)}{\varepsilon(\gamma + \beta)^2} + \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{(\gamma - \bar{\gamma}^h)^2} > 0$$

we obtain this kind of equilibrium as soon as $\gamma^M > \tilde{\gamma}$ where $\tilde{\gamma} > \bar{\gamma}^N$ is the unique solution of $F(\tilde{\gamma}) = 0$. Since $\gamma^M > \tilde{\gamma}$ is a necessary and sufficient condition to obtain a savers-spenders-hoarders society, the condition $\gamma^M \leq \tilde{\gamma}$ is a necessary and sufficient condition to obtain a spenders-hoarders society. As F'_γ is positive and for all $h > M+1$, F'_{p^h} , $F'_{\bar{\delta}^h}$ and $F'_{\bar{\gamma}^h}$ are negative, the threshold $\tilde{\gamma}$ increases with the p^h , γ^h and δ^h of the dynasties of hoarders but is independent of the p^h and γ^h of the dynasties of spenders.

Step 3: The long run capital stock.

Using (6), (11) and (13) we can rewrite the capital accumulation equation (8).

a – At the spenders-hoarders equilibrium, according to Step 1 and (7), the stationary capital stock k_* is the unique solution of $F[(1+n)k_*^{1-\alpha}/(\alpha A)] = 0$ such that $(1 +$

$n)k_\star^{1-\alpha}/(\alpha A) > \bar{\gamma}^N$. Since F'_{k_\star} is positive and since for all $h > M + 1$, F'_{p^h} , $F'_{\bar{\delta}^h}$ and $F'_{\bar{\gamma}^h}$ are negative, the capital stock k_\star increases with the p^h , γ^h and δ^h of the dynasties of hoarders but is independent of the γ^h of the dynasties of spenders.

b – According to (12), if x_\star^M is positive then the steady state capital stock k_\star corresponds to the M.G.R. capital stock (i.e., $f'^{-1}[(1+n)/\gamma^M]$) and we have: $k_\star = [\alpha A \gamma^M / (1+n)]^{1/(1-\alpha)} \equiv k_\star^M$. Obviously, the higher γ^M , the higher is k_\star^M and neither the proportion of each dynasty, nor the degree of altruism of both the spenders and the hoarders affect k_\star^M .

Step 4: Dynamic efficiency of the capital stock.

a – By definition, the GR capital stock is $k^G = f'^{-1}(1+n)$. This implies that $\phi(R^G) = 1$. Then k_\star is below, equal or above the GR level of capital if respectively $\phi(R_\star)$ is smaller, equal to or larger than 1. As $F(\cdot)$ is an increasing function (see Step 1) and since $F(\phi(R_\star)) = 0$, k_\star is below, equal or above the GR level of capital accumulation if respectively $F(1)$ is larger, equal to or smaller than zero. Since:

$$F(1) = \frac{\beta(1-\varepsilon)}{\varepsilon(1+\beta)} - \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h}{1-\bar{\gamma}^h} = \frac{1}{1+\beta} \left[\frac{\beta(1-\varepsilon)}{\varepsilon} - \sum_{h=M+1}^N \frac{p^h \delta^h}{1-\gamma^h} \right]$$

k_\star is below, equal or above the GR capital if respectively $\sum_{h=M+1}^N p^h \delta^h / (1-\gamma^h)$ is smaller, equal to or larger than $\beta(1-\varepsilon)/\varepsilon$.

b – The GR capital stock is $k^G = f'^{-1}(1+n)$ and the MGR capital stock is $k_\star^M = f'^{-1}[(1+n)/\gamma^M]$. Since $f(\cdot)$ is concave and $\gamma^M \in (0, 1)$ we have $k_\star^M < k^G$. Then, k_\star^M is in under-accumulation of capital. \square

APPENDIX B – PROOF OF PROPOSITION 2.

a – At the spenders-hoarders equilibrium, using the fact that $\alpha\phi(R_\star)w_\star = (1-\alpha)(1+n)k_\star$ and since the saving of the hoarders is given by (11), we have:

$$\begin{aligned} \mu^{HO} &= \sum_{h=M+1}^N \frac{p^h s^h}{(1+n)k_\star} = \frac{\varepsilon}{\phi(R_\star)} \left[\sum_{h=M+1}^N p^h + \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h (\beta + \phi(R_\star))}{\beta(\phi(R_\star) - \bar{\gamma}^h)} \right] \quad (34) \\ &= \frac{\varepsilon}{\phi(R_\star)} \left[\sum_{h=M+1}^N p^h + \frac{\phi(R_\star) - \varepsilon}{\varepsilon} \right] = \frac{\varepsilon}{\phi(R_\star)} \sum_{h=M+1}^N p^h + 1 - \frac{\varepsilon}{\phi(R_\star)} \end{aligned}$$

Using the same methodology, from the saving of the spenders given by (13), we obtain:

$$\mu^{SP} = \sum_{h=1}^M \frac{p^h s^h}{(1+n)k_\star} = \frac{\varepsilon}{\phi(R_\star)} \sum_{h=1}^M p^h$$

According to variations in k_\star (see Step 3 of Appendix A), the higher the parameters p^h , γ^h , δ^h of a dynasty h of hoarders, the higher is μ^{HO} and the lower is μ^{SP} .

b – At the savers-spenders-hoarders equilibrium, we have $\phi(R_\star) = \gamma^M$. Then, the methodology used to obtain (34) allows us to obtain:

$$\nu^{HO} = \sum_{h=M+1}^N \frac{p^h s^h}{(1+n)k_\star} = \frac{\varepsilon}{\gamma^M} \sum_{h=M+1}^N p^h \left[1 + \frac{\bar{\delta}^h(\beta + \gamma^M)}{\beta(\gamma^M - \bar{\gamma}^h)} \right]$$

Using the fact that $\alpha\phi(R_\star)w_\star = (1-\alpha)(1+n)k_\star$ and since the saving of the spenders is given by (13), we have:

$$\nu^{SP} = \sum_{h=1}^{M-1} \frac{p^h s^h}{(1+n)k_\star} = \frac{\varepsilon}{\gamma^M} \sum_{h=1}^{M-1} p^h$$

Given $\nu^{HO} + \nu^{SP} + \nu^{SA} = 1$ we have:

$$\nu^{SA} = 1 - \nu^{HO} - \nu^{SP} = 1 - \frac{\varepsilon}{\gamma^M} \left[1 - p^M + \sum_{h=M+1}^N \frac{p^h \bar{\delta}^h(\beta + \gamma^M)}{\beta(\gamma^M - \bar{\gamma}^h)} \right]$$

The higher γ^M , the higher is ν^{SA} and the lower are ν^{HO} and ν^{SP} . The higher the p^h , γ^h , δ^h of a dynasty h of hoarders, the higher is ν^{HO} . \square

APPENDIX C – PROOF OF PROPOSITION 3.

Saving by a dynasty h in the PAYG pension system case is given by:

$$\forall h \quad s_\star^h(\tau) = \frac{\beta}{1+\beta} w_\star(\tau) + \frac{\beta + \phi_\star(\tau)}{1+\beta} (x_\star^h(\tau) - \tau) \quad (35)$$

a – We first prove that $k_\star(\tau) < k_\star(0)$ for all positive τ . According to (19) and since $\vartheta(\cdot) > 0$, the spenders-hoarders equilibrium satisfies:

$$F[\phi(R_\star(\tau))] = -\frac{\tau}{w_\star(\tau)} \vartheta[\phi(R_\star(\tau))] < 0 = F[\phi(R_\star(0))]$$

According to Step 1 of Appendix A, $F(\cdot)$ is an increasing function of $\phi(R)$. Then, we have $\phi[R_\star(\tau)] < \phi[R_\star(0)]$ and consequently $k_\star(\tau) < k_\star(0)$ for all positive τ .

We now prove that for sufficiently low τ , this capital stock $k_\star(\tau)$ decreases with τ . According to (19), $k_\star(\tau)$ is defined by:

$$\psi(k_\star, \tau) = F\left(\frac{1+n}{\alpha A} k_\star^{1-\alpha}\right) + \frac{\tau}{(1-\alpha)A k_\star^\alpha} \vartheta\left(\frac{1+n}{\alpha A} k_\star^{1-\alpha}\right) = 0$$

Using the Implicit Function Theorem we have $k_\star = k_\star(\tau)$ and $k'_\star(\tau) = -\psi'_\tau(\cdot)/\psi'_k(\cdot)$. Since $\psi'_\tau(\cdot) = \vartheta(\cdot)/[(1-\alpha)A k_\star^\alpha] > 0$, $k'_\star(\tau)$ has the sign of $-\psi'_k(\cdot)$. Then:

$$\begin{aligned} \text{sign of } k'_\star(\tau) &= \text{sign of} \left[-\frac{(1+n)(1-\alpha)}{\alpha A k_\star^\alpha} F'(\cdot) + \frac{\tau\alpha}{(1-\alpha)A k_\star^{1+\alpha}} \vartheta(\cdot) - \frac{(1+n)\tau}{\alpha A^2 k_\star^{2\alpha}} \vartheta'(\cdot) \right] \\ &= \text{sign of} \left[-\frac{(1+n)(1-\alpha)}{\alpha} F'(\cdot) - \tau \left[\frac{1+n}{\alpha A k_\star^\alpha} \vartheta'(\cdot) - \frac{\alpha}{(1-\alpha)k_\star} \vartheta(\cdot) \right] \right] \end{aligned}$$

Since, $F'(\cdot)$ and $\vartheta(\cdot)$ are positive and $\vartheta'(\phi(R)) = -\sum_{h=M+1}^N [p^h \bar{\delta}^h (1 - \bar{\gamma}^h) / (\phi(R) - \bar{\gamma}^h)^2]$ is negative, there exists a threshold $\underline{\tau} > 0$ such that $k'_*(\tau)$ is negative if τ is lower than $\underline{\tau}$.

We now prove that the PAYG pension system reduces the saving of spenders. Using (35), the saving of the spenders is given by (17). Since $k_*(0) > k_*(\tau)$, we have $w_*(0) > w_*(\tau)$ which implies $s_*^h(0) > s_*^h(\tau)$ for all positive τ .

Finally, we prove that the savings and the bequests of hoarders are not necessarily themselves augmented by τ . Indeed, according to (18), the bequest of hoarders can be rewritten as:

$$\forall h > M \quad x_*^h \equiv \tilde{x}^h(k(\tau), \tau) = \frac{\bar{\delta}^h [w(k(\tau)) - [1 - \phi(k(\tau))]\tau]}{\phi(k(\tau)) - \bar{\gamma}^h}$$

Then we have:

$$\forall h > M \quad \frac{\partial x_*^h}{\partial \tau} = \frac{\partial \tilde{x}^h(k(\tau), \tau)}{\partial k(\tau)} \frac{\partial k(\tau)}{\partial \tau} - \frac{\bar{\delta}^h (1 - \phi(k(\tau)))}{\phi(k(\tau)) - \bar{\gamma}^h} \quad (36)$$

One notes that $\partial \tilde{x}^h(k(\tau), \tau) / \partial k(\tau)$ has the sign of $w'(k(\tau))[\phi(k(\tau)) - \bar{\gamma}^h] - w(k(\tau)) + \tau \phi'(k(\tau))(1 - \bar{\gamma}^h)$. Since $\phi'(k(\tau))$ is positive, for sufficiently low values of τ we have:

$$\begin{aligned} \text{sign of } \left[\frac{\partial \tilde{x}^h(k(\tau), \tau)}{\partial k(\tau)} \right] &= \text{sign of } \left[w'(k(\tau))[\phi(k(\tau)) - \bar{\gamma}^h] - w(k(\tau)) \right] \\ &= \text{sign of } \left[(1+n)(2\alpha-1) - (1-\alpha)A\alpha^2\bar{\gamma}^h k(\tau)^{\alpha-1} \right] \\ &= \text{sign of } \left[(2\alpha-1)\phi(k(\tau)) - \alpha(1-\alpha)\bar{\gamma}^h \right] \end{aligned}$$

Then, when $\alpha \leq 1/2$, $\partial \tilde{x}^h(k(\tau), \tau) / \partial k(\tau)$ is negative for low values of τ . As $k'_*(\tau)$ is negative for $0 < \tau < \underline{\tau}$, according to (36) the bequest of hoarders $x_*^h(\tau)$ is augmented by a PAYG pension system when τ , α and $k_*(\tau)$ are sufficiently low.

From (35) we can study the saving of hoarders. Since $k_*(0) > k_*(\tau)$, we have $w_*(0) > w_*(\tau)$ and $\phi(R_*(0)) > \phi(R_*(\tau))$ which obviously imply $s_*^h(0) > s_*^h(\tau)$ when $x_*^h(\tau) < x_*^h(0)$. Then, when (36) is such that $\partial x_*^h / \partial \tau$ is negative, an increase of τ leads to a decrease in the saving of hoarders. Note that even if the bequest of hoarders is increased by a PAYG pension system, the saving of hoarders does not necessarily increase. Indeed, when $x_*^h(\tau) > x_*^h(0)$, we have $s_*^h(\tau) > s_*^h(0)$ if and only if $w_*(0) - w_*(\tau) < \{[\beta + \phi(k_*(\tau))](x_*^h(\tau) - \tau) - [\beta + \phi(k_*(0))](x_*^h(0))\} / \beta$. To summarize, the impact of a PAYG system both on the bequest and the saving of hoarders is ambiguous.

b – Since the FOC (4) and (5) are not modified by the PAYG pension system, the equation (12) holds and, consequently, the economy is at the MGR capital stock k_*^M . Then, the PAYG pension system has no effects on the stationary capital stock of the savers-spenders-hoarders equilibrium.

Concerning the spenders, their bequests remain nil since (12) is unchanged. Using (17), we obtain for the saving of a dynasty h of spenders: $(1 + \beta)s_*^h(\tau) = \beta w^M - (\beta +$

$\gamma^M)\tau$. Then, the saving of the spenders decreases with τ . Using $s_\star^h(\tau)$, we can focus on the share of capital held by the spenders. After computation we obtain:

$$\nu^{SP}(\tau) = \nu^{SP}(0) - \frac{(\beta + \gamma^M)\tau}{(1+n)(1+\beta)k_\star^M} \sum_{h=1}^{M-1} p^h$$

Thus the share of capital $\nu^{SP}(\tau)$ held by the spenders decreases with τ .

According to (18), the bequest of a dynasty h of hoarders satisfies: $(\gamma^M - \bar{\gamma}^h)x_\star^h(\tau) = \bar{\delta}^h[w^M - (1 - \gamma^M)\tau]$. The bequest of hoarders decreases with τ . Using this value, we also obtain the saving of a dynasty h of hoarders: $(1 + \beta)(\gamma^M - \bar{\gamma}^h)s_\star^h(\tau) = [\beta(\gamma^M - \bar{\gamma}^h) + (\beta + \gamma^M)\bar{\delta}^h] - (\beta + \gamma^M)[\gamma^M - \bar{\gamma}^h + \bar{\delta}^h(1 - \gamma^M)]\tau$. Then, the saving of the hoarders decreases with τ . Using $s_\star^h(\tau)$ we can focus on the share of capital held by the spenders. After computation we obtain:

$$\nu^{HO}(\tau) = \nu^{HO}(0) - \frac{(\beta + \gamma^M)\tau}{(1+n)(1+\beta)k_\star^M} \sum_{h=M+1}^N p^h \left[1 + \frac{\bar{\delta}^h(1 - \gamma^M)}{\gamma^M - \bar{\gamma}^h} \right]$$

Hence, the share of capital $\nu^{HO}(\tau)$ held by the hoarders decreases with τ .

According to (20), the bequest $x_\star^M(\tau)$ of savers increases with τ . As $p^M s_\star^M(\tau) = (1+n)k_\star^M - \sum_{h=1}^{M-1} p^h s_\star^h(\tau) - \sum_{h=M+1}^N p^h s_\star^h(\tau)$ and since both the saving of spenders and hoarders decreases with τ , $s_\star^M(\tau)$ is an increasing function of τ . As the share of capital held by the savers is such that $\nu^{SA}(\tau) = 1 - \nu^{SP}(\tau) - \nu^{HO}(\tau)$ and as both $\nu^{SP}(\tau)$ and $\nu^{HO}(\tau)$ decrease with τ , the share $\nu^{SA}(\tau)$ increases with τ .

Finally, to compare the spenders and the savers we can use the fact that:

$$\nu^{HO}(\tau) - \nu^{SP}(\tau) = \nu^{HO}(0) - \nu^{SP}(0) + \frac{(\beta + \gamma^M)\tau}{(1+n)(1+\beta)k_\star^M} \left(\sum_{h=1}^{M-1} p^h - \sum_{h=M+1}^N p^h \left(1 + \frac{\bar{\delta}^h(1 - \gamma^M)}{\gamma^M - \bar{\gamma}^h} \right) \right)$$

Consequently, the PAYG pension system reduces (resp: increases) $\nu^{HO}(\tau) - \nu^{SP}(\tau)$ if the hoarders are (resp: are not) sufficiently numerous.

Finally, the proofs concerning results on welfare are given in the main text (end of section 4.1). \square

APPENDIX D – PROOF OF PROPOSITION 4.

The saving of a dynasty h in the debt case is given by:

$$\forall h \quad s_\star^h(b) = \frac{\beta}{1+\beta} \tilde{\omega}_\star(b) + \frac{\beta + \phi_\star(b)}{1+\beta} x_\star^h(b) \quad (37)$$

a – We first prove that $k_\star(b) < k_\star(0)$ for all positive b . According to (25) and since $\vartheta(\cdot) > 0$, the spenders-hoarders equilibrium satisfies:

$$F[\phi(R_\star(b))] = -\frac{(1+n)b}{\phi(R_\star(b))w_\star(b)} \vartheta[\phi(R_\star(b))] < 0 = F[\phi(R_\star(0))]$$

According to Step 1 of Appendix A, $F(\cdot)$ is an increasing function of $\phi(R)$. Then, we have $\phi[R_\star(b)] < \phi[R_\star(0)]$ and consequently $k_\star(b) < k_\star(0)$ for all positive b .

We now focus on the saving of the spenders. Using (37) we obtain the saving of the spenders described in (23). Since $k_\star(0) > k_\star(b)$, we have $w_\star(0) > w_\star(b)$ which obviously implies $s_\star^h(0) > s_\star^h(b)$ when $R_\star(b) > 1 + n$, i.e., when $k_\star(b)$ is in under-accumulation of capital. When $k_\star(b)$ is in over-accumulation of capital the saving of the spenders is reduced (resp: increased) by the public debt if $w_\star(0)$ is larger (resp: lower) than $\tilde{\omega}_\star(b)$.

The bequest of hoarders is given by (24). Since $\phi_\star(b) < \phi_\star(0)$, we have $x_\star^h(b) < x_\star^h(0)$ when $\tilde{\omega}_\star(b) < \tilde{\omega}_\star(0) = w_\star(0)$. Note that the bequest of hoarders is always reduced by public debt when $k_\star(b)$ (and consequently $k_\star(0)$) is in under-accumulation of capital. However, when $\tilde{\omega}_\star(b) > w_\star(0)$ (in this case, the capital stock $k_\star(0)$ is necessarily in over-accumulation) the bequest of hoarders is reduced (resp: increased) by public debt if $w_\star(0)$ is larger (resp: lower) than $\zeta_\star^h(b)\omega_\star(b)$ with $\zeta_\star^h(b) = (\phi_\star(0) - \bar{\gamma}^h)/(\phi_\star(b) - \bar{\gamma}^h) > 1$.

According to (37), the saving of hoarders is described by (24). Since $\zeta_\star^l(b) > 0$, we have $\zeta_\star^h(b) > \zeta_\star^h(0)$. Then, the saving of the hoarders is increased by public debt when $\tilde{\omega}_\star(b) > w_\star(0)$. When $\tilde{\omega}_\star(b) < w_\star(0)$ this saving is increased (resp: reduced) according to $\zeta_\star^h(b)\tilde{\omega}_\star(b)$ is larger (resp:) than $\zeta_\star^h(0)w_\star(0)$.

According to (25), k_\star is the capital stock of the spenders-hoarders equilibrium if and only if:

$$\Delta(k_\star, b) = F\left(\frac{1+n}{\alpha A}k_\star^{1-\alpha}\right) + \frac{\alpha b}{(1-\alpha)k_\star}\vartheta\left(\frac{1+n}{\alpha A}k_\star^{1-\alpha}\right) = 0$$

Since $\Delta'_b(\cdot) = \alpha\vartheta(\cdot)/[(1-\alpha)k_\star] > 0$, we can use the Implicit Function Theorem and, consequently, there exists a continuous function $k_\star(\cdot)$ of b such that $k_\star = k_\star(b)$. According to (12), this function is defined as long as k_\star is larger than k_\star^M . According to (26), we have $x_\star^M = 0$ when $b \equiv b^M = -F(\gamma^M)w^M\gamma^M/[(1+n)\vartheta(\gamma^M)]$. Then according to (25), when $b = b^M$, the equilibrium is such that $F[\phi(R_\star)] = w^M\gamma^M F(\gamma^M)\vartheta[\phi(R_\star)]/[w_\star\vartheta(\gamma^M)]$. Consequently, we have $k_\star(b^M) = k_\star^M$ and $k_\star(b)$ is a continuous function of b defined on $[0, b^M]$. Its maximum is at $b = 0$ and its minimum at $b = b^M$. Since the GR capital stock k^G is larger than k_\star^M , there exists (by continuity) a positive debt level $b^G \in (0, b^M)$ such that the economy with the constant debt level b^G converges towards the GR capital stock k^G whereas the economy without debt is dynamically inefficient (i.e., $k_\star(0) > k^G$).

b – Since the FOC (4) and (5) are not modified by public debt, (12) holds and, consequently, the economy is at the MGR capital stock k_\star^M . The public debt has no effects on the capital stock of the savers-spenders-hoarders equilibrium.

Concerning the spenders, their bequests remain nil since (12) is unchanged. Using (23), the saving of a dynasty h of spenders is such that $(1+\beta)s_\star^h(b) = \beta[w^M - (1+n)(1/\gamma^M - 1)b]$. Then, the saving of the spenders decreases with b . From this saving, we can focus on the share of wealth (capital plus bonds) held by the spenders. After

computation:

$$\nu^{SP}(b) = \sum_{h=1}^{M-1} \frac{p^h s_*^h(b)}{(1+n)(k_*^M + b)} = \frac{\beta [w^M - (1+n)(1/\gamma^M - 1)b]}{(1+n)(k_*^M + b)(1+\beta)} \sum_{h=1}^{M-1} p^h$$

Consequently, the share of wealth $\nu^{SP}(b)$ held by the spenders decreases with b .

According to (24), the bequest of a dynasty h of hoarders satisfies: $(\gamma^M - \bar{\gamma}^h)x_*^h(b) = \bar{\delta}^h [w^M - (1+n)(1/\gamma^M - 1)b]$. Consequently, the bequest of the hoarders decreases with b . Using (24) the saving of the hoarders is given by $(1+\beta)(\gamma^M - \bar{\gamma}^h)s_*^h(b) = [\beta(\gamma^M - \bar{\gamma}^h) + (\beta + \gamma^M)\bar{\delta}^h][w^M - (1+n)(1/\gamma^M - 1)b]$. Then, the saving of hoarders decreases with b . Consequently, we can focus on the share of wealth held by hoarders. After computation we obtain:

$$\nu^{HO}(b) = \sum_{h=M+1}^N \frac{p^h s_*^h(b)}{(1+n)(k_*^M + b)} = \frac{\beta [w^M - (1+n)(1/\gamma^M - 1)b]}{(1+n)(k_*^M + b)(1+\beta)} \sum_{h=M+1}^N p^h \left[1 + \frac{(\beta + \gamma^M)\bar{\delta}^h}{\beta(\gamma^M - \bar{\gamma}^h)} \right]$$

Hence, the share of wealth $\nu^{HO}(b)$ held by the hoarders decreases with b .

According to (26), the bequest $x_*^M(b)$ of savers increases with b . Since $p^M s_*^M(\tau) = (1+n)(k_*^M + b) - \sum_{h=1}^{M-1} p^h s_*^h(b) - \sum_{h=M+1}^N p^h s_*^h(b)$ and since the saving both of the spenders and the hoarders decrease with b , $s_*^M(b)$ is an increasing function of b . As the share of wealth held by the savers is such that $\nu^{SA}(b) = 1 - \nu^{SP}(b) - \nu^{HO}(b)$ and as both $\nu^{SP}(b)$ and $\nu^{HO}(b)$ decrease with b , the share $\nu^{SA}(b)$ increases with b .

Finally to compare the spenders and the savers we can use the fact that:

$$\nu^{HO}(b) - \nu^{SP}(b) = \frac{\beta [w^M - (1+n)(1/\gamma^M - 1)b]}{(1+n)(k_*^M + b)(1+\beta)} \left(\sum_{h=M+1}^N p^h \left[1 + \frac{(\beta + \gamma^M)\bar{\delta}^h}{\beta(\gamma^M - \bar{\gamma}^h)} \right] - \sum_{h=1}^{M-1} p^h \right)$$

As a result the public debt reduces (increases) $\nu^{HO}(b) - \nu^{SP}(b)$ if the hoarders are (are not) sufficiently numerous.

Finally, the proofs concerning results on welfare are given in the main text (end of section 4.2). \square

APPENDIX E – PROOF OF PROPOSITION 5.

The saving of a dynasty h in the estate taxation case is given by:

$$\forall h \quad s_*^h = \frac{1}{1+\beta} \left[\beta(w_* + \theta_*) + \left[\phi_*(\kappa) + \beta(1-\kappa) \right] x_*^h \right] \quad (38)$$

a – We first prove that $k_*(\kappa) < k_*(0)$ for all positive κ when the economy experiences only one dynasty of hoarders (indexed by N). In this setting, according to (32), the spenders-hoarders equilibrium satisfies:

$$\nabla(\phi_*(\kappa), \kappa) \equiv \frac{\beta(\phi_*(\kappa) - \varepsilon)}{\varepsilon(\phi_*(\kappa) + \beta) + \kappa\beta(\phi_*(\kappa) - \varepsilon)} - \frac{p^N \bar{\delta}^N}{\phi_*(\kappa) - \bar{\gamma}^N(1-\kappa)} = 0$$

Then, we have:

$$\begin{aligned}\nabla'_1 &= \frac{\partial \nabla(\phi_*(\kappa), \kappa)}{\partial \phi_*(\kappa)} = \frac{\beta \varepsilon (\beta + \varepsilon)}{[\varepsilon(\phi_*(\kappa) + \beta) + \kappa \beta (\phi_*(\kappa) - \varepsilon)]^2} + \frac{p^N \bar{\delta}^N}{[\phi_*(\kappa) - \bar{\gamma}^N (1 - \kappa)]^2} > 0 \\ \nabla'_2 &= \frac{\partial \nabla(\phi_*(\kappa), \kappa)}{\partial \kappa} = \frac{p^N \bar{\delta}^N (\bar{\gamma}^N - p^N \bar{\delta}^N)}{[\phi_*(\kappa) - \bar{\gamma}^N (1 - \kappa)]^2} > 0\end{aligned}$$

Using, the Implicit Function Theorem we have $\phi'_*(\kappa) = -\nabla'_2/\nabla'_1 < 0$. Then, we have $\phi[R_*(\kappa)] < \phi[R_*(0)]$ and consequently $k_*(\kappa) < k_*(0)$ for all positive κ .

With only one dynasty of hoarders, we have $\theta_*(\kappa) = \kappa p^N x_*^N(\kappa)$. Then, according to (31), $x_*^N(\kappa) = A(\kappa)/B(\kappa)$ with $A(\kappa) = \bar{\delta}^N w_*(\kappa)$ and $B(\kappa) = \phi_*(\kappa) - \bar{\gamma}^N + \kappa(\bar{\gamma}^N - p^N \bar{\delta}^N)$. Consequently $A'(\kappa) = \bar{\delta}^N w'_*(\kappa)$ is negative and $B'(\kappa) = \phi'_*(\kappa) + \bar{\gamma}^N - p^N \bar{\delta}^N$. Since $\phi'_*(\kappa) = -\nabla'_2/\nabla'_1$ we have $\phi'_*(\kappa) = [p^N \bar{\delta}^N - \bar{\gamma}^N]/[1 + e]$ where $e > 0$. Then, $\phi'_*(\kappa) + \bar{\gamma}^N - p^N \bar{\delta}^N > 0$ and, consequently, $B'(\kappa)$ is positive. Since $A(\cdot)$ decreases and $B(\cdot)$ increases with respect to κ , the bequest $x_*^N(\kappa)$ of the (single) dynasty of hoarders decreases with the estate tax κ .

To focus on the saving of the spenders and hoarders, we first determine how $\tilde{w}_*(\kappa) = w_*(\kappa) + \theta_*(\kappa)$ varies when κ varies. Since $\theta_*(\kappa) = \kappa p^N x_*^N(\kappa)$ and $x_*^N(\kappa) = A(\kappa)/B(\kappa)$ we have $\tilde{w}_*(\kappa) = [1 + \kappa p^N \bar{\delta}^N / B(\kappa)] w_*(\kappa)$. Then, $\tilde{w}'_*(0)$ has the sign of $p^N \bar{\delta}^N w'_*(0)/B(0) - w'_*(0)$. As $w'(\kappa)/w(\kappa) = \alpha k'(\kappa)/k(\kappa)$, we can show that $\tilde{w}'_*(0)$ has the sign of $\alpha k'_*(0)/k_*(0) + p^N \bar{\delta}^N / (\phi_*(0) - \bar{\gamma}^N)$. Thus the impact on \tilde{w}_* of the introduction of the estate tax is ambiguous since the first term of the R.H.S. is negative whereas the second is positive.

According to (30), the savings of a dynasty h of spenders are such that $s_*^h(0)$ has the sign of $\tilde{w}'_*(0)$. According to the previous paragraph, the impact of the estate taxation on the savings the spenders is ambiguous.

Concerning the savings of the dynasty of hoarders, according to (31), $s_*^N(0)$ has the sign of $\hat{\zeta}_*^N(0) \tilde{w}_*(0) + \hat{\zeta}_*^N(0) \tilde{w}'_*(0)$. Thus the impact of the estate taxation on the savings of hoarders is ambiguous. However, since $\hat{\zeta}_*^N(\kappa)$ has the sign of $-(1 - \kappa) \phi'_*(\kappa) - \phi_*(\kappa)$, $\hat{\zeta}_*^N(0)$ has the sign of $-\phi'_*(0) - \phi_*(0)$. Since $\phi'_*(0) + \bar{\gamma}^N - p^N \bar{\delta}^N > 0$ and $\phi(0) > \bar{\gamma}^N$, $-\phi'_*(0) - \phi_*(0) < -p^N \bar{\delta}^N < 0$. As $\hat{\zeta}_*^N(0)$ is negative, when the introduction of estate taxation increases (decreases) the savings of hoarders (spenders), it also increases (decreases) the savings of spenders (hoarders).

b – When the strong altruists are savers, according to (28) we have $\phi_*(\kappa) = \gamma^M (1 - \kappa)$. Then, the MGR capital stock is affected by the estate taxation since we have $k_* = k_*^M(\kappa) = [\alpha A \gamma^M (1 - \kappa) / (1 + n)]^{1/(1-\alpha)}$. Then, the larger the tax rate κ , the lower is the long run capital stock $k_*^M(\kappa)$.

Using this capital stock we can show after some tedious computations that $\tilde{w}_*(\kappa) = (1 - \kappa)^{\frac{1}{1-\alpha}} [\mathcal{A} \kappa^2 + \mathcal{B} \kappa + \mathcal{C}] \Pi / [\mathcal{A}' \kappa^2 + \mathcal{B}' \kappa + \mathcal{C}]$ where $\Pi = A(1 - \alpha) [\alpha A \gamma^M / (1 + n)]^{1/(1-\alpha)}$, $\mathcal{A} = -\beta \gamma^M (1 + \mathcal{E})$, $\mathcal{A}' = -\varepsilon \gamma^M (1 + \mathcal{E})$, $\mathcal{B} = \mathcal{B}' + \gamma^M (\beta + \varepsilon)$, $\mathcal{B}' = \varepsilon [\gamma^M - (\beta + \gamma^M) (1 + \mathcal{E})]$, $\mathcal{C} = \varepsilon (\beta + \gamma^M)$ and $\mathcal{E} = \sum_{h=M+1}^N [p^h \bar{\delta}^h / (\gamma^M - \bar{\gamma}^h)]$. Then, one can show that, $\tilde{w}'_*(0)$ has the sign of $\mathcal{B} - \mathcal{B}' - \mathcal{C} / (1 - \alpha)$ i.e. the sign of $\gamma^M (\beta + \varepsilon) - \varepsilon (\beta + \gamma^M) / (1 - \alpha)$, i.e. the sign of $\alpha \gamma^M - 1$. Consequently, $\tilde{w}'_*(0)$ is negative and the introduction of an estate taxation reduces the income \tilde{w}_* .

Note that $\hat{\zeta}_*^h(\kappa) = \bar{\delta}^h[\beta + \gamma^M]/[\gamma^M - \bar{\gamma}^h] + \beta \equiv o_M^h$ is independent of κ . According to (30), and (31), the savings of a dynasty h of spenders are such that $s_*^h(0)$ has the sign of $\tilde{\omega}'_*(0)$ whereas the savings of a dynasty h of hoarders has the sign of $\hat{\zeta}_*^h(0)\tilde{\omega}_*(0) + \hat{\zeta}_*^h(0)\tilde{\omega}'_*(0)$. Then, as $\tilde{\omega}'_*(0)$ is negative and $\hat{\zeta}_*^h(\kappa)$ is nil, the introduction of an estate taxation reduces the savings of all dynasties of spenders and of hoarders.

To analyze the effect of κ on the savings of the savers we begin by focus on the effect of κ on the bequests of the savers. According to (33), this bequest $x_*^M(\kappa)$ is such that $x_*^M(\kappa) = G(\kappa)w_*^M(\kappa)/H(\kappa)$ where $G(\kappa) = \beta[\gamma^M(1 - \kappa) - \varepsilon] - [(\gamma^M + \beta)\varepsilon + \kappa\beta\gamma^M] \sum_{h=M+1}^N [p^h \bar{\delta}^h / (\gamma^M - \bar{\gamma}^h)]$ and $H(\kappa) = \varepsilon(\gamma^M + \beta + \kappa\gamma^M)$. Since $G'(\kappa) = -\beta\gamma^M(1 + \sum_{h=M+1}^N [p^h \bar{\delta}^h / (\gamma^M - \bar{\gamma}^h)])$ is negative, $H'(\kappa) = \varepsilon\gamma^M$ is positive and $w_*^M(\kappa) = \alpha(1 - \alpha)A(k_*^M)^{\alpha-1}(\kappa)k_*^M(\kappa)$ is negative, $x_*^M(\kappa)$ decreases when κ increases.

Concerning the savings of savers, according to (38) we have $(1 + \beta)s^M(\kappa) = \beta\tilde{\omega}_*(\kappa) + (1 - \kappa)(\gamma^M + \beta)x_*^M(\kappa)$. Then, as the first term of the RHS decreases for low κ and the second term always decreases with κ , the introduction of an estate taxation depresses saving by the savers.

Concerning the bequests of a dynasty h of hoarders $x_*^h(\kappa)$, using (31) it is obvious that $x_*^h(\kappa)$ has the sign of $\ell'(\kappa)$ where $\ell(\kappa) = \tilde{\omega}_*(\kappa)/(1 - \kappa)$. Then, as $\ell'(0) = \tilde{\omega}'_*(0) + \tilde{\omega}_*(0)$ and $\tilde{\omega}'_*(0) = \tilde{\omega}_*(0)[\gamma^M(\beta + \varepsilon)/\mathcal{C} - 1/(1 - \alpha)]$, $x_*^h(0)$ is positive if and only if γ^M is larger than $\alpha\beta/[\alpha\beta + 1 - \alpha]$.

We now focus on the impact of κ on the wealth distribution. Using (30) we can focus on the share of wealth held by spenders. After computation we obtain:

$$\nu^{SP}(\kappa) = \sum_{h=1}^{M-1} \frac{p^h s_*^h(\kappa)}{(1+n)k_*(\kappa)} = \frac{\tilde{\omega}_*(\kappa)}{k_*^M(\kappa)} \sum_{h=1}^{M-1} \frac{\beta p^h}{(1+n)(1+\beta)}.$$

Using (31) we can also focus on the share of wealth held by the hoarders. We obtain:

$$\nu^{HO}(\kappa) = \sum_{h=M+1}^N \frac{p^h s_*^h(\kappa)}{(1+n)k_*(\kappa)} = \frac{\tilde{\omega}_*(\kappa)}{k_*^M(\kappa)} \sum_{h=M+1}^N \frac{o_M^h p^h}{(1+n)(1+\beta)}$$

To determine how $\nu^{SP}(\kappa)$ and $\nu^{HO}(\kappa)$ vary when κ increases we must study how vary $\psi(\kappa) = \tilde{\omega}_*(\kappa)/k_*^M(\kappa)$. As we have $\psi(\kappa) = e \times [\mathcal{A}\kappa^2 + \mathcal{B}\kappa + \mathcal{C}]/[\mathcal{A}'\kappa^2 + \mathcal{B}'\kappa + \mathcal{C}]$ with $e > 0$, $\psi'(0)$ has the sign of $\mathcal{B} - \mathcal{B}' = \gamma^M(\beta + \varepsilon)$. Then, $\psi'(0)$ is positive and consequently, the introduction of an estate taxation increases the shares of wealth ν^{SP} and ν^{HO} held respectively by the spenders and the hoarders. Conversely, the fact that $\nu^{SA} = 1 - (\nu^{SP} + \nu^{HO})$ implies that the introduction of an estate taxation decreases the share of wealth ν^{SA} held by the savers.

In the three-class equilibrium, the stock of capital is given by $k_*^M(\kappa)$ from which we obtain:

$$\mathcal{W}'(0) \equiv \left. \frac{\tilde{\omega}'_*(\kappa)}{\tilde{\omega}_*(\kappa)} \right|_{\kappa=0} = \frac{-\beta(1 - \alpha\gamma^M)}{(1 - \alpha)(\beta + \gamma^M)} \quad \text{and} \quad \mathcal{R}'(0) \equiv \left. \frac{R'_*(\kappa)}{R_*(\kappa)} \right|_{\kappa=0} = 1 \quad (39)$$

From these two equalities, we look for the effect of κ on the welfare of our three types of individuals. From the welfare of spenders and using (39), we have: $V_*^{SP}(0) = (1 + \beta)\mathcal{W}'(0) + \beta\mathcal{R}'(0) < 0$.

Turning to the savers, we know that $x_{\star}^M(0) < 0$ and $x_{\star}^M(0) > 0$. Then, $\Omega_{\star}^M(0) > \tilde{\omega}(0)$ and $\Omega_{\star}^M(0) < \tilde{\omega}'(0)$. Consequently, $\Omega_{\star}^{SA}(0)/\Omega_{\star}^{SA}(0) < \mathcal{W}'(0)$ which implies: $V_{\star}^{SA}(0) < V_{\star}^{SP}(0)$.

Concerning the hoarders, from $V_{\star}^{HO}(\kappa)$ we obtain: $V_{\star}^{HO}(0) = (1 + \beta + \delta^h)\mathcal{W}'(0) + \beta\mathcal{R}'(0) + \delta^h$. Then, using (39), $V_{\star}^{HO}(0)$ is positive if and only if γ^M is larger than $\beta[1 + \alpha(\beta + \delta^h)]/\{\beta[1 + \alpha(\beta + \delta^h)] + \delta^h(1 - \alpha)\}$.

Finally, the proofs concerning results on welfare are given in the main text (end of section 4.3). \square