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Retirement as a hedge

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Abstract

This paper explores the effect of letting individuals choose their retirement age in a world of uncertainty where there exist both defined benefit (DB) and defined contribution (DC) pension plans. The paper shows that giving individuals the flexibility to determine when to retire is an important tool for them when they are hedging against future uncertainty. It also finds that it is preferable to let people make their retirement decision after rather than before the uncertainty is lifted. Finally, it shows that shifting from DB to DC plans fosters the rate of activity of elderly workers.

Keywords: retirement decision, defined benefit defined contribution.

JEL Classification: H55, J26

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1 Introduction

There is quite a substantial literature that is concerned with the retirement age of individuals. Part of this interest has to do with the large changes in the retirement age that have occurred over the last twenty five years. It turns out that this change has been much larger in males than in females. The difference between the two may have a lot to do with the later entrance of women into the labor force in large numbers. Many of these papers try to determine the retirement age within the context of a life-cycle model where the incentive effects of both private and public pensions are very important in determining when people will retire. For example, the percent of employees with defined benefit pensions with full benefits prior to age 65 which increased from 1980 to 1989 as well as the unanticipated increase in the level of social security benefits during the 1970s (Anderson et al., (1999)) may have been contributing factors in inducing individuals to retire at a younger age and thus help account for the reduction in labor force participation by elderly males. On the other side, the more recent shift in pension plans from defined benefit to defined contribution plans (Poterba, Venti, and Wise (2004)), the elimination of mandatory retirement in 1986, as well as the step taken in 1983 to gradually increase the Social Security normal retirement age to 67 may all have been contributing factors in halting the trend to earlier retirement that occurred in the mid-1980s (Maestas (2007b) and Gustman and Steinmeier (2005)). More recent data suggest that the labor force participation rates by elderly males may even have started to increase. This increase is in addition to the increase in labor force participation by elderly women that has been occurring since the 1950s but has had its most dramatic rise since the mid-1880s (Cahill et. al. (2005), Gustman and Steinmeier (2008), and Maestas (2007b)). In addition to the above literature there is also a strand that is focusing on the increase in bridge jobs in recent years. These are short-duration or part-time jobs that individuals take after leaving a full-time career. Thus these are jobs that bridge the gap between full time employment and when individuals completely withdraw from the labor force (Cahill et. al. (2005). Finally, there is a literature that suggests that many people reverse their decision about retirement and return to work. It is argued that unretirement could be an intentional way of moving gradually out of the labor force but also a response to negative shocks. (Maestas (2007a)).

The evidence presented so far pertains to the United States where most of the empirical research is conducted. However, the issue of early retirement induced by distortive pension systems concerns other OECD countries and particularly the EU members at least as much or even more. See in this respect the work of Gruber and Wise (1999, 2004) on retirement around the world. They explain most of early retirement on pension systems that discourage working even well before the statutory age of retirement.

This paper takes a somewhat different approach to the above literature in looking at the issue of retirement. In a world of uncertainty individuals attempt to hedge against future events by using strategies that will insure that they have the wherewithal to consume in their old age. The standard ways of preparing for the future are through private saving, private defined benefit plans, private defined contribution plans, and social security. In this paper we add an additional element. Individuals are allowed to retire when they want, even beyond the statutory retirement age. The age at which they end up retiring will then depend on the circumstances they expect to face in the future. In the paper two cases are considered. In one case the decision about when to retire is made independent of which state occurs in the future and in the second case individuals are able to make their retirement decision contingent on which state occurs. Finally, to provide some context for the results obtained, in a third section the paper looks at the case of mandatory retirement and compares those results with the ones found in the sections where the retirement age is allowed to vary.

The paper uses an overlapping generations growth model where individuals face two stages in their life.[1] In the model it is assumed that individuals have several ways in which they can prepare for retirement. They can put money away for their later years through a defined contribution plan, they can participate in a private defined benefit plan, and they can continue working beyond their normal retirement age. While social security is not addressed in the paper, social security has many attributes that makes it similar to the defined benefit plan modeled here. The defined contribution plans are assumed to invest their funds solely in private capital.

The uncertainty considered here arises from macroeconomic fluctuations. In that situation there is no type of riskless pension plan. In the case of a defined contribution plan the risk is borne by the retirees[2]. As for the private defined benefit plan, even though the pensioners may view the plan as riskless, firms, or as in our model where the burden is shifted to the employed,

[1] There are other papers that study endogenous retirement in an overlapping generations model. However, those models do not view retirement as a hedge against uncertainty. An example without social security is Matsuyama (2008) and two examples that study social security in such a setting are Hu (1979) and Michel and Pestieau (2002-2003).

[2] "The pegging of benefits in DB plans to final average wage would appear to provide employees with a type of income-maintenance insurance not available in DC plans. This observation has been used to support the selection of these plans over DC plans. This conclusion, however, is not robust. If wage paths are unpredictable at the start of a career, then individuals may view it as very risky to have their retirement benefits depend so heavily on final salary." ((Bodie, Marcus, and Merton (1988), page 147.)

the young workders bear the burden of the risk[3],[4].

In this paper it is shown that allowing individuals to choose their age of retirement gives them an important tool to overcome the uncertainty they face during their golden years and raises their welfare since it gives them the flexibility to make their retirement decisions based on the circumstances they will be facing. In fact, we find that the labor market adjustments that individuals undertake at retirement may be even more important than changes in saving as individuals face increased uncertainty in the second phase of their lives. When the age of retirement is independent of the state of nature, we find in general that when individuals face greater uncertainty in the future, such as when the benefit rate on defined benefit pension plans is reduced or the difference in productivity across states increases they increase the amount of time they spend working during the second period of their lives. When we give individuals the additional tool to make their age of retirement state dependent, we find that the consumption gap across states when individuals are retired is in general reduced as compared to when the age of retirement is not allowed to vary across states. Thus not only is work effort in the second period an important stabilizer for individuals, allowing individuals to adjust that effort across states is an important additional tool as well. In addition, these tools turn out to be an important substitute to saving, particularly to defined contribution pension saving. In comparing labor adjustment in period 2 when that adjustment is state dependent versus when it is not, we find that saving through defined contribution plans is lower when the age of retirement is state dependent. Allowing individuals to make their retirement decision state contingent means that they do not have to save as much for the future and still have the wherewithal to consume an adequate amount in the second period.

The rest of the paper is organized as follows. The next section presents the basic theoretical model. Section 2.2 is devoted to discussing the implications of changing some of the exogenous variables. Unfortunately, for some of the outcomes and especially when two or more exogenous variables are changed simultaneously, the results end up being ambiguous. There-

[3] In terms of funding there is a big difference between defined contribution and defined benefit plans. "In defined contribution plans, by definition the value of the benefits equal that of the assets, so the plan is always exactly fully funded. But in defined benefit plans, there is a continuum of possibilities. There may be no separate fund, in which case the plan is said to be unfunded. When there is a separate fund with assets worth less than the present value of the promised benefits, the plan is underfunded. And if the plan's assets have a market value that exceeds the present value of the plan's liabilities, it is overfunded." (Bodie and Papke (1992), page 152.)

[4] Bodie, Marcus, and Merton (1988) discuss some of the trade-offs when comparing defined benefit and defined contribution plans.

fore section three presents a numerical simulation model and discusses the lessons learned from it. The section with the numerical example is divided into three parts. The first subsection assumes that the decision about when to retire is made independent of the state of nature. The second subsection provides individuals with an extra degree of freedom by allowing them to determine their retirement based on the state of nature they face. The third part tries to come close to replicating the standard model by assuming that there is a mandatory retirement age in effect. This last subsection tries to compare the results when variable retirement is allowed with the case when the retirement decision is externally set. Finally, the last section presents some conclusions.

2 The theoretical model.

2.1 Presentation

We use a standard overlapping generations model which has two states of the world. The population is assumed to grow at the rate n and productivity gets enhanced at the rate g. All individuals work in the first period of their lives and retire in the second. We distinguish two types of retirement. Statutory retirement that occurs at the end of the first period and after which pension benefits are awarded and effective retirement that occurs in the second period when the individual ceases to work. Individuals have access to private pensions which consist of a combination of a defined benefit and a defined contribution plan. The following equations specify the optimization problem and constraints faced by a typical person.

$$\max .5\left[u(c_{t1})+u\left(c_{t2}\right)\right]+\delta[.5u(c_{t+1,1})+.5u(c_{t+1,2})+\eta V(1-z_{t+1})] \qquad (1)$$

subject to:

$$c_{tj}+K_{t+1}=w_{tj}A_{tj}(1-\zeta_j)L_t; j=1,2 \qquad (2)$$

$$c_{t+1,j}=(1+r^K_{t+1,j})K_{t+1}+\theta\bar{w}_tL_t+w_{t+1,j}z_{t+1}L_tA_{t+1,j}; j=1,2 \qquad (3)$$

The lifetime utility of an individual depends upon his consumption in each period and state and the amount of time spent working in the second period. In equation (1) u(.) and V(.) are strictly concave functions and δ is a factor of time preference. Individuals maximize utility over the two periods and in each period there are two possible states of the world that can each occur with probability .5. Subscripts are used to denote first the period and second the state of nature. The argument of the utility function is consumption, c, in each period and the amount of time spent in retirement in the second period, $1 - z_{t+1}$. Each period having a unitary length, 1+z can be considered as the effective age of retirement. When z=0 effective and statutory retirement coincide. The parameter η measures the preference for leisure or retirement. In the simulations later in the paper the utility function is assumed to take the constant relative risk aversion functional form. Equation (2) denotes the budget constraint faced by the typical person of generation t when he/she is young. Individuals decide how much to consume when they are young, $c_{tj}, j = 1, 2$, and how much to save. Private saving can be done either through the defined benefit plan, where ζ_j represents the fraction of the average salary withheld from the employee and used to contribute to his defined benefit plan,[5] or through the purchase of private capital, K_{t+1}, that becomes part of his defined contribution plan. We assume that the value of K_{t+1} is determined by the individual. While in some defined contribution plans the employer may be a contributor, in almost all cases the worker can supplement the firm's contribution and thus the value of K_{t+1} at the margin is determined by the individual. To keep the model relatively simple we do not include bonds as one of the options for individuals in their defined contribution plan. By excluding bonds we do not have to include public goods or an income tax. In our model the defined benefit pension plan picks up some of the attributes of risk-free bonds. Moreover, for other types of bonds, capital can act as a partial proxy.

The net income of a typical individual when young consists of wage income from labor after defined benefit contributions, $w_{tj}(1-\zeta_j)A_{tj}L_t$. The amount set aside by workers for the defined benefit plan is assumed to be $\zeta_j w_{tj} A_{tj} L_t$. Not only is the amount contributed state dependent but the percentage of the wage paid out by the young, ζ_j, is state dependent. Even though the defined benefit plan is provided by the employer, we assume that the incidence is such that the burden of the pension plan falls on the worker. The labor

[5] In this paper we are not attempting to do a general equilibrium analysis of defined benefit plans. Ebrahim (2006) argues that if one wanted to do a general equilibrium analysis one would need to define them in terms of "not only the income but also expected portfolio payoffs. This is radically different from that observed in the 'real world', where it is defined strictly in terms of pensionable salary, which is derived only using the average of that in pre-retirement years (after incorporating the number of years of service)." (Ebrahim (2006), page 15.)

supply, L_t, can be viewed as efficiency labor. Uncertainty is introduced in this model through differences in productivity in the two states, $A_{tj}, j = 1, 2$. The variation in productivity in turn means that the wage rate, w_{tj}, is also state dependent. When individuals are old they face budget constraints (3). Their income consists of the payout from capital in the defined contribution plan, where the return on capital in period $t+1$ is $r^K_{t+1,j}, j = 1, 2$, the payout from the defined benefit plan, where θ is the payout rate from the defined benefit plan, and the amount received from wage income generated during the fraction of the time in period two that they spend working, z_{t+1}. It is assumed that the fraction θ is a parameter and thus its value is outside the control of the individual. The payout from the defined benefit plan is θ times the average wage income earned in period t. We define the average wage as:

$$\bar{w}_t = (w_{t1}A_{t1} + w_{t2}A_{t2})/2. \tag{4}$$

Defined benefit plans can be funded from a pool of safe assets purchased to sustain the plans. However, when there are macro shocks, at the margin, corporations need to cover the difference between the amount paid out by the plan and the amount one can get by selling the safe assets. Thus, with uncertainty, the amount corporations need to set aside to insure that the defined benefit plans can be funded will vary. However, to the extent that the incidence of the burden of the defined benefit pension falls on workers, the amount that workers need to contribute will fluctuate with the uncertainty. In our simplified model we try to address these issues by arguing that the amount workers pay in when young equals the amount paid out by the defined benefit plans to the retirees, i.e.,

$$\zeta_j w_{t+1,j} A_{t+1,j} L_{t+1} = \theta \bar{w}_t L_t; j = 1, 2. \tag{5}$$

In the steady state both the labor force and productivity grow at a constant rate and therefore the above equation can be rewritten as:

$$\zeta_j w_{t,j} A_{tj} L_t (1+n)(1+g) = \theta \bar{w}_t L_t; j = 1, 2. \tag{6}$$

Since the payout from the defined benefit plan is state independent, we also need the relationship,

$$\zeta_1 w_{t+1,1} A_{t+1,1} = \zeta_2 w_{t+1,2} A_{t+1,2} \tag{7}$$

It is quite clear that defined benefit pensions imply a harsh burden on workers in case of an adverse state of nature. This explains why flexible retirement will appear as a welcome substitute for such a burden.

Saving by the workers through their defined contribution plans can be found by substituting the budget constraints (2) and (3), into equation (1). Optimizing over the choice variable K_{t+1} yields the first order condition:

$$-.5[u'(c_{t1})+u'(c_{t2})]+.5\delta[u'(c_{t+1,1})(1+r^K_{t+1,1})+u'(c_{t+1,2})(1+r^K_{t+1,2})]=0 \quad (8)$$

To determine the amount of labor supplied by individuals in the second period of their lives, one needs to optimize over z_{t+1} to yield the first order condition,

$$\delta[.5u'(c_{t+1,1})w_{t+1,1}A_{t+1,1}L_t+.5u'(c_{t+1,2})w_{t+1,2}A_{t+1,2}L_t-\eta V'(1-z_{t+1})]=0 \quad (9)$$

For the production side we assume a standard constant returns to scale Cobb-Douglas production function that takes the form:

$$Y_{tj}=K_t^{\alpha}(A_{tj}L_t(1+z_{t+1}/(1+n)))^{1-\alpha}; j=1,2 \quad (10)$$

where Y_{tj} represents output in period t and state j. The rates of return on the factors of production can be written as:

$$\partial Y_{tj}/\partial K_t=\alpha K_t^{\alpha-1}(A_{tj}L_t(1+z_{t+1}/(1+n)))^{1-\alpha}=r^K_{tj}; j=1,2 \quad (11)$$

and

$$\partial Y_{tj}/\partial(A_{tj}L_t)=(1-\alpha)K_t^{\alpha}(A_{tj}L_t(1+z_{t+1}/(1+n)))^{-\alpha}=w_{tj}; j=1,2 \quad (12)$$

The variable w_{tj} represents the wage per unit of effective labor at time t in state j and the wage per unit of L_t would be $w_{tj}A_{tj}$. It should be noted that since the growth rates are constant, we also know that $w_{tj}=w_{t+1,j}$ and $r^K_{tj}=r^K_{t+1,j}$ for $j=1,2$.

To summarize, the model described in this section consists of the equations (2)-(4) and (6)-(12) in the unknowns, $c_{tj}, c_{t+1,j}, \bar{w}_t, Y_{tj}, K_t, w_{tj}$, $r^K_{tj}, \zeta_1, \zeta_2$, and z_{t+1}.

2.2 Implications of the Theoretical Model

In this subsection we use the theoretical model specified in the previous subsection to analyze the impact of a reduction in the payout from the defined benefit plans, a change in preference for leisure, and a change in the value of the discount factor. Unfortunately, the theory provides very few unambiguous results and thus we add examples that use specific functional forms to flush out more definitive results. Moreover, in the simulations section, in addition to replicating the impact of a reduction in the payout from the defined benefit plans, we look at other changes, such as changes in the amount of uncertainty, that are hard to analyze analytically.

Defined benefit pension plans place most of the risk on the workers whereas defined contribution pension plans place most of the risk associated with the plan on the retirees. Thus, a shift away from the reliance on private defined benefit plans places more risk on the retirees. When such a change occurs one finds that the contributions to the defined contribution plan rise but also the age of retirement rises as well. As the amount of uncertainty faced by the retirees rises they hedge their bets by postponing retirement. The increase in saving through the purchase of additional capital also means that the rate of return on capital falls and the wage rate rises.

When there is an increase in the discount factor so that individuals care more about the future, they increase the amount of capital that they purchase and are likely to reduce their retirement age. By placing greater weight on the second period, individuals would both like to consume more in the second period and also have longer retirement. However, the impact on z_{t+1}can go in the other direction since the result also depends on how much the increase in the capital stock lowers the rate of return on capital and raises the wage rate.

Finally, when there is an increase in preference for leisure there is a decrease in the age of retirement. The natural expectation when η increases is that there should be an increase in the capital stock and in turn a decrease in the rate of return on capital and an increase in the wage rate. However, an increase in capital means that there will be a reduction in consumption in period one. Thus, depending on the shape of the utility function, one could actually find cases where the capital stock actually declines in this situation.

Even though this model abstracts from many aspects of reality, as noted above, it is not possible to theoretically sign even relatively simple changes in the parameters of the model presented earlier.[6] To get some further insights

[6] An analysis of the model requires that it be differentiated and placed in matrix form. Since the results from the theoretical model tend to be ambiguous and the solution of the model is largely technical in nature we have omitted the solution from the paper. For

into the issues that are the focus of this paper, the next section presents some computer simulations on the version presented in the model section, one where the values of z_{t+1} are state dependent, as well as one where a mandatory retirement age is in effect.

3 Results from the Numerical Example

This section is intended to provide further insights into some of the issues discussed in the previous part of the paper as well as to explore some related topics. In the numerical example we use a utility function that is assumed to be isoelastic. It takes the form:

$$\begin{gathered} U(.) = .5[(c_{t1})^{(1-\gamma)} + (c_{t2})^{(1-\gamma)}]/(1-\gamma)]+ \\ \delta\{.5[(c_{t+1,1})^{(1-\gamma)} + (c_{t+1,2})^{(1-\gamma)}]/(1-\gamma) + \eta(1-z_{t+1})^{1-\beta}/(1-\beta)\} \end{gathered} \tag{13}$$

where γ is the Arrow-Pratt measure of relative risk aversion and β is the exponent on the leisure term.

In addition, for the simulations we use the somewhat more general CES production function that takes the form,

$$Y_{tj} = [(a_K K_t)^\rho + (a_L A_{tj} L_t(1 + z_{t+1}/(1+n)))^\rho]^{1/\rho}; j = 1, 2 \tag{14}$$

where a_K is the coefficient on capital in the CES production function, a_L is the coefficient on labor, and ρ takes on values from 1 to $-\infty$.

The simulation model in this section does not use parameter values that pretend to mimic the American or any other existing economy. Rather, the policy changes introduced here are intended to indicate the types of effects that would be expected to occur in an economy from the policy alterations that are analyzed in the example. Moreover, the policy changes introduced are relatively large so that one can easily see the various impacts. One should also note that we have collapsed the many years in the lives of workers and retirees into two periods. This section is divided into three subsections. The first analyzes policy changes when the age of retirement is independent of the state, the second subsection looks at the case where the age of retirement can vary with the state of nature, and the third assumes a mandatory retirement age.

3.1 Retirement is independent of the state

The simulations described in this section are summarized in Table 1. The first column of the table describes the base model. In the base model the

those interested, the solution can be obtained from the authors.

exponent of the utility of consumption is 3.5 and the exponent on the leisure term is 4. We assume that the rate of labor force growth, n, and the rate of technological change, g, are each 50%. The labor supply of individuals, L_t, takes a value of 10 and the discount factor, δ, takes on a value of .9. The level of productivity in state 1, A_{t1}, is 3 and in state 2, A_{t2}, is 1. In the CES production function the coefficient on capital takes on a value of .3, that of labor is set at .7, and ρ has a value of .65. Also the preference for leisure, η, takes on a value of 8. Finally, the benefit rate on private defined benefit pensions, θ, is .2. We did try other parameter values for the base model and found that the results below were robust to changes in the initial parameter values.

In Table 1 we summarize the values of the most important variables. The ones that are left out either do not add much to the explanation, or, as in the case of the utility of the participants, the direction of change can actually be inferred by seeing how individual consumptions move. Moreover, in those cases where first period and second period consumption move in opposite directions, what happens to the value of utility is very dependent on the magnitude of the exponent of consumption in the utility function, a parameter for which it is difficult to come up with a "true value".

The first policy change in Table 1 replicates the exercise of the theoretical section by lowering the benefit rate on defined benefit pensons from .2 to .1. As before, this change results in an increase in saving through defined contribution pensions and hence an increase in the capital stock and the average wage. In addition, the increase in the uncertainty faced by the older workers means that thay will increase the age of retirement from .120 to .126. The reduction in θ also means that consumption of the young rises but the increased uncertainty faced by the elderly means that the spread in consumption of the elderly between the good and bad states rises as well. For example, the consumption of the elderly in the good state rises from 9.67 to 9.87 but in the bad state it falls from 6.55 to 6.40.

The second exercise is to increase the discount factor from .9 to .95. By placing greater weight on the second period, saving is increased from 1.050 to 1.094 (as is the average wage) and the amount of time spent working in the second period is reduced from .120 to .117. This policy also reduces consumption in period one and tends to increase consumption in period two. The results for this example are consistent with wideheld concerns that stem from the view that people do not place sufficiently high weight on the future. For example, this exercise shows that if individuals tend to ignore the future they will consume too much in period one, not save enough for the future. and will be forced to postpone retirement beyond the statutory age.

The next exercise is to increase the preference for retirement from 8. to 9.

That change results in an increase in savings and hence the size of the capital stock from 1.050 to 1.104 and reduces the amount of time spent working in the second period from .120 to .107. The extra emphasis on retirement also means that consumption in both period one and period two is reduced. The extra saving in period one reduces consumption in period one and the lower age of retirement is the main factor in reducing consumption in period two.

The fourth change is to increase the gap in productivities between states one and two. Now productivity becomes 3.25 in state one and .75 in state two. The increase in spread in productivities also means that there will be a wider spread in consumption between the good and bad states in both consumption when young and consumption when retired. This change also results in a reduction in the amount saved through the defined contribution plans. The way individuals prepare for the future is by spending more time working in the second period. For example, the amount of time spent working rises from .120 to .185. Of course, if working during the second period were not possible, such a large downward adjustment in saving through the defined contribution plans would not be possible. Earlier it was noted that all pension funds in a world of macroeconomic uncertainty result in risk being placed on the shoulders of all the participants. Defined contribution plans place the uncertainty on individuals in the second period and defined benefit plans place it on them in period one. When uncertainty increases, a relatively large burden is borne by the workers in the state with the low productivity in order to pay for the defined benefit plans. The rate at which individuals need to contribute to fund the defined benefit plan in state 2 rises from .175 in the base case to .235 when the gap in productivities is increased. Thus, a large reduction in consumption in period 1 state 2 is the result. In order to mitigate that decrease to some extent, workers reduce their contributions to their defined contribution plans. In the second period individuals hope to be able to maintain a reasonable level of consumption by using the option to retire later.

As a point of comparison, the next change increases the gap in productivities between states one and two as well as decreases the benefit rate on the defined benefit pension plans. This policy change results, not surprisingly, in a combined effect of the two policies separately. The gap in consumption in periods one and two between states one and two is larger than for the policy that increases the gap in productivities alone. For example, for the combined policy in period 2 consumption in state one is 10.83 whereas it is 4.97 in state two. When only the productivity gap increases, the consumptions are 10.32 and 5.12 respectively. Also, for the combined policy saving is higher at .860 than for just the increase in the productivity gap where it is .288. The main adjustment for the increased uncertainty coming from the combined plan is through the age of retirement. For the combined policy z_{t+1} is .196, whereas it is .186 for just the increase in the gap and it is only .126 when the only change is the decrease in the benefit rate to .1. The reduction in the benefit rate on the defined benefit plans also means that the contribution rate by the young towards the defined benefit plan in state two will be reduced from .235 to .116. This large decrease means that consumption in period 1 state 2 can be higher even with an increase in saving. Of course the smaller defined benefit plan means more uncertainty for individuals in period 2 and hence the decision to retire later.

The next change explored is an increase in both the growth rate of the population and of productivity from .5 to .6. The increase in the growth rates means that the dependency ratio is reduced, i.e., the amount of effective labor available in period one is increased relative to the the number of retired individuals in period two. Note, our model collapses the many periods in an individual's life into two and thus the relatively large number for the growth rate. The higher growth rate means that there is more money available for each retiree through the defined benefit plans at the same defined contribution rate for these plans. Thus, the increase in growth rates mean that consumption in both periods one and two rise as does the capital stock. Since the contribution rate to the defined benefit plans falls, individuals are able to save more through their defined contribution plans. Finally, the amount of time spent working in the second period is reduced. The higher utility of the individual when the growth rate is increased comes not only from the higher levels of consumption but also the ability of individuals to increase their leisure time in the second period.

Table 1
Numerical example where work during retirement is independent of state

Policy Δ		$\theta = .1$	$\delta = .95$	$\eta = 9.$	$A_{t1} = 3.25$ $A_{t2} = .75$	$A_{t1} = 3.25$ $A_{t2}, \theta = .75, .1$	$n = .6$ $g = .6$
c_{t1}	19.09	19.20	19.05	19.05	21.38	21.62	19.18
c_{t2}	4.95	5.02	4.92	4.91	3.81	3.97	5.05
$c_{t+1,1}$	9.67	9.87	9.67	9.37	10.32	10.83	9.92
$c_{t+1,2}$	6.55	6.40	6.60	6.51	5.12	4.97	6.70
K_{t+1}	1.050	1.719	1.094	1.104	.288	.860	1.093
ζ_1	.060	.030	.060	.060	.055	.028	.052
ζ_2	.175	.087	.175	.175	.235	.116	.154
$\bar{w}_t$	1.43	1.45	1.44	1.44	1.42	1.43	1.43
z_{t+1}	.120	.126	.117	.107	.185	.196	.115

3.2 Retirement is State Dependent

In this section of the paper we give individuals an extra degree of freedom and allow them to make the age of retirement state dependent. In comparing the examples in this section with those in the last one, individuals will have higher utility here for comparable cases than when work effort is independent of state. The parameter values are the same for both Tables 1 and 2. In comparing the first column of both tables one will notice that the gap in consumption for individuals in period 2 is smaller in Table 2 than in Table 1. For example, in Table 1 the difference is 9.67-6.55=3.12 whereas in Table 2 the difference is 9.35-6.67=2.68. The reason for the smaller gap in Table 2 is that individuals are able to partially adjust for the difference in productivity across states by adjusting their work effort in period 2 across states. Individuals spend a fraction .1115 working in state 1 and .1343 in state 2 whereas, when they cannot adjust their work effort across states they work .1196. When the time worked in period two is not state dependent, the work effort takes on a value that is between the time worked in the two states when the work effort can be adjusted. Also, when work effort is state dependent the amount that needs to be saved through defined contribution plans is a little lower, the capital stock rising by 1.028 as compared to 1.050 when work effort is not state dependent. Thus, the work effort adjustment can offset required saving to some extent.

In the first example we look at the impact of a reduction in θ from .2 to .1. This policy shifts more of the uncertainty to the retirees. The consumption gap for the retirees when the age of retirement is state dependent rises to 9.36-6.58=2.78. However, this increase in the gap is smaller than in the case where work effort in period 2 is not state dependent, i.e., 9.87-6.40=3.47. Thus, part of the increased uncertainty is offset through changes in the age of retirement across states when that is possible. In state 2 time spent working in period 2 rises from .134 to .147 as can be seen in Table 2. In both cases individuals increase saving through their defined contribution plans. However, the increase is smaller when work effort is state dependent, 1.678-1.028=.650 than when it is not, 1.719-1.050=.669

The second case analyzed is an increase in the discount factor from .9 to .95. The higher discount factor means that individuals care more about the future. As in the previous example one notices that the consumption gap in period 2 is smaller, 9.38-6.71=2.67 when work effort in period 2 is state dependent than when it is not, 9.67-6.60=3.07. Similarly, the increase in capital is smaller at 1.073 when there is state dependency as compared to the 1.094 when work effort in period 2 is independent of state. While the higher discount factor tends to reduce work effort, the variation in work effort across states, .109 in state 1 and .130 in state 2, allows individuals to hedge the future through an adjustment in labor supply in period two.

The third change is to increase the preference for leisure during period two from .8 to .9. By placing greater weight on leisure, consumption is reduced in both periods one and two. However, even in this situation individuals use work effort in period 2 for smoothing purposes. For example the gap in consumption in period 2 is 9.37-6.51=2.86 when work effort in period 2 is independent of state but 9.16-6.60=2.50 when it is state dependent. The increased emphasis on leisure in period 2 also means that saving will be increased from the base case. However, saving is still smaller when the age of retirement is state dependent in period 2 as compared to when it is not. While individuals use the age of retirement to try to smooth consumption in period 2, the gap in work effort is smaller at η =.9 than for the base case. The gap is .117-.101=.016 at η=.9 and .134-.112=.022 for the base case.

The next example studies the impact of an increase in the gap in productivities between states one and two. When productivities are 3.25 in state one and .75 in state two, the increase in uncertainty means that the consumption gap across states increases. The increase in uncertainty also means that the gap in work effort in period 2 is now larger than for the base case, rising from .134-.112=.022 to .228-.152=.076 for the larger productivity gap. The increase in the productivity gap means that individuals try to use retirement to help reduce the second period consumption gap. In comparing the consumption gap in period 2 when the age of retirement is not state dependent, 10.32-5.12=5.20, with the gap when it is, 9.04-5.36=3.68, one notices quite a large reduction in the gap. In the good state individuals cut their age of retirement quite a bit if they can do so and they likewise increase it in the bad state to help increase consumption in state 2 period 2 as compared to the case where the retirement decision is not state dependent. Finally, saving when the age of retirement is state dependent is slightly lower than when it is not. Less saving is required since labor in period 2 is doing some of the smoothing. Also for this reason consumption in period 1 is slightly higher when one has state dependency in period 2 than when state dependency is not allowed.

To provide further insight into the impact of increased uncertainty the next case studied is one where the gap in productivities across states one and two is increased and at the same time there is a decrease in the benefit rate on the defined benefit pension plans. The results tend to reflect the combined effect of the two policies separately. When the age of retirement is state dependent, since labor supply in period two can offset some of the uncertainty faced by individuals, saving is reduced to .772 from .860. This reduction in saving means that consumption in period 1 is higher when labor is state dependent in period 2. Also, the large difference in the age of retirement between state 1 and state 2, .152 and .248 respectively, means that the consumption gap in period 2 is reduced quite a bit as well, falling from 10.83-4.97=5.86 to 9.06-5.23=3.83. Finally, consumption in period one state 2 will be higher in this case than both the case where the age of retirement is not state dependent and when there is no reduction in the defined benefit pension. The lower contribution rate to the defined benefit plan when that plan is reduced as well as the lower saving required when the retirement decision is state dependent both contribute to the higher level of consumption in period 1 state 2. Finally, consumption in period 2 state 2 will be higher. To see the important impact of allowing the retirement decision to be made state dependent, one can notice that consumption in period 2 state 2 is higher with state dependency than in the case where the defined benefit plan is kept unchanged and the retirement decision is not state dependent, i.e., 5.23 as compared to 5.12. On the other hand, if one wants to isolate on the effect of making the retirement state dependent, one sees an increase in consumption in period 2 state 2 from 4.97 when the retirement decision is independent of the state to 5.23 when the decision is state dependent.

The last experiment is to lower the dependency rate by increasing the growth rate of both the population and productivity from .5 to .6. As indicated in the section where the age of retirement is state independent, individuals are better off when the growth rate of population and productivity rise. However, when the age of retirement is state dependent we notice that in state 1 individuals lower labor supply and in state 2 they increase it as compared to when the retirement decision is independent of the state. This change means that the consumption gap in period two is lower when there is state dependency, 9.58-6.83=2.75 versus 9.92-6.70=3.22 when labor supply is not state dependent. In fact, consumption in period 2 state 1 is actually lower when labor is state dependent. This reduction in consumption occurs both because the age of retirement in state 1 is lower and because individuals lower the amount they feel they need to save through defined contribution plans when they can make period 2 state dependent labor adjustments. In state of 1 period 2 individuals are well enough off that they feel that they can afford to spend more time in retirement. The lower saving also means that consumption in period 1 is higher when the age of retirement is state dependent. Finally, with retirement being state dependent, consumption in period 2 state 2 will be higher than when the retirement decision is independent of the state even though the amount saved through the defined contribution plan is lower. This result again shows the importance of allowing state dependent retirement decisions.

In most standard models most of the adjustment across periods is done through saving behavior. In this paper, the last two sections have shown that when individuals are able to supply some labor in period 2, most of the adjustment is done through changes in labor supply during the second period. Moreover, when labor supply in period 2 is made state dependent, that dependency adds a very important factor in allowing individuals to smooth consumption across periods.

Table 2

Numerical example where Work During Retirement is State Dependent

Policy Δ		$\theta = .1$	$\delta = .95$	$\eta = 9.$	$A_{t1} = 3.25$ $A_{t2} = .75$	$A_{t1} = 3.25$ $A_{t2}, \theta = .75, .1$	$n = .6$ $g = .6$
c_{t1}	19.10	19.24	19.07	19.06	21.41	21.69	19.20
c_{t2}	4.97	5.05	4.93	4.92	3.84	4.04	5.07
$c_{t+1,1}$	9.35	9.36	9.38	9.16	9.04	9.06	9.58
$c_{t+1,2}$	6.67	6.58	6.71	6.60	5.36	5.23	6.83
K_{t+1}	1.028	1.678	1.073	1.088	.243	.772	1.069
ζ_1	.060	.030	.060	.060	.055	.028	.052
ζ_2	.175	.087	.175	.175	.235	.117	.154
$\bar{w}_t$	.143	1.45	1.43	1.44	1.41	1.43	1.43
$z_{t+1,1}$	.112	.113	.109	.101	.152	.152	.107
$z_{t+1,2}$	.134	.147	.130	.117	.288	.248	.129

3.3 Mandatory Retirement Age

To put our results into perspective, in this subsection we briefly discuss the case where individuals do not work in the second period. One can view this situation as one where a mandatory retirement age is in effect. Moreover, this situation comes closest to describing the results for the standard model and thus we will get a better understanding of how our results deviate from what is normally obtained. In Table 3 we use the same parameter values as those found in Tables 1 and 2. The big difference is that in period 2 individuals here spend all of their time in leisure activities.

There are several differences between the results of the previous two subsections and this one. First, utility is lower when individuals are constrained not to work. While leisure is larger when there is a mandatory retirement age, consumption in all states and in both periods is lower. Second, saving through the defined contribution pension plan is higher when individuals cannot offset future uncertainty by deferring their retirement. Moreover, saving is lower still when labor supply in period 2 is made state dependent. Third, while allowing individuals to retire at a later age provides another mechanism to increase consumption in period 2, to the extent that productivities vary across states, the gap in consumption in period 2 between states 1 and 2 is larger when labor in period 2 is permitted. This result holds even when second period labor supply is state dependent. Even though individuals work longer in the state with the lower productivity, that factor does not completely offset the difference in productivities between the good and bad states.

Looking more specifically at the changes in the policy parameters, we find that when the defined benefit rate is reduced from .2 to .1, in all three tables consumption in period 1 rises and saving through the defined contribution plans is increased. In period 2, the gap in consumption between states 1 and 2 increases the most when labor in period 2 is independent of state and rises the least when labor in period 2 can be adjusted across states. Thus, while in all three tables the gap in consumption in period 2 rises as one reduces the dependency on the defined benefit plans, making labor in period 2 state dependent is more effective in keeping the increase in the gap of consumption in period 2 from rising too much than relying solely on the increase in saving through the defined contribution plan.

When the discount is raised to .95, consumption in period 1 is reduced and saving is increased in all three situations. In period 2 consumption tends to rise in all three situations. However, one should note that when labor in period 2 can vary, leisure time is increased when the discount rate is raised and thus the increase in consumption in period 2 will be larger when there exists a mandatory retirement age.

An increase in the difference in productivities raises the gap in consumption between states 1 and 2 in both periods one and two when workers can adjust their retirement age, although the increase in the gap in period 2 consumption will be smaller when labor in period 2 is state dependent. However, when there is a mandatory retirement age and the difference in productivity across states increases, the gap in consumption in period 2 between the good and bad state actually falls. This fall in the gap is due to the reduction in saving through the defined contribution plan in this situation and the fact that individuals do not work in the second period. Relatively more saving is done through the defined benefit plan in this case and hence the reduction in the gap. One should note that consumption in state 2 period 1 is smaller when there is a mandatory retirement age in effect than in the cases where the retirement decision is flexible. Individuals reduce the amount they put into their defined contribution plan to try to keep consumption in state 1 period 1 from falling too far. At the same time, since individuals cannot supplement their second period income by working in the second period in the case discussed here, they cannot reduce savings too much if they wish to have the wherewithal to provide for adequate consumption in period two. Even so, consumption in period 2 when there is mandatory retirement is lower than in the other cases.

The next case increases the difference in productivities but at the same time lowers the benefits from the defined benefit pension program. In comparing the base case with the situation where the gap in productivity rises and there is a lower benefit rate on the defined benefit plan, one notices that the gap in consumption in period 2 across the two states increases by far the least when one has a mandatory retirement age in effect. When labor can vary in period 2, at the larger difference in productivity across the two states and the lower benefit rate, there is quite a large increase in work effort in period 2 and it is this increase in work effort in the two states that results in the larger consumption gap. In the case with mandatory retirement and the larger productivity gap, the increase in the gap in consumption in period 2 is a result of the switch from defined benefit to defined contribution plans. When the benefits from the defined benefit plan are reduced, individuals increase their saving through the defined contribution plan. This switch increases the gap in period 2 consumption but also results in a reduction in the average second period consumption. Also, the reduction in consumption in perod 2 state 2 is larger for the case with mandatory retirement than in the other cases. Indviduals are not able to offset the lower consumption by increasing work effort.

A reduction in the dependency ratio by raising both the rate of growth of productivity and labor to .6, raises consumption in all states and both periods as well as saving in all three tables. The only difference is that when labor in period two can vary, there is an increase in leisure time when the dependency ratio falls. It is, however, interesting that even with the increase in leisure in the cases where retirement can vary, consumption in period 2 rises more in those cases than in the case where there exists a mandatory retirement age.

Table 3

Numerical example where there is a Mandatory Retirement Age

Policy Δ		$\theta = .1$	$\delta = .95$	$A_{t1} = 3.25$ $A_{t2} = .75$	$A_{t1} = 3.25$ $A_{t2}, \theta = .75, .1$	$n = .6$ $g = .6$
c_{t1}	18.642	18.698	18.618	20.986	21.100	18.718
c_{t2}	4.45	4.46	4.43	3.32	3.36	4.54
$c_{t+1,1}$	7.05	7.05	7.11	5.46	5.54	7.25
$c_{t+1,2}$	6.30	6.10	6.35	4.81	4.59	6.44
K_{t+1}	1.648	2.384	1.678	.890	1.587	1.693
ζ_1	.060	.030	.060	.055	.028	.052
ζ_2	.174	.087	.174	.232	.115	.153
$\bar{w}_t$	1.45	1.46	1.45	1.43	1.45	1.45

4 Conclusion

One of the concerns raised over the last few years is that individuals are not saving sufficient amounts for their retirement. This concern is becoming even more important as life expectancy increases and individual health improves. While we do not argue in this paper that saving is not important, especially if unexpected events such as bad health occur that are hard to hedge against, we find that using a flexible retirement age as a means of hedging against the future may be a very important substitute to large amounts of saving. In fact we find that flexibility in the retirement age can be a very good supplement to saving in preparing for the future. As the amount of flexibility is increased the optimal amount of saving required is reduced. For example, when the flexible retirement age is independent of the state of nature the optimal amount of saving is lower than when a mandatory retirement age is in effect. However, when the retirement age is made state dependent, the optimal amount of saving is lower yet. Of course, if individuals, perhaps because they feel healthier, increase their preference for leisure one gets the expected result that individuals would increase their saving so that they can end up retiring at a younger age. But even in this case saving is lower when the retirement age is flexible, and when the retirement age is state dependent individuals end up retiring later in the low productivity state than in the higher productivity state.

Defined benefits and flexible retirement have in common the attribute that they both protect the retirees against adverse states of nature in the second period of their life. This appears clearly in Table 4 which presents the expected utility levels for 5 values of the parameter that provides the amount of defined benefits over the three retirement regimes. One first sees that the individual expected utility increases as we move from mandatory to flexible retirement. We also observe that for each retirement regime the defined benefit parameter bringing the highest utility varies. It decreases as we go from mandatory retirement (.25) to state independent retirement (.20) and then to state dependent retirement (.10). This table shows quite clearly that flexible retirement is a substitute for defined benefit plans.

Table 4
Utilty level for different retirement regimes and shares of defined benefit saving.

Policy	State Dependent	State Independent	Mandatory
$\theta = .3$	.902804	.902797	.894792
$\theta = .25$	.903876	.903837	.895219*
$\theta = .2$	.904408	.904319*	.895137
$\theta = .1$	.904413*	.904183	.893810
$\theta = .05$	.904040	.903717	.892662

where * indicates the highest level of expected utility for each of the three retirement scenarios.

Another issue that has been explored widely in the literature is the shift that has been occurring from defined benefit retirement plans to defined contribution ones. The paper finds that with that type of a shift one should expect to see an increase in saving and in turn an increase in the capital stock as well as an increase in the retirement age of individuals as more of the uncertainty is shifted to the retirees. In addition, when the retirement decision is state dependent, the gap between the retirement age in the good and bad states is increased.

The paper also looks at the impact of changing one's views of the future. For example, if individuals live more for today and put less weight on the future one finds that saving will be reduced and retirement will end up being postponed. This also means that consumption in period 2 will be higher when the retirement age is flexible than when a mandatory retirement age is in effect. The other important issue explored is the impact of an increase in uncertainty. When uncertainty increases individuals reduce the amount they save through their defined contribution plan so that they can keep consumption in the low productivity state of period 1 from falling too much. To offset the lower saving individuals are forced to retire later. Moreover, when the retirement decision is state dependent, the reduction in saving is even larger and hence the amount of consumption in period 1 state 2 will be higher than when the retirement is independent of the state. Individuals feel that they can reduce saving so much when the retirement decision is state dependent since they can adjust their retirement decision in period 2 based on the amount of uncertainty they will be facing. Thus, there will be a relatively large increase in the gap in the retirement age between the good and bad states when the degree of uncertainty increases.

This paper suggests that studying the retirement decision in a world of uncertainty is likely to provide insights that the literature on retirement that assumes all life-cycle decisions are made in a world of certainty are missing. Not only do macroeconomic shocks affect individual retirement decisions but also individual surprises such as changes in health are important factors that need to be considered. In this paper we have deliberately focused on a simple setting: a representative agent, no social security and two forms of pensions (defined benefits and defined contributions). In a forthcoming paper (Pestieau and Possen, 2009) introduce individual heterogeneity along with a social security system to tackle similar issues to those found in this paper, and in particular the effect that shifting from defined benefit to defined contribution pensions has on the age of retirement.

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