2009/45

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CORE DISCUSSION PAPER
2009/45

Residential equilibrium in a multifractal metropolitan area

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July 2009

Abstract

A residential location model derived from urban economics is combined with the geometry of a multifractal Sierpinski carpet to represent and model a metropolitan area. This area is made up of a system of built-up patches hierarchically organised around a city centre, and green areas arranged in an inverse hierarchical order (large open-spaces in the periphery). An analytical solution is obtained by using a specific geographic coding system for computing distances. The values of the parameters used in the model are based on the French medium sized metropolitan areas; a realistic benchmark is proposed and comparative-statics simulations are performed. The results show that the French peri-urbanisation process (which took place from 1970 onward) can be explained by an increase in income and a reduction in transport costs. Nevertheless, changes in household preferences, in particular an increased taste for open spaces, can also contribute to urban sprawl by making the gradient of land rents less steep and by making peripheral household locations more desirable.

Keywords: peri-urban, residential localisation, fractal geometry, amenities.

JEL Classification: R12, R21

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This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1. Introduction

Most modern metropolises spread out over a large area comprising a patchwork of, on the one hand, "built-up sites" providing housing, jobs, amenities and/or urban public goods, and, on the other hand, "green areas", which are open spaces such as public gardens, parks, fields, meadows, or forests offering a pleasant living environment. Casual observations reveal that the size and the composition of built-up and green areas vary considerably with distance to the central business district (CBD). Correspondingly, residents enjoy a mix of amenities that depends largely on their dwelling places. In this paper, we propose a model of urban equilibrium that encompasses these two aspects: households' desire for variety in their consumption of urban and rural amenities and the heterogeneity of the geographical space. While the former is incorporated in a classical residential location submodel, the latter is represented by a geometric fractal figure made up of hierarchically organised urban sites and green areas. The parameters specifying the model are selected as close as possible from real-world values, in order to simulate the effects of various factors: income, transport costs, changing taste of households for amenities, etc.

Numerous empirical works have already studied the role of "green" amenities in metropolitan areas, particularly by estimating their hedonic prices (see e.g. Bender et al., 1997; Bolitzer and Netusil, 2000; Cheshire and Sheppard, 1995; Geoghegan et al., 1997; Hobden et al., 2004; Irwin, 2002; Mooney and Eisgruber, 2001; Paterson and Boyle, 2002; Roe et al., 2004; Thorsnes, 2002; Tyrvainen and Miettinen, 2000). Yet, analytical studies of this topic are quite rare. Microeconomic models of urban economics focussing on amenities have recently been formulated (Brueckner et al., 1999; Marshall, 2004; Turner, 2005; Wu and Plantinga, 2003), some being calibrated or estimated on structural equations (Bates and Santerre, 2001; Cavailhès et al., 2004b; Cheshire and Sheppard, 2002). The underlying geographical setting of these models is often a Thünian space with interlocking rings. Such a geometry is hardly suitable for modelling real-world settlements, which are made up of heterogeneous objects. Moreover, if it accommodates well classical microeconomics, it is ill-adapted to the recent developments of industrial economics (d'Aspremont et al., 1979; d'Aspremont et al., 1996) and of geographical economics (Krugman, 1991), which insist on product differentiation and consumers' preference for variety as fundamental characteristics of the modern world. The celebrated Dixit-Stiglitz (1977) and the Ottaviano, Tabuchi and Thisse (2001) models provide alternative analytical formulations of the preference for variety and allow to better specify the concept of monopolistic competition due to Chamberlin (1933). Moreover, a geography of variety seems better fitted by a Christaller (1933)-like setting, as differentiated bundles of commodities are proposed according to the hierarchical level of Christaller's urban centres.

If the « New industrial economics » and the « New geographical economics » have undergone important developments during the last twenty years, these paradigms have seldom been mobilized in Christallerian and related geographies. Now, if goods are differentiated to meet the consumers' preference for variety, the corresponding market space should also be adapted. We here propose to embed the behaviour of a consumer endowed with a preference for variety in a heterogeneous geographical space, so that economics and geography match. In the present stage of our research, which is only in its infancy because the concordance between economics and geography is new in urban and regional economics literature, the modeller must start from postulates. The selected geographical setting is a multifractal Sierpinski carpet. Of course, there is an infinity of alternative geometrical shapes (multiple variations on the Sierpinski carpet, teragons, Fourier dusts, etc.), or other 2D patterns (such as Christaller hexagonal organization, etc.), or irregular forms, such as in actual world. Because of this infinity, it is impossible for a social planner to have in mind all the possible forms, to compare their welfare and to choose the one that maximizes the social welfare. It is also impossible to imagine a self-generating procedure where the market yields the optimal pattern due to externalities that entails market failures. Thus, the social planner has to resort to an *a priori* spatial organization, just as we do here. The microeconomic model of residential location closely follows Cavailhès et al. (2004a). However, its most restrictive hypotheses are relaxed in the present paper. Indeed, on the one hand, the size of the residential plot depends on land price, which offers the household a trade-off between costs of land and costs of transport. On the other hand, because the Sierpinski carpet is here multifractal, we obtain larger residential sites in the centre of the carpet than in its periphery, and the opposite for the green areas that are larger in the periphery, as expected. This analytical model allows us to simulate the effects of changes in the economic parameters or in the household's preferences.

Section 2 of this paper presents the analytical models in which we develop the geometric model of the multifractal Sierpinski carpet, the spatial model (coding the coordinates of the sites and computing distances), and the microeconomic model of residential location. The results for a benchmark situation with parameters close to real-world values are presented in Section 3. Simulations of comparative statics are presented in Section 4, which enables us to discuss the properties of this economic-fractal representation. Section 5 concludes.

2. The models

2.1. The geometric model

2.1.1. The multifractal Sierpinski carpet

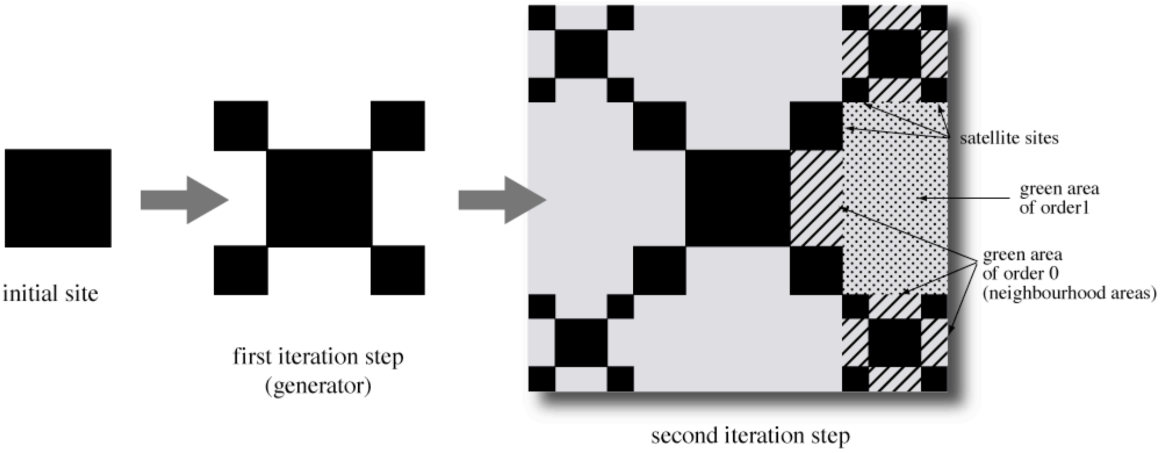
The objective is to model an urban area made up of a central city (the CBD) where jobs are concentrated, surrounded by suburban and peri-urban subcentres of various levels, themselves circumscribed by villages. Open spaces (green areas) of different compositions fill in the gaps left by the urbanisation process. The extent of the built-up and green patches depends on their locations. A multifractal Sierpinski carpet is a possible way to represent such a hierarchy.

Fractal geometry is frequently used for describing real world urban patterns or for verifying whether urban patterns follow fractal laws (see e.g. Arlinghaus and Arlinghaus, 1989; MacLennan et al., 1991; White and Engelen, 1993; 1994; Batty and Longley, 1994; Batty and Xie, 1996; Frankhauser, 1994; 1998; De Keersmaecker et al., 2003; Batty, 2005; Thomas et al., 2007 and 2008), as well as for describing urban dynamics by using, for example, cellular automata (see e.g. Batty, 1991 and 2005; Batty and Longley, 1994; Bailly, 1999). Unfortunately, most of these approaches have a loose theoretical background, even if some of them are grounded on economic foundations (for instance White and Engelen, 1993; 1994). They are mainly inductive (estimating a fractal dimension from the real world, for example) and it seems quite difficult to induce general laws from these works. It follows that hypothetico-deductive approaches must also be placed on the research agenda to explain the observed evolution of urban morphologies. Advances in knowledge proceed with the combination of empirico-inductive and hypothetico-deductive approaches, and both should not be opposed.

The present contribution pertains to the second approach. Nevertheless, despite a high level of abstraction, attention is paid to linking the abstract and the real worlds by means of simulations. In so doing, the properties of a fractal-economic model are examined using realistic parameters rather than arbitrary values.

The multifractal Sierpinski carpet used in this paper is constructed by an iterative outer mapping procedure.[1] It is initiated by drawing a square (called *centre*) and by appending four replicates (called *satellites*) scaled by a factor $\theta < 1$ at its corners. This defines the *generator* of the carpet. At the k^{th} iteration, we complete the pattern obtained at the previous iteration by placing its copy at the scale θ at each corner, so that the total number of squares (of various sizes) amounts to 5^k. Figure 1 allows one to visualize the two first steps. Figure 2 shows a branch of the pattern generated at the third step. The procedure stops after a given finite number K of iterations, K being the *order* of the carpet.

Figure 1: Multifractal Sierpinski carpet with the first two iterations



The geometric construct of squares and complementary empty areas is interpreted as follows. Each square of the Sierpinski carpet represents simultaneously a residential area and a place where public goods and urban amenities are available, but the range of the latter varies according to the relative position of the square in the multifractal, with a strong Christaller-like flavour. The initial square is called CBD (Central Business District) or "urban centre of order K". The CBD is considered as offering a complete range of urban amenities and public goods. It is also the place where all employments are concentrated. The central squares of each peripheral patterns appended at the final iteration K are supposed to offer a lower range of amenities; they are referred to as "urban centres of order K-1"; obviously there are 4 such centres. The 4 central squares of the appended patterns at iteration K-2 as well as their 16 reduced copies arising in the next iteration are the "urban centres of size K-2", etc. The initial four satellite sites created at iteration 1 and their subsequent counterparts offer only basic local public goods (considered as being the 0-level).

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[1] The more usual inner mapping procedure is not suitable for modelling a metropolitan area. Since the iterations are performed by fragmenting the initial site, this procedure would lead to an area for the urban sites that would shrink in the course of the iterations. We are aware that the Sierpinski carpet is oversimplistic with regard to the real world and we do not exclude that other fractal models such as "dragons" (teragons), Fournier dusts can also be of great interest for modelling urban patterns

The complementary areas are interpreted as non-residential zones offering goods, facilities and "rural" amenities. They are of different sizes, as well as regularly and hierarchically organized in accordance with the Sierpinski carpet: green areas close to the CBD are smaller than those located in the periphery.[2] We suppose that the rural amenities can be ranked into $K+1$ classes. First, we consider the four rectangles adjacent to the initial square and its four satellites. We assume they only offer the lowest range of rural amenities, say order 0. There are 5^{K-1} similar local patterns, thus $4\times5^{K-1}$ rectangles of different sizes offering basic level rural amenities. Turning to the pattern generated at the second iteration, we see on Figure 1 that there are 4 rectangles lying between the core pattern and its reduced replicates. Each of these rectangles provides rural amenities of order 1. In the final structure, we find $4\times5^{K-2}$ analogues. Proceeding in this way, we eventually identify four interstitial rural areas of order K-1. Finally, the area outside the convex hull of the carpet is the zone of order K offering the largest range of rural amenities.

Finally, we assume that the variety-seeking consumers are obliged to travel to enjoy each type of urban and rural amenities. This implies a communication network. In the present framework, movements are restricted to urbanized areas. Complex trips are then a composition of straight lines included in the squares of the carpet.

In summary, a social planner faces an infinity of possible geographical patterns. We postulate that he or she is aware of the consumer's taste for variety, as modern economics suggests. Therefore, he or she opts for a multifractal Sierpinski carpet because this form offers a range of urban and rural goods, fulfilling this taste for variety. The drawback of this exogenous geographical shape is that it cannot answer the optimality issue, due to the infinity of families of parameters that command the urban patterns. Moreover, the multifractal Sierpinki carpet also allows to fulfil three requirements: (i) yielding a continuous network to access the CBD and any urban centre through the residential sites (which would not be the case for instance with a Fourier dust); (ii) being coherent with the utility function introduced in the next section, more precisely a CES subutility function expressing the preference for variety; (iii) allowing the highest sites in the urban hierarchy to be also the largest in size.

Let us emphasize that we assume implicitly a perfect coherence between the transportation network and market areas, which is not the case for Christallers' network. It is evident that the choice of this particular spatial model is in some sense arbitrary. It would indeed be possible to introduce an equivalent multifractal pattern based on a hexagonal logic like that of central place theory (Frankhauser 2008). However in such a pattern the shape of interstitial free space and particularly its hierarchical organization is rather complex and doesn't allow identifying in a simple way different level of free space according to their surface. Let us be aware that for fractal models the situation is rather different from that of spatial models referring to Euclidean geometry. In Euclidean geometry only two types of figures allow covering space uniformly, squares and hexagons. This argument becomes obsolete for fractals, since they do no longer cover space uniformly, by definition. Hence it seems indeed impossible to determine an optimal spatial pattern what justifies to refer to the simplest one, which is square-like (cf. footnote 2).

This makes evident too, the crucial difference between Christallers' network, which refers to Euclidean geometry and a fractal (even hexagonal) network. In the last one, central places are distributed in a *non-uniform* way and this is why free areas separating central places form a hierarchically organized spatial system offering a high diversity of green areas with respect to their size. This allows introducing, as shown, a panel of leisure areas corresponding to

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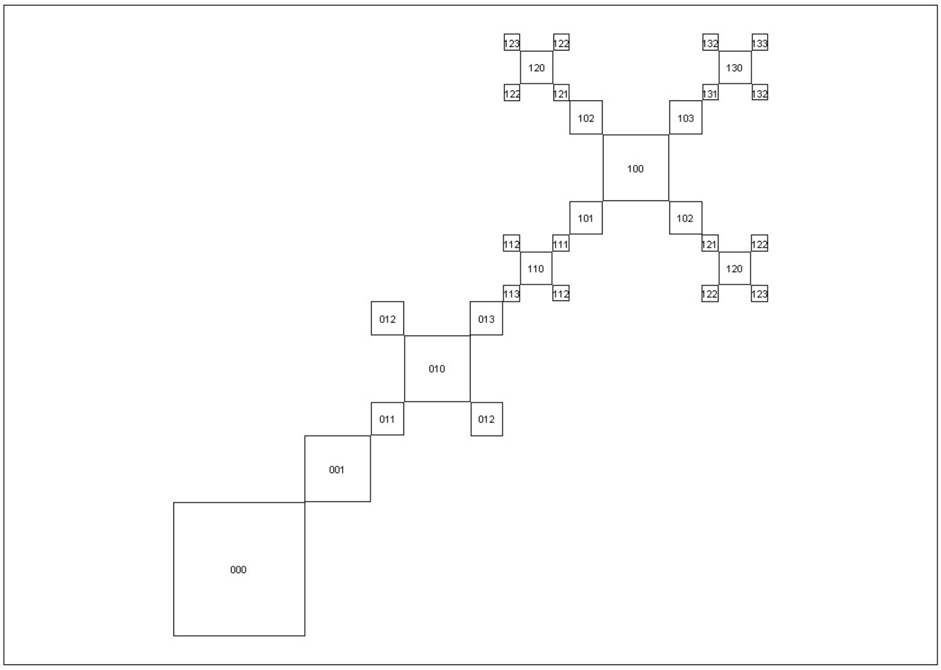
[2] The largest ones are, for instance, forests, while the smallest ones are public gardens or squares.

different types of amenities.

2.1.2. Computing distances

Households commute to the CBD for working and to urban and rural areas for enjoying amenities that are implicitly supposed not transferable. The aim of this section is to produce formulae for computing the distances between each residential area and its closest urban centre of order k. We limit ourselves to the case $K = 3$. The formulae rely on a coding scheme introduced in Cavailhès et al. (2004a). In short, a code composed of three digits ($C_3C_2C_1$) is created for each residential site. C_1 is set equal to 0 for each urban centre, whatever its order k. The value of C_1 for the four satellites is conditional on their positions in the structure: $C_1 = 1$ is attributed to the satellite closest to the urban centre of immediately higher order $k + 1$, $C_1 = 2$ to the two satellites on the lateral branches, and $C_1 = 3$ to the satellite opposite to the one labelled $C_1 = 1$. Next, we set $C_2 = 0$ for the centres of level at least 2. The subordinate centres of order 1 receive their C_2 label exactly in the same way as proceeded above for the satellites, while each satellite inherits the value C_2 of the centre of order 1 from which it depends. Finally, we set $C_3 = 0$ for the CBD, $C_3 = 1$ for the four urban centres of order 2, and each site inherits the value of C_3 of its tributary centre of order 2. The coding scheme is illustrated in Figure 2.

Figure 2: Coding of residential sites (north-eastern part of the Sierpinski carpet)



The distance between a site ($C_3C_2C_1$) and a site ($C_3'C_2'C_1'$) is noted $d(C_3C_2C_1, C_3'C_2'C_1')$. If ($C_3'C_2'C_1'$) is a centre of order k, we simplify the notation by writing $d_k(C_lC_mC_n) \equiv d(C_lC_mC_n, C_3'C_2'C_1')$ for each site ($C_lC_mC_n$) in its supply area. The computation of the distances between these points must take into account that the sites can be of different sizes in a multifractal Sierpinski carpet.[3]

Firstly, we consider the generator (Figure 3). Without loss of generality, we take the half-diagonal of the initial site as the unit of distance. At the first iteration, the half-diagonal of the

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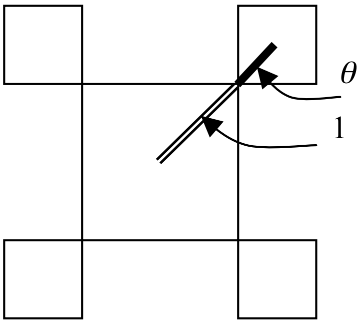
[3] For technical reasons, we must introduce a small intra-site distance; this does not affect the simulation results.

four satellite sites of the generator is reduced by the factor θ. In the same way, at each following iteration, smaller sites are generated whose half-diagonals are multiplied each time by the factor θ.[4]

Figure 3: Measuring the distances



For computational purposes, distances are decomposed into partial distances. The principle of the method can be demonstrated on the itinerary linking the sites labelled (132) to the CBD, that is $d(132, 000) \equiv d_3(132)$ (Figure 2). (132) is subordinate to the urban centre of order 2 (100), directly linked to (000). Moreover, sites (132) are satellites of the centre of order 1 (130), itself tributary to (100). So:

$$d_3\,(132) \equiv d(132, 000) = d(100, 000) + d(130, 100) + d(132, 130)$$

$$\equiv d^{(3)}(100) + d^{(2)}(130) + d^{(1)}(132).$$

Here we have introduced the partial distance $d^{(k)}$ whose index indicates that it refers to the distance to the closest centre of order k. Thus:

$$d^{(3)}(C_3 00) = d(C_3 00, 000);$$

$$d^{(2)}(C_3 C_2 0) = d(C_3 C_2 0, C_3 00);$$

$$d^{(1)}(C_3 C_2 C_1) = d(C_3 C_2 C_1, C_3 C_2 0).$$

Summing up, we have:

$$d(C_3 C_2 C_1, 000) = d^{(3)}(C_3 00) + d^{(2)}(C_3 C_2 0) + d^{(1)}(C_3 C_2 C_1).$$

More generally, the partial distances can be written in general as

$$d^{(k)}(C_3 C_2 C_1) = \zeta^{(k)} \theta^{l(k)}\, (1+\theta)^{m(k)}\, (1 + 2\,\theta)^{n(k)},$$

where the exponents $l(k)$, $m(k)$, $n(k)$ depend on the code $(C_3 C_2 C_1)$ and on the order k. The prefactor $\zeta^{(k)}$ is an operator whose values depend on order k allowing for the fact that one does not necessarily always have to pass through the centre with the higher rank: for example, in order to get to site (000) from sites (111), (110), (113), etc., it is not necessary to go through site (100). This factor has a constant value of $\zeta^{(3)} = (1 - \delta_{C3,0})$ for $k = 3$ where the function $\delta_{i,j}$ is defined as

$$\delta_{i,j} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$$

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[4] In order to guarantee that, for all residential areas, the distance to the nearest centre of order k really corresponds to that we expect according to the coding (i.e. the logic of the Sierpinski carpet) we must assume that $\theta \geq 0.5$. If this condition did not hold it would no longer be true that the logical centre was the closest (e.g. the distance from (101) to (100) would be less than the distance from (101) to (110). Hence, for goods of order k =1, consumers would prefer to go to (110).

For the other k-values, the operator can take the values –1, 0 and +1. By introducing the value 0, the terms that are not present for certain sites can be eliminated: for example, the distances $d^{(3)}(C_300)$ and $d^{(2)}(C_3C_20)$ for site (130) would be the same as for site (133), but the term $d^{(3)}(C_3C_2C_1)$ vanishes for site (130). On the other hand, the value –1 reflects the logic of accessibility: the distance d_3 (133) differs from d_3 (131) because site (131) is located on the axis which links (130) to the centre (100), and so is closer to the centre (000) than sites (132) and (133). For order $k = 2$ this gives:

$$\zeta^{(2)} = (-1)^{(\delta_{C2,1}-\delta_{C3,0})}(1-\delta_{C2,0})(1-\delta_{C2,0}\delta_{C1,0}).$$

For $k = 1$, certain sites change their character: for example, site (111) is closer to site (100) than site (113), but it is further from site (000) than site (113). In general

$$\zeta^{(1)} = (1-\delta_{C1,0})(-1)^{\delta_{C1,1}(1-\delta_{C2,0}\delta_{C3,0})+\delta_{C2,1}(1-\delta_{C3,0})(\delta_{C1,1}+\delta_{C1,3})}.$$

Up to now we have only considered the distances from the residential areas to the CBD. A similar formulation may be given for centres of other orders. For example, the distances to the centres of order 2 are given by

$$d_{k=2}\,(C_3C_2C_1) \equiv d(C_3C_2C_1, C_300) = d^{(2)}(C_3C_20) + d^{(1)}(C_3C_2C_1).$$

The general formula for the partial distances remains valid, but the formulae for the prefactors change. Indeed the partial distances can no longer be negative for $k = 2$, which yields:

$$\zeta^{(2)} = (1-\delta_{C2,0})(1-\delta_{C2,0}\delta_{C1,0}).$$

For similar reasons, the formula for $\zeta^{(1)}$ is also simpler:

$$\zeta^{(1)} = (1-\delta_{C1,0})(-1)^{\delta_{C1,1}(1-\delta_{C2,0})}.$$

To reach centres of order 1, only one partial distance remains. The prefactor is then always +1 or 0, and

$$\zeta^{(1)} = (1-\delta_{C1,0}).$$

It is also possible to find a general formula for the indices $l(k)$, $m(k)$, $n(k)$ by introducing a new code $\widetilde{C}_i$ (e.g. $\widetilde{C}_i = 1$) for all $C_i \neq 0$. With this binary coding, all the satellite sites are then equalised (this is possible because we have taken account of the reductions in distances of certain sites with the value of the prefactor $\zeta^{(k)}$). By computing the differences $\Delta\widetilde{C}_i = \widetilde{C}_i - \widetilde{C}_i' \quad i = 1,2,3$, the exponents $l(k)$, $m(k)$, $n(k)$ of the partial distances $d^{(k)}(C_3C_2C_1)$, $k = 1,2,3$, are linked to the codes $\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3$ and $\widetilde{C}_1', \widetilde{C}_2', \widetilde{C}_3'$ as follows:

$l(3) = 0$	$m(3) = \Delta\widetilde{C}_3$	$n(3) = 2\Delta\widetilde{C}_3$
$l(2) = \widetilde{C}_3\Delta\widetilde{C}_2$	$m(2) = \Delta\widetilde{C}_2$	$n(2) = \Delta\widetilde{C}_2$
$l(1) = (\widetilde{C}_2 + \widetilde{C}_3)\Delta\widetilde{C}_1$	$m(1) = \Delta\widetilde{C}_1$	$n(1) = 0$

A few remarks close this section. First, the formulation remains valid if we consider the distances between sites (C_1, C_2, C_3) and centres of different hierarchical levels such as (010) or

(110). Second, a similar formula can be used for computing distances to the rural areas, but the expressions are extremely cumbersome and not reported here. Third, notice that the distances depend on the surface unit and on θ. The estimation of these parameters is described in Section 3.2. Finally, although we limited ourselves to three levels of urban and rural amenities, the coding scheme and the procedure for computing the distances can readily be extended to any level $K > 3$.

2.2. Microeconomic model of residential localisation

We consider a household residing at the site i of the Sierpinski carpet. Its annual income W is allocated:

- to renting a residential plot of size Z_i at a cost of $R_i + R_A$ (R_A is the agricultural opportunity rent, while R_i is the differential rent at i);

- to commuting to work to the CBD located at a distance d_{Ki} (with N being the annual number of trips and t the unit commuting cost);

- to purchasing a quantity X_i of a composite good made up of all the goods other than housing and travel; the price of this composite good is used as the numeraire in this study;

- to acquiring a differentiated range of urban and rural amenities. Since we are not interested in the provision of public goods,[5] we will consider that the latter are obtained free of charge, but that they are local in the sense that the household must travel to enjoy them. Therefore, only trip costs are incurred here. The geometrical setting is the Sierpinski carpet of order K introduced previously. The urban (rural) amenities of order $k = 0, \dots, K$ are available at urban (rural) spots situated at distances $d_{0i}, \dots, d_{Ki}$ ($e_{0i}, \dots, e_{Ki}$) from the household respectively. Section 2.2.2 was devoted to the computation of these distances. The (endogenous) annual number of trips for each class of urban (rural) amenity are $a_{0i}, \dots, a_{Ki}$ ($b_{0i}, \dots, b_{Ki}$). The unit cost of transport is t, no matter what the type of trip.

The household budgetary constraint can be written as:

$$W = (R_i + R_A)Z_i + tNd_{Ki} + X_i + t\sum_{k=0}^{K} d_{ki}a_{ki} + t\sum_{k=0}^{K} e_{ki}b_{ki} \tag{1}$$

To keep notation simple, we omit the index i in the rest of this section. The available income is $W_D = W - tNd_K$, i.e., the gross income minus the cost of commuting to work.

Household's preferences are expressed by a mixed Cobb-Douglas / CES utility function (see, e.g., Dixit and Stiglitz, 1977; Fujita and Thisse, 2002; Krugman, 1991), so that the household's problem can be written as:

$$\mathrm{Max}_{X, a_1, \dots, a_K, b_1, \dots, b_K} U = \frac{1}{\alpha^\alpha \beta^\beta \gamma^\gamma \delta^\delta} X^\alpha Z^\beta \left(\sum_{k=0}^{K} a_k^\rho \right)^{\frac{\gamma}{\rho}} \left(\sum_{k=0}^{K} b_k^\sigma \right)^{\frac{\delta}{\sigma}} \tag{2}$$

subject to (1) and the obvious positivity requirements. We can also specify, without loss of generality, that $\alpha + \beta + \gamma + \delta = 1$, $\alpha, \beta, \gamma, \delta > 0$. The parameters $\rho < 1$ and $\sigma < 1$ are generally interpreted as indicators of the household' desire for variety. Choosing such a utility function implies that the household acquires each level of urban and rural amenities.

The problem can be solved by two-step method suggested by Fujita et al. (1999). Let

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[5] The quantity of open spaces and urban public goods is determined by the choice of the multifractal Sierpinski carpet. Their management by the public authority is independent of the visits: this is a fixed cost for the municipal budget. Taking it into account in the budget constraint of the households by a tax would complicate the model without bringing substantial inlights.

$$D = \left[\sum_{k=0}^{K} d_k^{\mu}\right]^{\frac{1}{\mu}}, \quad E = \left[\sum_{k=0}^{K} e_k^{\nu}\right]^{\frac{1}{\nu}}, \quad \mu = \frac{\rho}{\rho - 1}, \quad \nu = \frac{\sigma}{\sigma - 1} \tag{3}$$

where the coefficients D and E can be respectively interpreted as global indices of remoteness from the urban and rural amenities. We then compute the optimum consumption of goods

$$X^* = \alpha W_D, \quad Z^* = \frac{\beta W_D}{R + R_A}, \quad A^* = \frac{\gamma W_D}{tD}, \quad B^* = \frac{\delta W_D}{tE} \tag{4}$$

and the optimal number of household's trips for the different types of amenities:

$$a_k^* = \left(\frac{d_k}{D}\right)^{\frac{1}{\rho-1}} A^*, \quad b_k^* = \left(\frac{e_k}{E}\right)^{\frac{1}{\rho-1}} B^*, \qquad k = 0, \ldots, K \tag{5}$$

Consequently, the optimal level of utility, U^* is given by

$$U^* = \frac{1}{(R + R_A)^{\beta} t^{\gamma+\delta} D^{\gamma} E^{\delta}} W_D \tag{6}$$

At the equilibrium, each household, no matter what its residential location, reaches the same level of utility. Moreover, we assume an open city situation, so that this level of utility is given by its value in the rest of the world:

$$U^* = \overline{U} \tag{7}$$

In this open city framework, the equilibrium is an optimum, providing the households with the maximum level of utility, $\overline{U}$. Nevertheless, it is impossible to determine if it the *optimum optimorum*, that maximizes the aggregated social welfare, due to the reason given in Section 2.1.

We can then determine the amount of available income in an area i at equilibrium and, in this way, the land rent in this area:

$$R = R_A + \left(\frac{W - tNd_K}{\overline{U} t^{\gamma+\delta} D^{\gamma} E^{\delta}}\right)^{\frac{1}{\beta}} \tag{8}$$

The optimal size of a plot is given by:

$$Z = \frac{\beta(W - tNd_K)}{R_A + \left(\frac{W - tNd_K}{\overline{U} t^{\gamma+\delta} D^{\gamma} E^{\delta}}\right)^{\frac{1}{\beta}}} \tag{9}$$

Based on the available household income and using (4), (5), (7) and (9), we obtain the optimal decision at equilibrium as function of the parameters (W, t, $\overline{U}$, R_A, N, α, β, γ, δ, ρ, σ).

3. Simulations: a benchmark

The equilibrium outcome described in the previous section has been obtained at the expense of many simplifying assumptions. In order to assess the relevance of the model, we perform some simulations inspired from real data, even if our setting remains very stylized: (i) the geography of the 2D-Sierpinski carpet is exogenous, although richer than the usual Thünen's rings; (ii) it is a regular/hierarchical geography, while in the real-world, the seedling of towns and villages results from history and geography and does not follow such a regular distribution; (iii) the economy is hardly stylised: we consider only individual houses, households are identical, consumption of green amenities only entails transport costs, etc. The present simulations are thus far from a calibration of the model.

3.1. Method

We are interested in three outputs of the model: the land rent (Equation 8), the size of the residential plots (Equation 9) and the population of the metropolitan area Q. This latter is equal to the inverse of the size of the residential plots, given by (9), multiplied by Z, the area of the accommodating residential sites, which can be deduced from Section 2.1.2 (some further adjustments will be described later):

$$Q = \sum_n \frac{Z_n}{S_n} \tag{10}$$

where n are the residential sites ($n \in [1,125]$ when $K = 3$). Land rents, as given by (8), depend on the common utility, which is unobservable. For our purpose, we can more conveniently express the utility as a function of $R(0)$, the observable land rent in the CBD, since the indirect utility function of the households located in site (000) is

$$\overline{U} = \frac{W}{t^{\gamma+\delta} D(0)^{\gamma} E(0)^{\delta} \left[R(0) + R_A\right]^{\beta}}, \tag{11}$$

Substituting $\overline{U}$ from (11) into (8) and (9), we obtain expressions for land rent and the residential plot sizes that only depend on observable values. Together with (10), these are the results of our simulations.

In the real world, the land rents $R^o(x)$ and the sizes of the residential plots $Z^o(x)$ are functions of the distance from the centre of the urban area (the superscript o is used for observed values, while x is the distance from the centre of the urban area). The radius of the metropolitan area, f^o, and its population, Q^o, are also known. We have chosen parameters for the theoretical model in such a way that the predicted values $R^p(x)$, $Z^p(x)$, f^p and Q^p (the superscript p is used for predicted values) are close to the observed values. Some parameters (such as household income W) are known and available; others are more difficult to obtain and require some computation (e.g. the generalised transport cost t) or experts' opinion (e.g. R_A). The remaining parameters, such as those characterising preferences (β, γ, δ, etc.), are unobservable.

3.2. Parameters

The French Housing Survey carried out in 1996 by the INSEE (National Institute of Statistics and Economic Studies) contains most of the variables necessary for estimating/computing the parameters included in our model. We selected 23 urban areas in France between 100,000 and 200,000 inhabitants.[6,7] On average, a city centre is surrounded by (i) 15.3 suburban *communes* (the commune is the French lowest administrative level) containing 8200 inhabitants (median: 5,200), and (ii) a peri-urban belt containing 69,000 inhabitants spread over 74 communes (mean: 930 inhabitants/commune; median: 550). Hence, each urban area includes – on

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[6] A French urban area is a densely built up urban 'pole' (city-center + suburbs) offering more than 5,000 jobs, and a 'peri-urban belt' made up of communes which are not adjacent to the pole (separated by agriculture, forests, etc.), from which 40% or more of the working population commutes daily to another commune, generally to the urban pole.

[7] The scope of our investigation is limited to owner-occupied houses; this corresponds to 1,706 observations in the 1996 Housing Survey. We have eliminated (i) apartment buildings because the theoretical model applies to residential plots, and (ii) rented housing (not frequent for houses) since their rent is not commensurate with the price of purchasing a house.

average – 90 communes, that is to say a number quite close to the 125 sites of the Sierpinski carpet with $K = 3$. This explains our selection of cities.

However, the correspondence between the communes and the residential sites of a Sierpinski carpet requires some further adjustments. We assimilated the 5 generator sites to the city-centre of the urban area, the 20 sites of iteration 2 to the 15.3 suburban communes, and the 100 sites of step 3 to the 74 communes of the peri-urban belt. The unit of distance was chosen in relation to the size of the selected urban areas, by setting distance $d_{K=3}(133{,}000) = 24$ kilometres. Therefore, we derive both the value of the parameter θ and the surface unit, especially for the residential sites. A proportion of the area of the residential sites is not available for housing: it is dedicated to road networks and economic activities, which represent 40% or so of the urban space (based on French national data on land use).

The other parameters are derived from the 1996 French Housing Survey (further explanations are available from the authors upon request):

- household income: $W = 30{,}300$ €/household/year;
- discounting rate for housing prices: 5%;
- urban land rent at the CBD: $R(0) = 5.06$ €/m^2/year (see Appendix 1);
- agricultural rent $R_A = 0.015$ €/m^2/year (source: experts' opinion);
- generalised unit transport cost (t): the sum of the direct monetary cost (computed on the basis of the French Fiscal Administration at 0.30 €/km in 1996) and the opportunity cost of time which is evaluated by experts at 0.15 €/min. On the basis of a round-trip and a speed computed from the Housing Survey, we obtain an annual generalised transport cost of 0.9 €/km;
- we estimate that a worker commutes to work 200 times per year; this computation assumes 5 weeks of paid holidays, 5 working days per week, 10 days of public holidays and 25 other days that were not worked (part-time, flexitime, sick leave, maternity leave, etc.). We assume that a household is composed of 1.5 workers commuting separately. Hence, $N = 300$.
- all members of a household travel together to urban/green amenities (one single trip by household and visit).

As for the parameters of the utility function, the following values have been chosen. β is the proportion of available income (after commuting costs) devoted to residential consumption; here it is set at 0.20.[8] We have selected identical initial values for the parameters which characterise taste in urban and green amenities: $\gamma = 0.065$ and $\delta = 0.065$ (these values vary in the static comparative simulations). Substitutability between amenities are fixed at $\rho = -30$ and $\sigma = -30$; these values are a compromise between complementarity and unitary substitutability. The multifractal parameter is equal to 0.50, which means that during the first iteration, the central site represents 50% of the built-up area while each of the four peripheral sites represents 12.5%.

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[8] The French Housing Survey indicates that, for households that have recently become landowners, repaying the loan accounts for 22% of the gross income (or about 25% of the income, net of commuting). Nevertheless, the theoretical model takes into account the land rent (equal to the price of the land, whereas the building itself is included in the composite a-spatial good). The price of land represents roughly 28.4% of the total cost, which results in a value of β close to 10%, higher than that generally found in the literature. Hence, β is fixed at 20%.

3.3. Results

Figure 4. Lot size

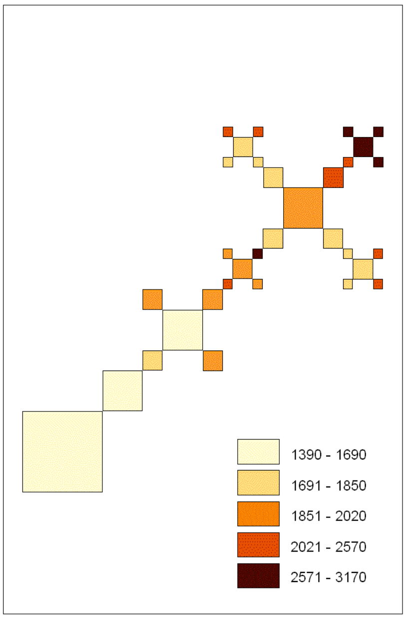


Figure 5. Population

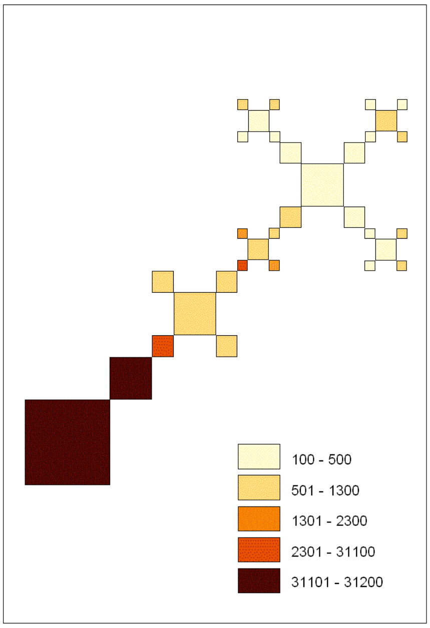
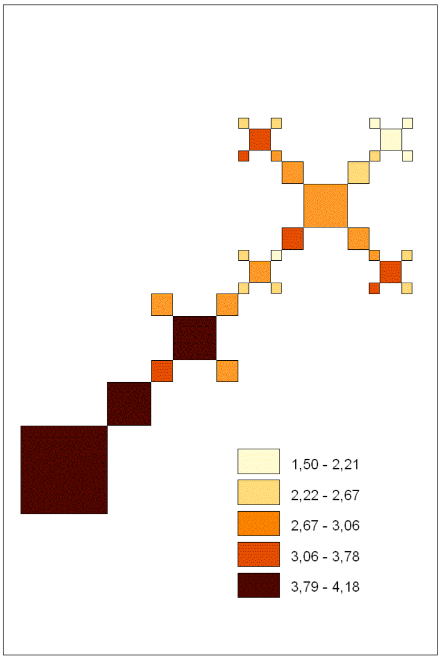


Figure 6. Land rent



Figures 4, 5 and 6 show the spatial distribution of the residential plot sizes, the land rent and the population. For reasons of clarity, the graphical representation is restricted to the northeast quadrant.[9] As expected, an increase in plot size, a decrease in land rent, and a reduction in population with distance from the CBD are observed (in some simulations, some of the peripheral sites could be uninhabited). The size of the residential plots is 1390 m^2 in the central site and 3170 m^2 on the periphery. However, the progression is not monotonic: for instance, the sites located on the principal diagonal are poorly located both in relation to urban public goods in the central site and to green amenities on the periphery; hence, large residential plots compensate for this poor accessibility. Obviously, land rent has an inverse relationship with plot size.

The residential plots derived from the model are considerably larger than those observed in the real world (400 m² at the centre of the cities and 1000 m² at the periphery, that is to say 3 times larger or so); land rent is also higher in the model, but by a smaller proportion (it is the same at the centre, and near 3 times higher in the periphery). The population of the model is too large in the centre of the commune (31,000 versus 26,000 in the actual world), too low in the suburban communes (230 inhabitants versus 280) and again higher than the actual value in the peri-urban communes. However, bearing in mind the high level of abstraction of the model, the order of magnitude of most of the figures give rough estimates that allow for some comparative static simulations to be undertaken by varying the parameters characterising households' behaviour and the economy. This is the objective of Section 4 below.

4. Comparative statics

Common wisdom attributes urban sprawl to increase in population and income and to decrease in transport costs. However, for several decades, urban expansion has taken on a particular pattern. In 1980, the American House of Representatives said that it was "like Swiss cheeses with more holes than cheese" (cited by Burchfield et al., 2006). Therefore we are interested in identifying factors that may explain this discontinuous development, or "leapfrogging". Numerous authors have added a shift in household preferences for a "green" living environment to the just mentioned three factors to explain these new forms of metropolisation (see, e.g., Brueckner et al., 1999). Here, we rather examine how the benchmark situation developed in the preceding section evolves as these factors change.

4.1. Changes in preferences

Figures 7 and 8 illustrate the variation in the size of residential plots (in m^2) when the preference for green amenities is greater than for urban public goods ($\gamma = 0.03$ and $\delta = 0.1$) and conversely ($\gamma = 0.1$ and $\delta = 0.03$). In the benchmark simulation these parameters were equal. Land rent is not presented here because the pattern is almost exactly the inverse of that of the size of residential plots.

When the preference for green amenities is greater than for urban public goods (Figure 7), the size of residential plots is often smaller than in the benchmark situation, particularly on the periphery of the Sierpinski carpet. This indicates a substitution between residential consumption and the consumption of green amenities. Nevertheless, there is an increase in the size of residential plots for most of the sites on the principal diagonal, particularly in the centre of the carpet: poor accessibility to green amenities is compensated by larger parcels of land. The changes for the population distribution (not illustrated here) show a shift from the centre towards the periphery of the Sierpinski carpet.

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[9] We are aware that, strictly speaking, population should be mapped by proportional symbols; it is represented here by a choropleth map to facilitate visual comparisons.

Figure 7. Simulations Increased preference for green amenities: Plot size

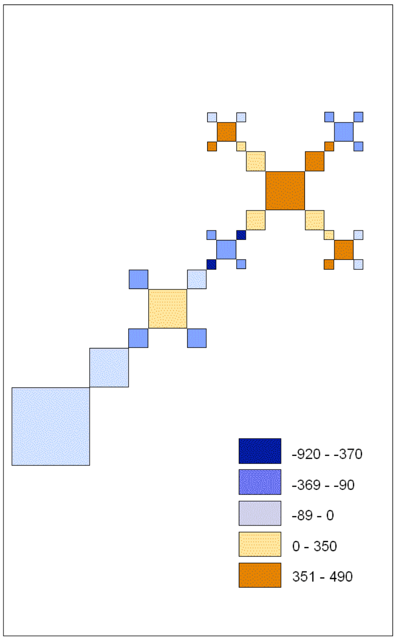
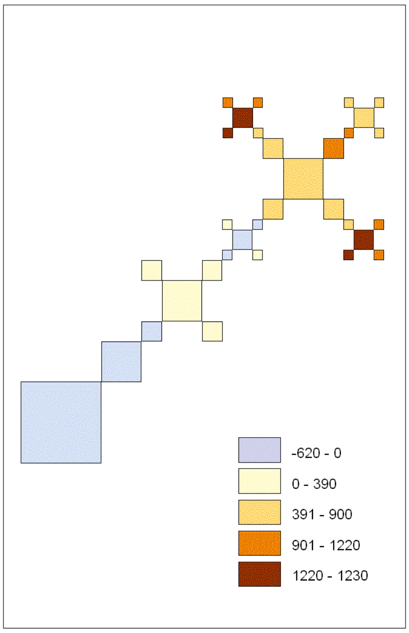


Figure 8. Simulations Increased preference for urban public goods: Plot size



When urban amenities are preferred to green amenities (Figure 8), households living on the periphery have larger residential plots to compensate for longer trips to urban sites, including the CBD. When the preference for urban amenities further increases, the tropism of the CBD becomes stronger: even in the urban centres with a high level in the urban hierarchy (such as those labelled 100) the size of the residential plots decreases because they are not central.

We have also analysed the effects of changes in the substitutability of green and urban amenities (parameters ρ and ρ). In both cases, the plot size decreases when substitutability between amenities increases: close green (or urban) amenities are consumed rather than more remote alternative types of green (or urban) amenities. This indicates that the larger the desire for variety, the better a geometric pattern such as the Sierpinski carpet for improving the welfare of inhabitants, as it offers a variety of urban and green sites through its interlocking scales.

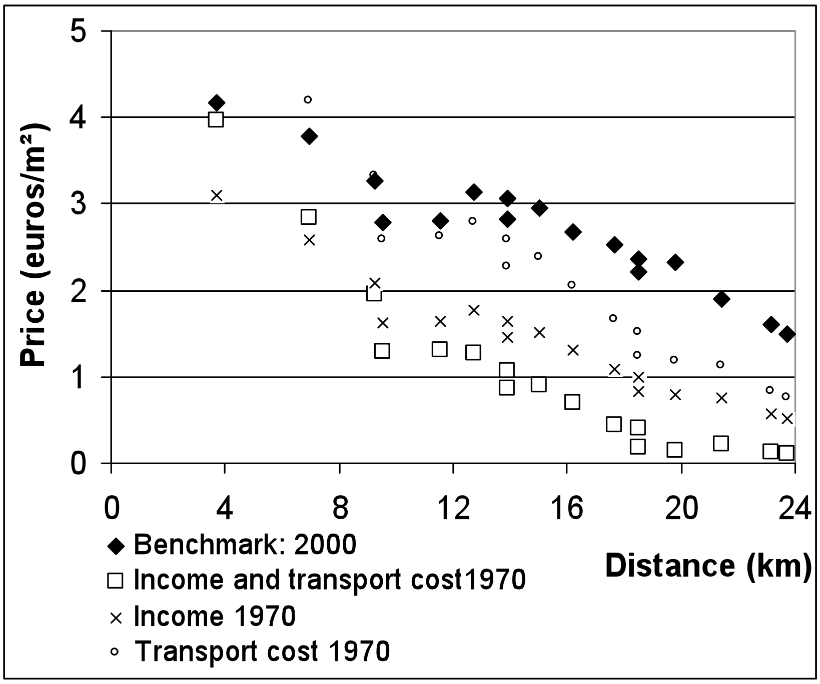
4.2. Increase in income and decrease in transport costs

Let us now consider a situation where the average income is lower than presently but the transport costs are higher. We know that individual income in France increased by 60% between 1970 and 2000 (adjusted for inflation). Simultaneously, transport costs decreased by 60%, which is justified as follows. According to the INSEE price index adjusted for inflation and for the quality effect, a car costs half as much in 2000 as it did in 1975 (INSEE, 1990 and calculations by the authors for the 1990s). The price of gasoline (adjusted for inflation) was in 1990 at the same level than in 1970 and 40% cheaper than in 1960 (INSEE, 1990). The travel speed also increased, particularly because of improvements in the road network, which halves the monetary cost of transport between 1970 and 2000 (calculation by the authors from the INSEE housing survey; INSEE, 2001). Accordingly, surveys by the INSEE suggest that commuting time remained constant from 1984 until 2001, despite an increase in trip length (increase by 52%; authors' computation). On the whole, we estimate the decrease in the unit transport cost by 60%. We can then reconstruct the situation in the early 1970s by examining the effects of changes in real income and transport cost, separately and simultaneously. Figure 9 shows their impacts on land rents.

Considering the two effects together, we see that land rents in 1970 were effectively zero at more than roughly 18 kilometres from the CBD, whereas by the benchmark date (2000) they were about 1.5 € per square meter at 24 kilometres. The low rent in 1970 means that the population was also approximately zero and the residential plots were huge: obviously, the front line of peri-urbanisation at this time had not reached twenty kilometres. The economic changes in income and transport costs seem to have played an important role in the urban spread that characterised the end of the twentieth century.

Figure 9 shows that income and transport costs do not have the same impact on rents at different distances. If income remains constant and transport costs decrease, land rents decrease by similar proportions no matter what the distance. In the opposite case, if only income increases, land rents are hardly affected up to twelve kilometres from the city centre, but decrease sharply beyond that. It is intuitively obvious, and well established (Wheaton, 1974), that a decrease in transport costs produces an increase in land rents on the periphery of urban areas. However, between 1970 and 2000, the effect of the decreases in transport costs is smaller than the effect of the increase in income.

Figure 9: Simulation of the effect of changes in income and transport costs on land rent



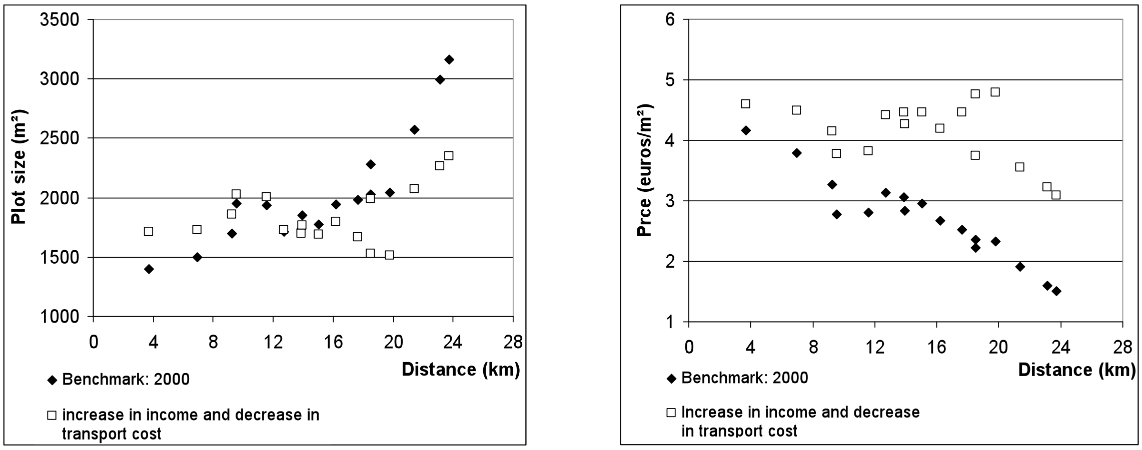
All in all, it appears that the peri-urbanisation movement observed in France over the last thirty years can be explained by the combination of two economic factors: the increase in income and the decrease in the cost of transport, without reference to changes in preferences. If these have also changed in favour of rural amenities, their effects can only be added to the two previous ones.

4.3. A continuing rise in income and decrease in transport costs

Let us now simulate a new increase in income (from an average of 30,300 to 40,000 € per household per year) and/or a new decrease in the unit transport cost (from 0.9 to 0.5 €/km/year). The results of these changes are illustrated in Figure 10. This shows that the effect of these changes on the average size of residential plots agrees with our expectations:

the size increases in the central sites and decreases in the periphery. However, the changes are smaller than those simulated above.

Figure 10: Simulations of the effects of an increase in income and a decrease in transport costs



on plot sizes

on land rents

Land rents flatten, increasing everywhere but particularly for the sites at some distance from the CBD, so that the rent at a distance of about 20 kilometres from the centre is as high as, if not slightly higher than, that in the centre. The effect on the population is not shown, but is very small; there is only a small additional migration towards the periphery.

5. Conclusions

Economists and geographers often stress the heterogeneity of metropolitan areas, which does not exhibit the homogeneity of interlocking Thünian rings, but is made up of interlaced built-up sites and recreational/agricultural open spaces. These urban patterns are poorly integrated into economic models because of the difficulty to model heterogeneity in a two-dimensional space (see e.g. Ogawa and Fujita, 1982; Lucas and Rossi-Hansberg, 2002). Geography is often better at representing the heterogeneous spaces that make up the core of the discipline, but this is often done without any economic or sociological theoretical background, even though the challenge here is to explain how human actions create these spaces. Anas et al. (1998) have already stressed this division between the two disciplines; nevertheless both are necessary to explain urban structures and their use should be complementary. The present paper tries to fill a gap in this respect.

Here we have used an urban microeconomics model of residential localisation, with the geometry (and hence the geography!) of a multifractal Sierpinski carpet. From an economic point of view, households consume differentiated urban public goods and green amenities, with a preference for diversified bundles. The variety of amenities reflects the variety of urban sites, which offer urban goods ranked according to the Sierpinski carpet hierarchy; the green lacunas separating urban areas are also hierarchically structured. The economic model leads to an analytical solution by using a coding system for the sites of the Sierpinski carpet, which allows the computation of distances between any two sites.

In order to study the properties of this model, we used a set of parameters derived from the observed statistical reality of medium-sized urban areas in France. We simulated retrospective or future changes, starting with a benchmark situation for the year 2000. Findings show that an increase in preferences for green amenities flattens land rents, residential plot sizes and population gradients, and lead to an overall extension of the metropolitan areas towards more outlying residential locations. Moreover, when the substitutability between urban and green amenities decreases, some complex patterns (such as the level of interlocking on the scale of the multifractal Sierpinski carpet) result, because they offer households a variety of differentiated public goods close to their homes.

The simulations performed on highly abstract models incorporating parameters resulting from observations are too coarse to draw clear-cut conclusions. However, regarding the economic aspects, it seems possible that, in a country like France, the sharp increase in real income over the last thirty years, combined with a decrease in transport costs, should explain the extent of peri-urbanisation, independent of possible changes in preferences.

The analyses performed in this paper illustrate the value of approaches combining economics and geography for representing spaces formed by heterogeneous objects. However, the level of abstraction is much too high to allow straightforward applications to real world urban patterns. Future studies should try to reduce the gap between the abstract world of models and the real world. Several directions are conceivable. First, we can think of a model where a 2D-settlement pattern made of residential and green cells (and also of network cells to meet the CBD) endogenously emerges in the market, instead of exogenous/hierarchical/fractal geometry. This framework is completely different from ours. Second, it is possible to combine several fractals (Sierpinski carpet, teragons and Fournier dusts) to generate the underlying geometrical structure, which would allow more complex geometric representations of urban areas while maintaining their theoretical intelligibility. Third, it might also be possible to achieve an econometric calibration based on a richer set of equations than the ones we have used. Finally, we can think that the rules of economic behaviour generating fractal patterns could be defined by other methods (cellular automata, agent-based modelling) rather than starting with an *a priori* geometry. This paper is merely a first attempt in this direction.

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Appendix 1: Estimating the land rent function

The logarithm of the housing price (real estate price) for households who had recently purchased a house in a medium-sized French city in 1996 was estimated by regression (Source: Housing Survey; 292 observations available). The independent variables were the living space in the house and the area of the garden (the sum of which is *Z*, the size of the residential plot in the model), the distance from the urban area (commune centre) to the CBD, the age of the building, the number of bathrooms, a dummy variable indicating if the commune belongs (1) or odes not belong (0) to the Mediterranean region, and the fiscal income of the commune (see Cavailhès, 2005 for more details).

Hausman's test shows that only the living space is endogenous. We used the 2SLS method with instrumental variables as our estimation technique. The instruments are here the characteristics of the household and the location; Sargan's test shows that they are exogenous (see Table A1).

Table A1: Regression coefficients for the variables in the regression on housing prices

Variable	**Parameter**	**Student *t***
Constant	8.03030	17.73
Living space (m²)	0.96448	9.53
Number of bathrooms	0.14404	6.44
Date of construction: ≥ 1990	0.39303	6.10
1982-89	0.27416	3.91
1975-81	0.29466	3.53
1968-74	0.27461	2.58
1962-67	0.49101	5.19
1949-61	0.03585	0.40
1915-48	0.04277	0.61
Area of the garden (m²)	9.354 E-5	4.02
Average communal income	0.00308	2.74
Mediterranean region	0.10190	1.94
Distance from the centre	– 0.08523	– 4.13
Distance from the centre (squared)	0.00636	2.47
Distance from the centre (cubed)	– 1.618 E-4	– 1.92

The total predicted real estate value was computed at the average point of the independent variables other than distance, and was annualised at a rate of 5%. The value obtained at the origin is 17.8 €/m²/year. For the 118 cases in which the plot of land and the building were purchased separately, we found that the land price averaged 28.4% of the total cost of housing. The land rent in the CBD (denoted *R*(0)) is then 5.06 €/m²/year. The parameters of distance are used in the computation of the error formula.

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