Competition among the big and the small

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Abstract

Armchair evidence shows that many industries are made of a few big commercial or manufacturing firms, which are able to affect the market outcome, and of a myriad of small family-run businesses with very few employees, each of which has a negligible impact on the market. Examples can be found in apparel, catering, publishers and bookstores, retailing, finance and insurances, and IT industries. We provide a new general equilibrium framework that encapsulates both market structures. Due to the higher toughness of the market, the entry of big firms leads them to sell more through a market expansion effect, which is generated by the exit of small firms. Furthermore, the level of social welfare increases with the number of oligopolistic firms because the procompetitive effect associated with the entry of a big firm dominates the resulting decrease in product variety.

Keywords: oligopoly, monopolistic competition, product differentiation, welfare.

JEL Classification: L13, L40

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1 Introduction

Armchair evidence shows that many industries are made of a few large commercial or manufacturing firms, which are able to affect the market outcome, and of a myriad of small family-run businesses with very few employees, each of which has a negligible impact on the market. Examples can be found in apparel, catering, publishers and bookstores, retailing, finance and insurances, and IT industries. To the best of our knowledge, such a mixed market structure has been overlooked in the literature.\footnote{The main noticeable exception we are aware of is provided by the dominant firm model in which one large firm and a competitive fringe coexist (Markham, 1951). Another set of contributions deal with “big agents” (formally, atoms) whose role in exchange economies have been studied within the context of cooperative game theory (Gabszewicz and Shi-tovicz, 1992). Finally, Neary (2003) uses a framework that combines a large number of sectors, each having a small number of firms. It is worth stressing that, despite some resemblance, all these models are very different from ours.} This is surprising because such a configuration of firms is both widespread and old. Indeed, although firms started operating at a very low scale at the beginning of the Industrial Revolution, the growth of larger and larger establishments, due to technological innovations, was progressive and slow, involving both large and small firms in various proportions according to the industry. Average plant size increased in most industries, but small and medium-sized firms continued to be, and still are, the rule in many sectors (Mokyr, 2002). It is also worth mentioning that business analysts stress the fact that firms within the same industry often form strategic groups that operate differently. In particular, an industry is typically divided into a group of big firms and a competitive fringe of much smaller firms (Porter, 1982).

Such a neglect is unwarranted from the viewpoint of competition policy because several countries have passed bills that restrict the entry of large firms or the expansion of existing ones, forbid price discounts or regulate the hours of operations in order to permit small firms to remain active. To illustrate, consider the case of the retailing sector, which has attracted a lot of attention in several countries. In France, the Royer-Raffarin Law imposes severe restrictions on the entry of department stores whose surface exceeds 300 square meters, the purpose being that small shops provide various convenience services. The Net Book Agreement in the United Kingdom between book publishers and retailers forbids discounts on books with the aim of preserving a large network of small bookstores, whereas in France the Lang Law, which also prevents price discounting, is argued by the publishers and small book sellers to be justifiable on the same grounds. It would be easy to mention more real-world examples.
Even though the objective of such laws and regulations was often to gain the political support of small-business associations, popular thinking in developed countries has it that small firms allow for a wider array of varieties and services, thus contributing to consumers’ welfare. Very much in the same spirit, starting with Sears Roebuck and the emergence of mail order firms in the late 19th century, the American public has proven wary of retail innovations. Wal-Mart, the largest retailer in America and the pioneer of the large discount chain store, experiences the same public wariness regarding its business practices. Even though several of the criticisms raised might be justified, we find it fair to say that the public often dismisses the fact that the presence of large retailers fosters much lower prices than small ones, thus making a wider array of goods accessible to a larger population of customers (Basker, 2007).

As a second motivation, we should add that the field of industrial organization is dominated by partial equilibrium models of oligopoly in which strategic interactions between firms appear to be the central ingredient. They now serve as the cornerstone of many competition policy studies of real-world markets (Motta, 2004). By contrast, monopolistic competition has been extensively employed as the main building-block in the analyses of imperfect competition within general equilibrium models developed in various economic fields. Examples include economic policy, growth and innovation, international trade, and economic geography (Matsuyama, 1995). It is fair to say that both approaches are useful to analyze different economic issues and have their own merits. However, as said in the foregoing, it is a fact that many industries consist of a few big firms and many small firms. In such industries, the big firms behave strategically, whereas small firms maximize their profits on the residual demand in the absence of strategic interactions. This paper may then be viewed as an attempt at providing a reconciliation between such different approaches to market competition.

The purpose of this paper is to provide a unified approach that embodies both big/strategic and small/nonstrategic firms and to assess the relevance of the above arguments. More precisely, we use this framework to study (i) how those two types of firms interact to shape the market outcome and (ii) whether or not it is socially desirable to have large and/or small firms in business. To reach our goal, we blend two standard models of imperfect competition, namely the oligopoly model à la Cournot with differentiated products and the monopolistic competition model of the Chamberlin-type.²

²The bulk of the literature on general equilibrium with oligopolistic firms focuses on Cournot competition. Therefore, to facilitate comparison we have chosen to focus on quantity-setting firms. Note, in passing, that the difference between price-setting and quantity-setting firms is inconsequential for the behavior of small firms.
On the production side, we consider a differentiated product market in which both oligopolistic and monopolistically competitive firms coexist. In measure theoretic-terms, each oligopolistic firm is an \textit{atom}, whereas each monopolistically competitive firm is \textit{negligible}. Because small firms typically exhibit more volatility than big firms in their entry behavior, we find it reasonable to assume that the mass of monopolistically competitive firms adjust to the number of oligopolistic firms until profits in the competitive fringe are zero, as in Chamberlin (1933). On the consumption side, we consider a utility function with a symmetric CES subutility, as in Spence (1976) and Dixit and Stiglitz (1977). However, unlike these contributions, our model involves both \textit{discrete} and \textit{negligible} varieties of the differentiated product. By so doing, we capture within the same preference framework the two specifications of the CES model that have been used in the literature (Vives, 1999; Matsuyama, 1995).

Our main findings are as follows. First, \textit{the entry of an oligopolistic firm extends the market of these firms at the expense of monopolistically competitive firms}. Hence, deregulating mixed markets is likely to lead to a progressive disappearance of small firms.\footnote{Recall that Dixit and Stiglitz assumed a continuum of varieties in their initial discussion paper reprinted in Brakman and Heijdra (2004).} Furthermore, this market expansion effect is sufficiently strong for the output of each big firm to rise. Similarly, it leads to a decrease in the price index of the industry as a whole, and so despite the fact that the entry of a big firm sparks the exit of small firms. This agrees with Bertrand and Kramarz (2002), who show that the enforcement of the Royer-Raffarin Law has had a negative impact on job creation in France. This in turn suggests that this regulation has lowered the output and increased the price index of the French retail sector, as suggested by our model.\footnote{In this respect, it is worth noting that there has been in the UK a sharp decline in the number of small groceries after the passage of the Resale Prices Act in 1964 abolishing resale price maintenance (Everton, 1993).} Conversely, a smaller number of large producers fosters the entry of small firms. This is illustrated by the recent evolution of the beer industry, which is characterized by the concentration of large producers as well as by the growing emergence of small breweries. Finally, our analysis reveals the existence of a new general equilibrium effect: because of the market expansion effect, big firms are better off when entry arises under the concrete form

\footnote{By restricting the entry of large retailers, the Raffarin Law has led to higher prices for food products. This has been estimated to a loss of €9 billion for French consumers in 2002 (Askenazy and Weindenfell, 2007).}
of a new large firm and the exit of a range of small ones.

In terms of welfare, we show the unexpected (at least to us) result that, despite the exit of small firms it triggers, the entry of a big firm is always beneficial to consumers. In other words, the loss of variety that the smaller mass of monopolistically competitive firms generates is more than compensated by the fact the whole industry becomes more competitive when more oligopolistic firms operate. Because it imposes no specific restriction on the structural parameters of the economy, this result casts serious doubt on the welfare foundations of the various laws and regulations that tend to keep active a large number of small businesses. Note, in passing, that our analysis provides analytical foundations to what Schumpeter wrote more than 60 years ago:

“In the case of retail trade the competition that matters arises not from additional shops of the same type, but from the department store, the chain store, the mail-order house and the supermarket which are bound to destroy those pyramids sooner or later.” (Schumpeter 1942, p.85)

Admittedly, our results are obtained in the case of a specific model, namely the CES. Being aware of its limits, we want to stress the fact that this model is the workhorse of many contributions dealing with imperfect competition in modern economic theory. So our results cannot be dismissed on that basis only. Although more work is called for, we believe that our analysis, by providing a unified framework encapsulating different market structures, uncovers new and unsuspected insights about a topic that has been so far neglected. In addition, it is worth stressing that we depart from standard partial equilibrium models in that the income level is endogenous in our framework. Finally, our approach reveals how the emergence of a mixed market structure depends on the size of economy and the level of fixed costs in the two subsectors.

That said, our analysis also sheds light on some of the major trends characterizing the market dynamics of developed economies. Considering a traditional economy which is typically populated with small businesses, our results show that more affluent societies and technological progress have combined to facilitate the entry of a growing number of big firms. This in turn has triggered the decline of the small business subsector in mixed markets endowed with old and small firms as well as modern and big firms. Such a result concurs with the prediction made by many observers, ranging from Karl Marx to Robert Lucas. However, our analysis also suggests that the fall in small firms' fixed costs sparked by the development of the new information technologies has permitted the revival of SMEs. Indeed, as predicted
by our model, the launching of small firms became again profitable from the 1980s, which has led to the progressive emergence of new mixed markets. The evolution of markets, therefore, seems to be a non-monotonic process, involving the transition from monopolistic competition to mixed markets through markets dominated by large oligopolistic firms. It is worth stressing that this agrees with a well-documented fact stressed in the economic and business literature on entrepreneurship, that is, the existence of a U-shaped relationship between entrepreneurship and economic development (see Wennekers et al., 2009 for a survey and empirical evidence).

The remaining of the paper is organized as follows. The details of the model are provided in Section 2. Section 3 deals with the main properties of a mixed market equilibrium. The welfare analysis is taken up in Section 4, whereas Section 5 concludes.

2 The model

2.1 The representative consumer

The economy involves two goods, two sectors, and one production factor - labor. The first good is homogenous and produced by the traditional sector under constant returns to scale and perfect competition. This good is chosen as the numéraire. Without loss of generality, we assume that one unit of the homogenous good is produced by using one unit of labor. Without loss of generality, we may assume that one unit of the homogenous good is produced by using one unit of labor. The equilibrium wage is thus equal to 1.

The other good is a horizontally differentiated good produced in the modern sector. It is supplied both by oligopolistic firms and by monopolistically competitive firms (MC-firms). Variables associated with oligopolistic firms are described by capital letters and those corresponding to MC-firms by lower case letters (this should help the reader to remember that an MC-firm is smaller than an oligopolistic firm). Each firm supplies a single variety, thus implying that oligopolistic firms do not contribute to product variety by supplying a product line, as in Brander and Eaton (1984). Let \( N \geq 2 \) be the number of varieties supplied by oligopolistic firms and \( M > 0 \) the mass of varieties supplied by MC-firms. In other words, the differentiated sector is

\[ \text{The choice of the numéraire in general equilibrium models with imperfect competition is often a critical issue because it may affect the nature of the market outcome (Bonanno, 1990). Since our purpose is to focus on the interactions between different types of firms belonging to the same industry, we find it reasonable to retain the homogeneous good as the numéraire (Ginsburgh and Keyzer, 1997).} \]
mixed in that it is constituted by two subsectors governed by distinct forms of competition, which interact according to rules that will be made precise below.

There exists a representative consumer who describes the aggregated behavior of a population of consumers having different tastes (Anderson et al., 1992). This agent is endowed with $L$ units of labor, holds the shares of all firms, and has a preference relation represented by the following utility function:

$$U = \left( \sum_{j=1}^{N} Q_j^\alpha + \int_{0}^{M} [q(i)]^\rho di \right)^{\frac{1-\alpha}{\rho}} \cdot X^\alpha$$

where $Q_j$ is the output level of oligopolistic firm $j = 1, ..., N$, $q(i)$ the output level of the MC-firm $i \in [0, M]$, $X$ the aggregate consumption of the homogeneous good, whereas $\alpha$ and $\rho$ are given parameters satisfying the inequalities $0 < \alpha < 1$ and $0 < \rho < 1$. The upper-tie utility being of the Cobb-Douglas type, the homogenous good is always produced. Hence, the equilibrium wage remains equal to 1 regardless of firm’s behavior in the modern sector. This highlights the role played by the traditional sector in our setting.

The novel feature of preferences (1) is that they incorporate both “discrete” varieties, $Q_j$, and “negligible” varieties, $q(i)$. Such a modeling strategy allows us to capture two important characteristics of a mixed market. First, each discrete variety has a positive impact on consumers’ well-being, whereas each negligible variety has a zero impact. This asymmetry reflects the idea that big firms provide additional attributes appended to their product - and, perhaps, a better access to other goods and services -, while small firms supply basic attributes only. Second, whereas there is a finite number of discrete varieties, there is a continuum of negligible varieties. Altogether small firms have, therefore, a positive impact on consumers’ utility that is comparable to big firms. These two features may be viewed as the demand counterpart of the assumptions made below about firms’ market behavior.⁷

Yet, the representative consumer has a priori no preference on the type of firms from which she buys, meaning that her choices are not biased in favor of small against big firms, or vice versa. However, the process of substitution between the two types of varieties is more involved than in standard oligopoly or monopolistic competitive models. To illustrate how it works, consider the situation in which the quantities of discrete varieties $j = 1, ..., N$ are the same and equal to $Q$, whereas the quantity density of negligible varieties

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⁷Alternately, we could have assumed that large firms supply a positive segment of varieties. We have chosen to use mass-points instead of intervals because this formulation would bias the result in favor of the large firms by allowing them to increase product variety. We will return to this issue in our concluding section.
is uniform and equal to $q$ over $[0, M]$. Let us now assume that there is a $(N+1)$th discrete variety and consider the variation of the total mass of negligible varieties that leaves the utility level unchanged. It is readily verified that $M$ must decrease by $\Delta M = (Q/q)^\rho$. Hence, despite the fact that consumers have a love for variety, for the utility level to remain the same, the entry of a new discrete variety is to be compensated by the exit of a positive range of negligible varieties.

The representative consumer solves the following maximization problem:

\[
\begin{align*}
\text{Maximize} & \quad \left( \sum_{j=1}^{N} Q_j^\rho + \int_0^M [q(i)]^\rho \, di \right)^{\frac{1-\alpha}{\rho}} \cdot X^\alpha \\
\text{subject to} & \quad \sum_{j=1}^{N} P_j Q_j + \int_0^M p(i)q(i) \, di + X \leq Y
\end{align*}
\]

where $P_j$ is the price of variety $j = 1, \ldots, N$, $p(i)$ the price of variety $i \in [0, M]$, and $Y$ the income level given by her wage plus the sum of distributed profits.

It is useful to decompose this problem into two steps. In the first one, we solve the following minimization problem:

\[
\begin{align*}
\text{Minimize} & \quad \int_0^M p(i)q(i) \, di \quad \text{subject to} \quad \left[ \int_0^M q(i)^\rho \, di \right]^{\frac{1}{\rho}} \equiv Q_0
\end{align*}
\]

where we interpret $Q_0$ as the output index of the MC-subsector. The first order conditions for an interior maximum are as follows:

\[
\begin{align*}
p(i) &= \mu \rho [q(i)]^{-(1-\rho)Q_0^{1-\rho}} \\
\int_0^M q(i)^\rho \, di &= Q_0^\rho
\end{align*}
\]

where $\mu$ is the Lagrangian multiplier. Set $R \equiv q(i)[p(i)]^{1/(1-\rho)}$ and let

\[
P_0 \equiv \left[ \int_0^M (p(i))^{-\frac{1-\rho}{\rho}} \, di \right]^{-\frac{1-\rho}{\rho}}
\]

be the price index of the monopolistically competitive varieties. We may then rewrite $Q_0$ as follows:

\[
Q_0 = R \cdot \left[ \int_0^M (p(i))^{-\frac{1-\rho}{\rho}} \, di \right]^{\frac{1}{\rho}} = R \cdot (P_0)^{-\frac{1}{1-\rho}}
\]
so that \( R = P_0^{1/(1-\rho)} Q_0 \), which in turn implies that

\[
q(i) = R \cdot [p(i)]^{-\frac{1-\rho}{\rho}} = Q_0 \cdot [p(i)]^{-\frac{1-\rho}{\rho}} \cdot P_0^{\frac{1}{1-\rho}} \quad \text{for all } i \in [0, M]. \tag{4}
\]

Hence, the demand function of an MC-firm decreases with its own price but increases with two aggregate statistics, \( Q_0 \) and \( P_0 \), which encapsulate the aggregate behavior of the competitive fringe.

Substituting (3) and (4) into the original maximization problem yields the following reduced maximization problem:

\[
\text{Maximize} \quad \left( \sum_{j=0}^{N} Q_j^\rho \right)^{\frac{1-\alpha}{\rho}} \cdot X^\alpha \quad \text{subject to} \quad \sum_{j=0}^{N} P_j Q_j + X \leq Y.
\]

The corresponding first order conditions imply that

\[
(1 - \alpha) \left( \sum_{j=0}^{N} Q_j^\rho \right)^{\frac{1-\alpha}{\rho} - 1} Q_j^{-(1-\rho)} X^\alpha = \lambda P_j \quad j = 0, 1, \ldots, N \tag{5}
\]

\[
\alpha \left( \sum_{j=0}^{n} Q_j^\rho \right)^{\frac{1-\alpha}{\rho}} X^{-(1-\alpha)} = \lambda \tag{6}
\]

\[
Y - \sum_{j=0}^{N} P_j Q_j - X = 0 \tag{7}
\]

where \( \lambda \) is the Lagrangian multiplier. Let \( P \) be the *industry price index*, which we define as follows:

\[
P = \left( \sum_{j=0}^{N} P_j^{-\frac{1-\rho}{1-\rho}} \right)^{-\frac{1-\rho}{1-\rho}} \tag{8}
\]

so that \( P \) increases with the price \( P_j \) of any discrete variety \( j = 1, \ldots, N \) as well as with the price index of the MC-subsector \( P_0 \). It is readily verified that the system (5)-(7) imply that the demand functions are given by

\[
Q_j = (1 - \alpha) Y (P_j)^{-1/(1-\rho)} P_0^{\rho/(1-\rho)} \equiv D(P_j, P, Y) \quad j = 1, \ldots, N \tag{9}
\]

\[
X = \alpha Y
\]

\[
q(i) = (1 - \alpha) Y [p(i)]^{-1/(1-\rho)} P_0^{\rho/(1-\rho)} \equiv d[p(i), P, Y] \quad i \in [0, M] \tag{10}
\]

where \( D(P_j, P, Y) \) is the demand function of the oligopolistic variety \( j = 1, \ldots, N \) and \( d[p(i), P, Y] \) that of the MC-variety \( i \in [0, M] \). The fact that
the functional forms \( D \) (resp., \( d \)) is independent of \( j \) (resp., \( i \)) reflects the symmetry of preferences on the varieties supplied by the corresponding subsector. There is a major difference, however. Small firms face demands having the same constant elasticity, whereas big firms’ demands displays a variable elasticity since \( P \) changes with \( Q_i \). As to be expected, both \( D \) and \( d \) are decreasing in their own price when \( Y \) is fixed. Under the same condition, using (2) and (9), it is easy to show that \( \partial D(P_j,P,Y)/\partial P_k > 0 \) for all \( j,k = 1, \ldots, N \) and \( k \neq j \), thus implying that the varieties provided by the large firms are strong gross substitutes. As the same holds for \( j = 0 \), we may conclude that the output index of the MC-subsector plays the same role in consumption as any variety of the oligopolistic subsector. Recall that the fraction of the income spent on the differentiated good is constant and given by \( 1 - \alpha \). To ease the burden of notation, we set \( Y_d \equiv (1 - \alpha)Y \) but use \( Y \) each time ambiguity could arise.

2.2 Oligopolistic firms

Each oligopolistic firm selects its output level to maximize its profit. Hence, the solution to the interactive profit-maximizing problem is given by a Nash equilibrium of the following strategic-form game: (i) the players are the \( N \) oligopolistic firms; (ii) the strategy of firm \( j = 1, \ldots, N \) is its output level \( Q_j \); and (iii) the payoff for player \( j \) is given by its profit function

\[
\Pi_j(Q_1, \ldots, Q_N; Y, Q_0) = \Psi_j(Q_1, \ldots, Q_N; Y, Q_0)Q_j - CQ_j - F
\]

where \( \Psi_j(\cdot) \) is the inverse demand function for the product of oligopolistic firm \( j \), \( C > 0 \) the constant marginal cost and \( F > 0 \) the fixed cost of an oligopolistic firm. In order to allow for at least two big firms to be active, we assume throughout this paper that \( L > 2F \).

The demand functions (9) and (10) allow us to describe the market behavior of both types of firms. First, an oligopolistic firm is aware that its strategy affects the industry price index \( P \) and is, therefore, involved in a game-theoretic environment. It also understands that the price index \( P \) is influenced by the aggregate behavior of the MC-firms, as expressed by \( Q_0 \). Finally, each oligopolistic firm should also account for the fact that its strategic choice generates an income effect, through profit distribution, which affects itself the level of demand for its variety. However, for reasons discussed below, we assume that these firms treat the income level as a parameter, even though the income will be endogenously determined. Thus, the profit maximization problem of oligopolistic firm \( j \) is given by

\[
\text{Maximize}_{Q_j} \quad \Pi_j(Q_1, \ldots, Q_N; Y, Q_0).
\]
By contrast, being negligible to the market, each MC-firm may accurately treat the price index $P$ and the income $Y$ as parameters when selecting its profit-maximizing output. Hence, unlike the oligopolistic firms, the MC-firms do not behave strategically.

Solving (9) for $P_j$ yields $P_j = Y_d^{1-\rho} Q_j^{-(1-\rho)} P^\rho$ and substituting this expression into (8) yields the price index as function of the consumption levels:

$$P = Y_d \left( Q_j^\rho + \sum_{k \neq j} Q_k^\rho \right)^{-\frac{1}{\rho}}. \quad (11)$$

Substituting (11) into $P_j = Y_d^{1-\rho} Q_j^{-(1-\rho)} P^\rho$ then yields the inverse demand function for variety $j = 1, ..., N$:

$$\Psi_j(Q_1, \cdots, Q_N; Y, Q_0) = \frac{Y_d Q_j^{-(1-\rho)}}{Q_j^\rho + \sum_{k \neq j} Q_k^\rho}$$

which depends on the output of each oligopolistic firm as well as on the aggregate output of the MC-subsector. Consequently, the profit function of firm $j$ may be written as follows:

$$\Pi_j(Q_1, \cdots, Q_N; Y, Q_0) = \frac{Y_d Q_j^\rho}{Q_j^\rho + \sum_{k \neq j} Q_k^\rho} - CQ_j - F. \quad (12)$$

Let $Q_{-j} \equiv (Q_1, ..., Q_j, ..., Q_N)$ be the vector of all outputs but that of firm $j$. Because $\partial \Pi_j / \partial Q_j$ is strictly decreasing in $Q_j$, we have:

**Lemma 1.** For any $j = 1, ..., N$ and any given $Q_{-j}$ and $Q_0$, $\Pi_j$ is strictly concave with respect to $Q_j$ over $[0, \infty)$.

Hence, the best reply function $Q_j^*(Q_{-j}; Y, Q_0)$ of firm $j$ is the unique solution to the following first order condition:

$$\frac{\partial \Pi_j}{\partial Q_j} = \rho Y_d \frac{\sum_{k \neq j} Q_k^\rho}{Q_j^{1-\rho} \left( Q_j^\rho + \sum_{k \neq j} Q_k^\rho \right)^2} - C = 0. \quad (13)$$

Despite the fact that oligopolistic varieties are substitutes, $Q_j$ and $Q_k$ need not be strategic complements or substitutes. Furthermore, even though $Q_0$ is not chosen by a player per se, the output index of the MC-subsector may also be a strategic substitute or a strategic complement of oligopolistic firms’ output. Among other things, this implies that an expansion of the MC-subsector (through an increase of $M$) does not necessarily imply that
oligopolistic firms lower their output. However, we will see below that we have a “well-behaved model” in that an increase in the number of firms of each type leads to lower prices for these firms as well as a decrease of the overall price index.

2.3 Monopolistically competitive firms

The profit maximization problem of the MC-firm \( i \in [0, M] \) is defined as follows:

\[
\text{Maximize} \quad p(i)q(i) - cq(i) - f \quad \text{subject to} \quad q(i) = d[p(i), P, Y]
\]

where \( c > 0 \) is the constant marginal cost and \( f > 0 \) the fixed cost of a MC-firm. Note that, for the economy to supply the homogenous good, it must be that

\[
L > NF + Mf. \quad (14)
\]

This implies the existence of the upper bound \((L - NF)/f\) on the size of the competitive fringe that depends on the number of oligopolistic firms.

It follows from (10) that the inverse demand function is given by

\[
p(i) = Y d^{1-\rho} q(i)^{-(1-\rho)} P^{\rho}.
\]

As a result, firm \( i \)'s profit function becomes

\[
\pi[q(i); P, Y] = Y d^{1-\rho} q(i)^{\rho} P^{\rho} - cq(i) - f
\]

where, unlike oligopolistic firms, MC-firms accurately treat the price index \( P \) parametrically. This difference in firms’ behavior reflects the difference in the underlying market structure that characterizes each subsector.

The profit maximization problem of the MC-firm \( i \in [0, M] \) may then be rewritten as

\[
\text{Maximize} \quad \pi[q(i); P, Y].
\]

Since \( \rho < 1 \), \( \pi(i) \) is strictly concave in \( q(i) \). The first order condition for profit maximization leads to

\[
q(i) = \left( \frac{c}{\rho} \right)^{-\frac{1}{1-\rho}} Y d P^{\frac{\rho}{1-\rho}}.
\]

This allows us to determine the common equilibrium price and output:

\[
p = \frac{c}{\rho} \quad \text{and} \quad q = \left( \frac{c}{\rho} \right)^{-\frac{1}{1-\rho}} Y d P^{\frac{\rho}{1-\rho}}. \quad (15)
\]
This equilibrium is unique and symmetric for any given vector \((Q_1, \cdots, Q_N)\). Whereas the equilibrium price is constant, the equilibrium size of an MC-firm is a function of the price index \(P\) and, therefore, depends on the quantities chosen by the oligopolistic firms. For any given \(M\) and \(N\), the equilibrium profit of an MC-firm is thus:

\[
\pi \equiv (1 - \rho) \left( \frac{c}{\rho} \right)^{\bar{\rho} \rho} Y_d P^{1/\bar{\rho}} - f
\]

while the profit earned by an oligopolistic firm at a symmetric equilibrium is:

\[
\Pi \equiv Y_d^{1-\rho} P^\rho Q^\rho - CQ - F.
\]

Finally, the mass \(M\) of MC-firms given \(N\) is determined by the zero-profit condition \(\pi = 0\).

Observe that, in all the foregoing expressions, the total income given by

\[
Y = L + N\Pi + M\pi
\]

is inherently variable because profits are endogenous and redistributed to the representative consumer.

## 3 The market outcome

A *market equilibrium* is defined as a state in which the following conditions simultaneously hold: (i) the representative consumer maximizes her utility subject to the budget constraint, (ii) both oligopolistic and MC-firms maximize their own profits with respect to output, (iii) oligopolistic firms earn positive profits, (iv) the mass of MC-firms is such that \(M^*\pi^* = 0\), and (v) all markets clear. Hence, for a given number \(N\) of oligopolistic firms,\(^8\) the mass of MC-firms is adjusted until their profits are zero or there is no MC-subsector:

\[
M^* > 0 \quad \Rightarrow \quad \pi^* = 0
\]

\[
\pi^* < 0 \quad \Rightarrow \quad M^* = 0.
\]

When \(N > 0\) and \(M^* > 0\), we say that the market equilibrium is mixed. Clearly, the resource constraint (14) implies that \(L - NF - M^*f > 0\).

\(^8\)Our aim being to study how the market and consumer welfare react to the entry of a big firm, we treat \(N\) as exogeneous.
Even though the output index of the MC-subsector and the total income are endogenous, when choosing its own output level, each oligopolistic firm treats them as parameters. The first assumption accounts for the fact that the market outcome is given by a Nash equilibrium of a game in which both big and small firms move simultaneously. The second implies that the big firms behave as *income-takers* in that they neglect the impact that their output decisions have on the total income, hence on their demands, through the distribution of profits (Gabszewicz and Vial, 1972; d’Aspremont *et al.*, 1989, 1996). In contrast, MC-firms accurately treat the total income as a parameter. However, both types of firms understand that a higher - or lower - income influences positively - or negatively - the level of their demands. More precisely, in equilibrium all firms anticipate correctly what the total income will be. That said, we want to stress the fact that our model is *not* a partial equilibrium one, the difference being that the income level is exogenous in a partial equilibrium analysis whereas it is endogenous in our setting. Accordingly, although our model does not capture all feedback effects, it is a full-fledged general equilibrium model in which oligopolistic firms account for (i) strategic interactions within their group, (ii) the aggregate behavior of the small firms, and (iii) endogenous income generated by profit distribution.\(^9\) Stated differently, firms have rational conjectures about consumers’ behavior.

For any \( N \geq 2 \), we may characterize the market equilibrium with \( M^* > 0 \) by means of the following four conditions: (i) the demand functions, (ii) the profit-maximization conditions of MC-firms, (iii) the profit-maximization conditions of oligopolistic firms, and (iv) the zero-profit condition of MC-firms. In this way, we may view 0 as a “pseudo-player” who chooses the mass of MC-firms for their profits to be zero.

### 3.1 Existence of a mixed market equilibrium

For the moment, we assume the existence of a *symmetric* mixed market equilibrium in which all oligopolistic firms choose the same output \( Q^* \) sold at the same price \( P^* \), whereas all MC-firms have the same output given by (15). Hence, symmetry prevails within each group of firms but not between groups.

The equilibrium analysis involves two steps: (i) we compute the equilibrium conditions when the size \( M \) of the MC-subsector is fixed and (ii) we determine the equilibrium value of \( M \).

(i) The profit \( \Pi \) of an oligopolistic firm at this market outcome is:

\(^9\)See, e.g. Marschak and Selten (1974) and Hart (1985) for a similar modeling strategy.
\[ \Pi = Y_1 \rho Q \rho \bar{P} \rho - CQ - F. \]  

Moreover, the income of the representative consumer is:

\[ Y = L + N \Pi + M \pi \]

and determined in equilibrium by

\[ Y = L + N \left[ Y_1 \rho Q \rho \bar{P} \rho - CQ - F \right] + M \left[ \left( \frac{c}{\rho} \right)^{\frac{1}{1-\rho}} (1 - \rho) Y_1 \bar{P}^{\frac{\rho}{1-\rho}} - f \right]. \]  

It then follows from (11) that

\[ P = Y_d \left( Q_0^\rho + N Q^\rho \right)^{-\frac{1}{\rho}}. \]  

Substituting (15) into (2) and (4), we obtain the equilibrium values of the price and output indices of the MC-firms:

\[ P_0 = \frac{c}{\rho} M^{-\frac{1-\rho}{\rho}} \]  

\[ Q_0 = Y_d \left( \frac{c}{\rho} \right)^{\frac{1}{1-\rho}} M^{\frac{1}{1-\rho}} \bar{P}^{\frac{\rho}{1-\rho}}. \]  

Although the equilibrium price of each variety is constant, (21) implies that a larger mass of MC-firms makes competition in this subsector tougher through more fragmented individual demands, thus leading to a lower industry price index. This shows how varying the size of the MC-subsector affects the intensity of competition in the whole industry.

Finally, setting \( Q = Q_j \) for all \( j = 1, ..., N \) and using (20) shows that (13) can be rewritten as follows:

\[ Y_1^{1-\rho} = \frac{C}{\rho} P^{-\rho} Q^{1-\rho} + Y_1^{1-2\rho} \bar{P}^{\rho} Q^\rho. \]  

The system of four equations, (19), (20), (22) and (23), gives the equilibrium outcome \( Q(M; N), Y(M; N), P(M; N) \), and \( Q_0(M; N) \) conditional to the size \( M \) of the MC-subsector and the number \( N \) of the oligopolistic firms. By studying how this outcome changes with \( M \), we are able to see how the two subsectors interact at the mixed market equilibrium.

We start with the following result, the proof of which is given in Appendix A.
Lemma 2. For any given value of $N$, the equilibrium profit of an MC-firm $\pi(M; N)$ is a decreasing function of $M$.

This implies that the entry and exit process in the MC-subsector yields a unique outcome, which is globally stable. In other words, for any given value of $N$, there exists at most one value $M^*(N)$ such that $\pi(M; N) = 0$. Unfortunately, the fact that the outputs $Q_j$ may be either strategic complements or substitutes does not allow us to determine how $Q$ and $Q_0$ are affected when $M$ varies. Nevertheless, we are able to characterize the impact on equilibrium prices. First, (21) implies that the price index $P_0$ of the MC-subsector decreases as the size of this subsector rises. Then, as proven in Steps 1 and 3 of Appendix A, increasing $M$ has a similar impact upon $P$ and $P$.

Proposition 1 Consider a symmetric equilibrium outcome in which the size of the MC-subsector is exogenous and the number of oligopolistic firms is fixed. Then, both the industry price index and the price at which oligopolistic firms sell their output decrease when the mass of MC-firms increases.

Hence, the entry of MC-firms makes the industry more competitive. In other words, the market reacts as if the MC-subsector were a single big firm whose size is given by $Q_0$. This confirms the idea that the agent 0 may be interpreted as a pseudo-player. It should be kept in mind, however, that $Q_0$ is the output index of the MC-subsector. It stems from the aggregation of production decisions made by a continuum of small firms, and is not the total output chosen by this pseudo-player.

(ii) To provide a full characterization of a symmetric market equilibrium, we still have to determine the size $M^*$ of the MC-subsector. Using (16), the zero-profit condition $\pi = 0$ is equivalent to

$$Y_d = \frac{f}{1 - \rho} \left( \frac{c}{\rho} \right) P^{-\frac{\rho}{1-\rho}}. \quad (24)$$

Substituting (24) into (22), we obtain

$$Q_0 = \frac{f}{1 - \rho} \left( \frac{c}{\rho} \right)^{-1} M^{\frac{1}{\rho}}. \quad (25)$$

As to be expected, both the equilibrium mass and output index of the MC-subsector move in the same direction. The market outcome $Q^*(N), Q_0^*(N), P^*(N), Y^*(N)$, and $M^*(N)$ corresponding to $N$ is then described by the five simultaneous equations (23), (19), (20), (24) and (25) whose unknowns are $Q, Q_0, P, Y, and M$. 

17
Having determined all the equilibrium conditions, we are now equipped to show the existence and uniqueness of a symmetric mixed equilibrium. In Appendix B.1, we show that, provided that oligopolistic firms earn positive profits, a symmetric market outcome with \( M^* > 0 \) exists if and only if

\[
\frac{f}{L} < S \left( N; \frac{F}{L} \right) \equiv \frac{NC}{(N-1)c} \left[ \frac{1}{\alpha N + (1 - \alpha)\rho(N-1)} \left( 1 - \frac{NF}{L} \right) \right] \quad (26)
\]

where \( S \) is linear and downward sloping in \( F/L \), \( S(N;0) > 0 \) and \( S(N;F/L) \to 0 \) when \( F/L \to 1/N \).

In Appendix B.2, we show that there exists a function \( B(F/L) \), where \( B \) is strictly increasing, \( B(0) = 0 \) and \( B(F/L) \to \infty \) when \( F/L \to (1 - \alpha)\rho \), such that oligopolistic firms’ profits are positive if and only if \( B(F/L) < f/L \). As to be expected, high values of \( F/L \) prevent big firms to be active at the market outcome.

Consequently, the domain of the \((F/L,f/L)\)-plane for which a mixed market equilibrium prevails is, therefore, defined by the intersection of the two sets delineated by \( B(F/L) < f/L \) and \( f/L < S(N;F/L) \). It is non-empty because \( S \) is strictly decreasing with \( S(N;0) > 0 \), while \( B \) is strictly increasing with \( B(0) = 0 \). Consequently, we have:

**Proposition 2** If \( F/L < 1/N \) for \( N \geq 2 \), then there exists a unique symmetric mixed market equilibrium if and only if

\[
B \left( \frac{F}{L} \right) < \frac{f}{L} < S \left( N; \frac{F}{L} \right).
\]

Thus, the existence of a mixed market equilibrium depends on the ratios of fixed costs, \( F \) and \( f \), and the size of the economy, \( L \). In Figure 1, we depict the domain of parameters in which a mixed market exists. Depending on the relative values of \( F/L \) and \( f/L \), the economy may have a handful of big firms and/or a myriad of small firms. In particular, increasing the value of \( f/L \) leads to the widening of the range of \((F/L)\)-values for which the market involves oligopolistic firms only. This is because the entry of MC-firms becomes harder. Regarding the impact of a steadily increase in \( F/L \), the range of \((f,L)\)-values for which the market involves MC-firms shrinks. This is due to an *income effect* that stems from the general equilibrium nature of our setting. As shown by the bottom triangle delineated by the locus \( B(F/L) = f/L \) in Figure 1, when \( F/L \) rises there is room for a decreasing mass of MC-firms. Indeed, the income \( Y \) decreases because oligopolistic firms earn lower profits. Hence, (16) implies that the equilibrium profits of MC-firms decrease. More generally, as shown in Appendix B, increasing \( f \)
or $F$ allows a smaller mass of MC-firms to operate, which in turn leads each oligopolistic firm to sell more. Note that the uniqueness of the symmetric equilibrium implies that we stay on the same equilibrium path when the number of oligopolistic firms varies.

Insert Figure 1 about here

Let us now briefly consider the case of a single large firm ($N = 1$). As long as there exists a competitive fringe ($Q_0 > 0$ and $P_0 > 0$), firm 1’s demand function is not iso-elastic, which allows for the precise determination of $Q_1^* > 0$ and $P_1^* < \infty$. However, the model remains asymmetric because the market involves one big firm and a continuum of small firms. This makes the analysis of the case where $N = 1$ very similar to that developed when $N \geq 2$. We have chosen, therefore, to skip it. As shown below, however, the condition $f/L < S(1; F/L)$ is always satisfied.\textsuperscript{10}

### 3.2 The Industry Structure

First, as to be expected, there is no competitive fringe when the product is homogeneous (see equation (C.3) in Appendix C). In other words, product differentiation is necessary for the MC-firms to have a positive demand. That said, our aim is to study how the two subsectors are affected by the entry of an oligopolistic firm.

Our first two results follow from (25) and Appendix C.

**Proposition 3** At the symmetric mixed market equilibrium, the equilibrium output of an oligopolistic firm increases when the number of oligopolistic firms rises.

**Proposition 4** At the symmetric mixed market equilibrium, the equilibrium mass and quantity index of the MC-subsector decrease when the number of oligopolistic firms increases.

Thus, the entry of a large firm leads to the exit of a range of small businesses and, therefore, to a contraction of the MC-subsector. To illustrate, observe that Basker (2007) finds that, in the U.S. retail sector, Wall-Mart’s competitive pressure has caused other stores, especially small ones, to shut down. Combining the last two propositions then show that the shrinking of the MC-subsector generated by the entry of a big firm is sufficiently strong to permit the expansion of each oligopolistic firm’s output.

\textsuperscript{10}Observe that the case $N = 1$ is different from Markham (1951) because unlike him we assume that big and small firms move simultaneously.
Proposition 5 has another important implication: the MC-subsector disappears when the number of oligopolistic firms is sufficiently large. Indeed, using (C.3) in Appendix C, we see that there exists a single real number \( N_O \) such that \( M^* = 0 \) and \( Q^*_o = 0 \) once \( N \) is larger than or equal to \( N_O \). Observe that \( N_O \) is the unique and positive solution to the equation

\[
S \left( N; \frac{F}{L} \right) = \frac{f}{L}
\]

because the left hand side is decreasing in \( N \geq 2 \), arbitrarily large at \( N = 1 \), and negative when \( N \) is arbitrarily large. As to be expected, more big firms are needed to trigger the exit of all small firms when the level of fixed and/or marginal costs in the MC-subsector is lower.

All of this suggests how the market structure evolves as more big firms enter the market. First, when the number of large firms is much smaller than \( N_O \), the economy involves a large mass of small firms; then, once \( N \) rises while remaining lower than \( N_O \), the mass of small firms shrinks but remains positive; last, when \( N \) exceeds \( N_O \), small firms disappear from the market which becomes purely oligopolistic. During the whole process, the resource constraint (14) is always satisfied, thus implying that the above dynamics describes feasible allocations. Furthermore, the transition from the mixed to the oligopolistic market does not involve any discontinuity in the market equilibrium (see Appendix D). Finally, it is readily verified that \( N_O > 1 \), which means that the market outcome is always mixed in the presence of a unique big firm. In other words, the case of a pure monopoly, in which the big firm’s demand has a unit elasticity, never emerges.

In the foregoing, we have uncovered the existence of a trade-off between the two subsectors: as the oligopolistic subsector expands, the MC-subsector shrinks. This in turn allows us to determine the impact of an increase in the number of oligopolistic firms on the equilibrium industry price index. All else equal, Proposition 5 implies that the equilibrium price \( P^* \) at which the oligopolistic firms sell their varieties decreases when \( N \) rises. However, by (21), the corresponding decrease in the mass of MC-firms leads to an increase of \( P^*_o \). Thus, the total impact on \( P \) is a priori undetermined. Yet, we are able to show the following result in Appendix C.2.

**Proposition 5** At the symmetric mixed market equilibrium, the industry price index decreases when the number of oligopolistic firms rises.

In other words, despite the fact that the entry of a new oligopolistic firm triggers the exit of some MC-firms, the entry of a new oligopolistic firm makes the market more competitive. Thus, even though the exit of MC-firms
tends to make the market less competitive (see Proposition 1), this effect is dominated by the pro-competitive effect generated by the entry of a big firm, thus making competition fiercer and prices lower. This is reminiscent of Wall-Mart whose entry has led the U.S. retail sector to become “more efficient at providing consumers with the goods they want at better prices and with increased convenience” (Basker, 2007, p.195).

Propositions 4 and 5 open the door to new welfare questions that we investigate below.

4 Welfare

The purpose of this section is not to conduct a first best analysis of the industry structure. Instead, we aim at determining whether or not the entry of an oligopolistic firm is welfare-enhancing. Because preferences satisfy the Gorman polar form, the level of social welfare may be described by the indirect utility corresponding to the utility of the representative consumer.11

Introducing (9)-(10) into (1), we obtain the indirect utility:

\[ W = \alpha^\alpha (1 - \alpha)^{1-\alpha} Y^* (P^*)^{-(1-\alpha)} \]  

(27)

and determine the impact of increasing \( N \) upon both \( P^* \) and \( Y^* \) to determine how entry affects total welfare. We already know from Proposition 5 that \( P^* \) goes down. It remains to consider how \( Y^* \) is affected. Using (24), we immediately see that a lower value \( P^* \) leads to a higher value of \( Y^* \). Proposition 5 thus implies:

**Proposition 6** At the symmetric mixed market equilibrium, the consumer income increases when the number of oligopolistic firms increases.

Using (27), the following result immediately follows from the fact that, when \( N \) increases, \( P^* \) goes down while \( Y^* \) goes up.

**Proposition 7** At the symmetric mixed market equilibrium, the social welfare increases when the number of oligopolistic firms rises.

---

11For the welfare analysis developed below to be meaningful, preferences must be defined over a given space of varieties. This can be accomplished by assuming that preferences (1) are a priori defined on the Cartesian product of (i) the vector space of dimension equal to the largest integer smaller than or equal to \( L/N \), and (ii) the functional space of measurable functions defined on \([0,L/f] \). The varieties not supplied by the market are then given a weight equal to zero in (1).
In words, this result has the following major implication: a differentiated market with several big firms and a small number of small firms is more efficient than a market with fewer big firms and a larger number of small firms. This runs against the conventional wisdom according to which a multitude of small firms does better in terms of efficiency than a handful of large ones. This contrast in results is due to the fact that the mixed market model allows for direct comparisons of different market structures within a unified framework, thus shedding new light on their relative merits. It is also worth stressing that the above proposition is obtained in the case of a differentiated industry in which consumers have a preference for variety. Proposition 7 thus shows that the pro-competitive effect associated with the presence of large firms dominates the decrease in variety generated by the exit of several small firms. Note also that Proposition 7 imposes no specific restriction on the parameters of the economy, apart from those stated in Proposition 2 that guarantee the existence of a mixed market outcome.

Given that $Y^*(N) = L + N\Pi^*(N)$, Proposition 6 also implies that total profits in the economy rise with the number of oligopolistic firms. It should be stressed that this result is the outcome of the interplay of several intertwined effects. First, since the profit of MC-firms decreases with $N$ and $M$, the mass of MC-firms decreases as more large firms enter the market. The exit of MC-firms, which are less competitive than oligopolistic firms, generates a market expansion effect that allows the output index $Q^*$ to increase. This in turn leads to a lower price index $P^*$. Consequently, both consumer income and social welfare increase when $N$ rises. This not the end of the story, however. The income effect stressed by Proposition 6 also fuels the expansion of the market for each type of firms, thus allowing more MC-firms to stay in business. However, as shown by Proposition 1, even though this effect slows down the exit of small firms, it is not sufficiently strong to break it off.

5 Concluding remarks

Mixed markets are plentiful in the real world, one reason being that keeping a competitive fringe seems to be a political concern in several countries. Yet, our analysis suggests that consumers gain from the presence of large firms that make the market more competitive. Nevertheless, both in the public and the general press, it is customary to find the idea that the “small business” world of yesterday was more appealing than the “large business” world of today. Although sectors dominated by a few big firms were often more standardized than those involving many small producers (think of the “Model T” developed by Henry Ford), our analysis shows that consumers
need not be better off under many small producers rather than under a handful of large ones. This is because the variety argument put forward by interest groups ignores (deliberately!) the pro-competitive effect that the entry of big firms brings about. Furthermore, the fact that large firms are now able to exploit scope economies through flexible manufacturing to supply a large array of varieties invalidates, at least to a large extent, the variety loss argument (Eaton and Schmitt, 1994). Even though it is a priori unclear how the entry of a multi-product firm affects the total supply of varieties, this should make the case for our welfare result even stronger because the entry of such a firm will increase even more the degree of competition within the industry.

Note that collusion may be easier to enforce under a small number of producers than under a large one. This seems to run against the desirability of oligopolistic markets. It should be kept in mind, however, that effective competition policies allow one to dampen such a possibility. Furthermore, the case of Japan, studied in detail by Garon and Mochizuki (1993), shows that small-business associations aim to exchange their political influence for governmental policies that compensate for their weakness in the marketplace. The same is likely to hold in several other countries. Whatever these considerations as well as others not discussed here, our analysis suffices to cast serious doubts on the idea that “small is beautiful” in modern market economies. Although we would be the last to claim that “large is beautiful”, it is our contention that “small need not be beautiful”.

To conclude, observe that our setting can be applied to study various issues that have been investigated using the framework of monopolistic competition only. The first question that comes to mind is the opening to trade of two economies that have different market structures. Our analysis suggests that, by exacerbating competition with big firms, economic integration might trigger the progressive disappearance of small firms. This need not affect the two countries in the same way. Another example is the impact of large department stores or shopping malls that locate at the outskirts of a city, while competing with a large number of small shops located at the city center. In such a context, it seems reasonable to conjecture that the exit of small shops might make consumers living downtown worse-off when they have a bad access to the shopping malls. We leave these topics for future research.
Appendix A

The argument involves six steps.

1. Consider the impact on \( P \) of increasing \( M \). Substituting (22) into (20) and simplifying, we obtain

\[
Y^\rho_d = P^\rho \left[ \left( Y_d \left( \frac{c}{\rho} \right)^{\frac{1}{1-p}} M^p P^{\frac{p}{1-p}} \right)^\rho + NQ^\rho \right] = Y^\rho_d \left( \frac{c}{\rho} \right)^{\frac{1}{1-p}} M^p P^{\frac{p}{1-p}} + N^\rho P^\rho Q^\rho.
\]

Hence,

\[
Q = Y_d N^{-\frac{1}{p}} P^{-1} \left[ 1 - \left( \frac{\rho}{c} \right)^{\frac{p}{1-p}} M^p P^{\frac{p}{1-p}} \right]^\frac{1}{p}.
\]

(A.1)

Substituting (A.1) into (23), we obtain

\[
P \left[ N - 1 + \left( \frac{c}{\rho} \right)^{\frac{1}{1-p}} M^p P^{\frac{p}{1-p}} \right] = C^\rho N^{-\frac{2}{p}} P^{\frac{p}{1-p}} \left[ 1 - \left( \frac{\rho}{c} \right)^{\frac{p}{1-p}} M^p P^{\frac{p}{1-p}} \right]^{\frac{1-p}{p}}
\]

which implies

\[
P^{\frac{p}{1-p}} \left( \frac{c}{\rho} \right)^{\frac{p}{1-p}} \left[ N - 1 + \left( \frac{c}{\rho} \right)^{\frac{1}{1-p}} M^p P^{\frac{p}{1-p}} \right]^{\frac{p}{1-p}} = N^{-\frac{2}{p}} P^{\frac{p}{1-p}} \left[ 1 - \left( \frac{\rho}{c} \right)^{\frac{p}{1-p}} M^p P^{\frac{p}{1-p}} \right]^{\frac{1-p}{p}}.
\]

(A.2)

Setting

\[
H \equiv \left( \frac{c}{\rho} \right)^{\frac{p}{1-p}} M^p P^{\frac{p}{1-p}}
\]

(A.3)

(A.2) may be rewritten as follows:

\[
G(H; N) \equiv N^{\frac{1-2p}{1-p}} \frac{H(N - 1 + H)^{\frac{p}{1-p}}}{1 - H} = 1.
\]

Given \( M \) and \( N \), \( G \) is increasing in \( H \), while \( G(0) = 0 \) and \( G(H) \to \infty \) when \( H \to 1 \). Therefore, \( G(H) = 1 \) has a unique solution in \( H \in ]0, 1[ \), so that (A.2) has a unique solution \( P(M; N) > 0 \). Because both \( G(H) \) and \( H \) are strictly increasing in \( M \), \( P(M; N) \) is strictly decreasing in \( M \).

2. Using (A.3), (A.2) may be rewritten as follows:

\[
N^{\frac{1-2p}{1-p}} \frac{H(N - 1 + H)^{\frac{p}{1-p}}}{1 - H} = M.
\]

(A.4)
As long as \( Q > 0 \), (A.1) implies that \( H < 1 \). Step 2 implies that (A.4) has a unique solution \( H(M; N) \), which is strictly increasing in \( M \).

3. Since \( P = Y_d^{1-\rho}Q^{\rho-1}P^\rho \) and \( Q = Y_dN^{-\frac{1}{\rho}}P^{-1}(1 - H)^{\frac{1}{\rho}} \), the equilibrium price set by an oligopolistic firm is given by

\[
P = N^{\frac{1}{1-\rho}}P(1 - H)^{\frac{1}{1-\rho}}.
\] (A.5)

Because \( P \) is strictly decreasing (Step 1) and \( H \) is strictly increasing in \( M \) (Step 2), it must be that \( P \) strictly decreases with \( M \).

4. Using (A.5), the profit of an oligopolistic firm is given by

\[
\Pi = Y_d^{1-\rho}Q^{\rho}P^\rho - CQ - F = Y_dN^{-1}(1 - H) - CN^{-\frac{1}{\rho}}Y_dP^{-1}(1 - H)^{\frac{1}{\rho}} - F.
\] (A.6)

Substituting (A.6) and (16) into (17) yields, after some manipulations, the following expression:

\[
Y\left[\alpha + (1 - \alpha)\rho H + (1 - \alpha)CN^{-\frac{1}{\rho}}P^{-1}(1 - H)^{\frac{1}{\rho}} \right] = L - NF - Mf.
\] (A.7)

Using (A.2) and (A.3), we obtain

\[
\left(\frac{C}{\rho}\right)^{\frac{1}{1-\rho}}N^{\frac{2\rho-1}{\rho}}(1 - H) = P^{\frac{\rho}{1-\rho}}(N - 1 + H)^{\frac{1}{1-\rho}}
\]

so that

\[
(1 - H)^{\frac{1}{\rho}} = \left(\frac{C}{\rho}\right)^{\frac{1}{1-\rho}}N^{\frac{1-2\rho}{\rho}}P^{\frac{1}{1-\rho}}(N - 1 + H)^{\frac{1}{1-\rho}}
\]

which in turn amounts to

\[
N^{-\frac{\rho}{1-\rho}}P^{-1}(1 - H)^{\frac{1}{\rho}} = \left(\frac{C}{\rho}\right)^{\frac{1}{1-\rho}}N^{\frac{\rho}{1-\rho}}P^{\frac{\rho}{1-\rho}}(N - 1 + H)^{\frac{1}{1-\rho}}.
\]

Replacing in (A.7) leads to

\[
Y = \frac{L - NF - Mf}{\alpha + \rho(1 - \alpha)H + (1 - \alpha)\left(\frac{C}{\rho}\right)^{\frac{1}{1-\rho}}CN^{\frac{\rho}{1-\rho}}P^{\frac{\rho}{1-\rho}}(N - 1 + H)^{\frac{1}{1-\rho}}}.
\] (A.8)
5. It follows from (16) that 
\[ \pi(M; N) = \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} (1 - \rho) Y_d P^{\frac{\rho}{1-\rho}} - f. \]

Substituting (A.8) into this expression yields:

\[ \pi(M; N) = \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} \frac{(1 - \rho)(1 - \alpha)(L - NF - Mf)}{I(M; N)} - f \]

where

\[ I(M; N) \equiv \alpha[P(M; N)]^{-\frac{\rho}{1-\rho}} + \rho(1 - \alpha) \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} M \]

\[ + (1 - \alpha) \left( \frac{C}{\rho} \right)^{\frac{1}{1-\rho}} CN^{\frac{1}{1-\rho}} [N - 1 + H(M; N)]^{\frac{1}{1-\rho}}. \]

6. The numerator \( L - NF - Mf \) is decreasing in \( M \), while the denominator \( I \) is increasing in \( M \) because \( P \) is strictly decreasing (Step 1) and \( H \) is strictly increasing (Step 2) in \( M \). Therefore, for any given value of \( N \), the equilibrium profit of an MC-firm \( \pi(M; N) \) is a strictly decreasing function of \( M \).

**Appendix B**

We determine a necessary and sufficient condition for a symmetric market equilibrium to be mixed.

1. To do this, we first describe the equilibrium conditions for a given value \( M \geq 0 \) as a function of \( H \) (see (A.3)):

\[ H \equiv \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} M P^{\frac{\rho}{1-\rho}}. \]

Conditions (17) and (23) may then be rewritten as follows:

\[ L - NF - Mf = Y_d \left[ \frac{\alpha}{1-\alpha} + \rho H + CN^{1-\frac{1}{\rho}}(1 - H)^{\frac{1}{\rho} - 1} P^{-1} \right] \]

\[ 1 = \frac{C}{\rho} N^{1-\frac{1}{\rho}}(1 - H)^{\frac{1}{\rho} - 1} P^{-1} + \frac{1 - H}{N}. \]

We now account for the equilibrium of the MC-subsector:

\[ H \geq 0 \quad \pi \leq 0 \quad H\pi = 0 \]
where \( \pi \) is given (16):

\[
\pi = \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-\rho}} (1-\rho) Y_d P^{\frac{\rho}{1-\rho}} - f.
\]

Solve (B.1) for \( Y_d \) and (B.2) for \( P \). Substituting the resulting expressions into \( \pi \) yields a new expression for \( \pi \) such that

\[
\text{sign } \pi = \text{sign } J(H; N)
\]

where, for any given \( N \),

\[
J(H; N) = \left( \frac{C}{c} \right)^{\frac{\rho}{1-\rho}} \frac{1}{(1 - \frac{1}{N})^{\frac{\rho}{1-\rho}}} \frac{(1-\rho)(1-\alpha)(L - NF)(1-H)}{\{N[1-(1-\rho)(1-\alpha)(1-H)] - \rho(1-\alpha)(1-H)^2\}} - f
\]

for \( H \in [0,1] \). Observe that \( J \) is strictly decreasing in \( H \) and satisfies

\[
J(0; N) = \left( \frac{C}{c} \right)^{\frac{\rho}{1-\rho}} \left( \frac{N}{N-1} \right)^{\frac{\rho}{1-\rho}} \frac{(1-\rho)(1-\alpha)(L - NF)}{\alpha N + \rho(1-\alpha)(N-1)} - f
\]

\[
J(1; N) = -f.
\]

Hence, there exists a unique solution \( H^*(N) \in [0,1] \) to \( J(H; N) = 0 \) if and only if

\[
f < \left( \frac{C}{c} \right)^{\frac{\rho}{1-\rho}} \left( \frac{N}{N-1} \right)^{\frac{\rho}{1-\rho}} \frac{(1-\rho)(1-\alpha)(L - NF)}{\alpha N + \rho(1-\alpha)(N-1)}.
\]

which implies that \( M^*(N) > 0 \).

Accordingly, provided that oligopolistic firms earn positive profits, there exists a unique symmetric mixed equilibrium given by

\[
\begin{align*}
P^*(N) &= \left( \frac{C}{\rho} \right) \frac{N^{2\rho-1} [1 - H^*(N)]^{\frac{1-\rho}{\rho}}}{N - 1 + H^*(N)} \quad (B.3) \\
M^*(N) &= \left( \frac{c}{C} \right)^{\frac{\rho}{1-\rho}} \frac{NH^*(N)}{1 - H^*(N)} \left[ \frac{N - 1 + H^*(N)}{N} \right]^{\frac{\rho}{1-\rho}} \\
Y^*(N) &= \frac{N \left[ L - NF - M^*(N)f \right]}{\alpha N + (1-\alpha)\rho \left[ N - [1 - H^*(N)]^2 \right]} \\
Q^*(N) &= \frac{(1-\alpha)\rho \left[ N - 1 + H^*(N) \right] \left[ 1 - H^*(N) \right] \left[ L - NF - M^*(N)f \right]}{CN \left[ \alpha N + (1-\alpha)\rho \left[ N - [1 - H^*(N)]^2 \right] \right]}
\end{align*}
\]
It follows from (18) that the equilibrium profits of an oligopolistic firm are given by

$$
\Pi^*(N) = \frac{(1 - \alpha) \{(1 - \rho)N + \rho[1 - H^*(N)]\} \{1 - H^*(N)\} [L - NF - M^*(N)f]}{N \{\alpha N + (1 - \alpha)\rho \{N - [1 - H^*(N)]^2\}\}} - F.
$$

In addition, the social welfare is:

$$
W^*(N) = \left(\frac{\rho}{C}\right)^{1-\alpha} (\alpha)^{\alpha} (1 - \alpha)^{2-\alpha} N^{1/2 (1-\alpha)+\alpha} \left\{ L - NF - M^*(N)f \right\} \frac{\alpha N + (1 - \alpha)\rho \{N - (1 - H^*(N))^2\}}{\rho \{N - (1 - H^*(N))^2\}} \cdot \left[ N - 1 + H^*(N) \right]^{1-\alpha} \left(\frac{1}{1 - H^*(N)}\right)^{F}. \tag{B.7}
$$

Note, finally, that $J(H; N)$ is shifted downward when $f$ and/or $F$ increases, which implies that $H$ decreases. Using (B.3) then shows that $M^*(N)$ also decreases with these two parameters.

2. It remains to determine under which condition oligopolistic firms earn positive profits $\Pi^* > 0$. Since $P_j = Y_d^{1-\rho}Q_j^{(1-\rho)}\rho$, we have

$$
\Pi(Q_j) = Y_d^{1-\rho}Q_j^{(1-\rho)}\rho - CQ_j - F \tag{B.4}
$$

whereas (24) leads to

$$
P^\rho = \frac{\gamma}{\rho} Y_d^{1-\rho} \tag{B.5}
$$

where

$$
\gamma \equiv c \left[ \frac{\rho f}{(1 - \rho)c} \right]^{1-\rho} > 0
$$

is a bundle of parameters. Substituting (B.5) into (B.4) and using symmetry yields the equilibrium profit of an oligopolistic firm

$$
\Pi^* = \frac{\gamma}{\rho} (Q^*)^\rho - CQ^* - F
$$

so that the equilibrium income is given by

$$
Y^* = L + N \left[ \frac{\gamma}{\rho} (Q^*)^\rho - CQ^* - F \right].
$$

Substituting (B.5) into (23) leads to

$$
1 = \frac{C}{\gamma}(Q^*)^{1-\rho} + (Y_d^{\rho})^{-1} \frac{\gamma}{\rho} (Q^*)^\rho.
$$
Hence, $Q^*$ is a solution to

$$N\Pi(Q) - \Gamma(Q) = 0 \quad (B.6)$$

where

$$\Pi(Q) = \frac{\gamma}{\rho} Q^\rho - CQ - F \quad \text{and} \quad \Gamma(Q) \equiv \frac{\gamma}{\rho} \frac{Q^\rho}{(1 - \alpha)(1 - (C/\gamma)Q^{1 - \rho})} - L$$

are both strictly increasing in $Q$ over $]0, Q[\]$ where

$$Q \equiv \left( \frac{\gamma}{C} \right)^{\frac{1}{1 - \rho}}.$$

Note that $Q^* < Q\bar{Q}$.

Insert Figure 2 about here

Since $N\Pi(Q)$ is concave and strictly increasing, $\Gamma(Q)$ is convex and strictly increasing, and $\Gamma(0) < N\Pi(0) < 0$ (see Figure 2), (B.6) has a unique positive solution. Therefore, it must be that

$$N\Pi(Q) > \Gamma(Q) \iff Q < Q^*$$
$$N\Pi(Q) = \Gamma(Q) \iff Q = Q^*$$
$$N\Pi(Q) < \Gamma(Q) \iff Q > Q^*.$$ 

Furthermore, because $\Gamma(0) = -L$ and $\Gamma(Q) \to \infty$ when $Q \to Q\bar{Q}$, $\Gamma(Q) = 0$ has a unique solution $Q_s \in ]0, Q\bar{Q}[.$

The comparison of $Q^*$ and $Q_s$ involves three cases.

(i) If $\Pi(Q_s) < 0$, then $N\Pi(Q_s) < \Gamma(Q_s) = 0$, so that $Q_s > Q^*$. Since $\Pi(Q)$ is increasing, it must be that $\Pi(Q^*) < \Pi(Q_s) < 0$.

(ii) If $\Pi(Q_s) = 0$, then $N\Pi(Q_s) = \Gamma(Q_s) = 0$, so that $Q_s = Q^*$ since (B.6) has a unique positive solution.

(iii) If $\Pi(Q_s) > 0$, then $N\Pi(Q_s) > \Gamma(Q_s) = 0$, so that $Q_s < Q^*$. In this case, it must be that $\Pi(Q^*) > \Pi(Q_s) > 0$ because $\Pi$ is increasing. All of this implies that

$$\Pi(Q^*) > 0 \iff \Pi(Q_s) > 0.$$ 

Therefore, a necessary and sufficient condition for the oligopolistic firms’ profits to be positive at the mixed market equilibrium may be obtained by studying the sign of $\Pi(Q_s)$.

Note that

$$\Gamma(Q) \overset{\geq}{\sim} 0 \iff G(Q^\rho) \equiv \gamma^2 Q^{2\rho} + (1 - \alpha)(1 - \rho)L\gamma Q^\rho - (1 - \alpha)\rho LF \overset{\leq}{\sim} 0.$$
Replacing $-CQ_s$ in $\Pi(Q_s)$ shows that $\Pi(Q_s) > 0$ is equivalent to $G(Q^\rho) > 0$. This inequality holds if and only if $Q_r < Q_s$ where

$$Q^\rho_r \equiv \frac{(1 - \alpha)(1 - \rho)L}{2\gamma} \left[ \sqrt{\frac{4\gamma F}{(1 - \alpha)(1 - \rho)^2 L} + 1} - 1 \right]$$

is the positive solution of the quadratic equation $G(Q^\rho) = 0$ in which $Q^\rho$ is the unknown. Since $\Gamma$ is increasing and $\Gamma(Q_s) = 0$, we have:

$$Q_r < Q_s \iff \Gamma(Q_r) < 0 \iff \gamma Q^\rho_r \left[ 1 - \frac{\gamma}{(1 - \alpha)\rho L}Q^\rho_r \right] > CQ_r.$$ 

Substituting $Q_r$ in the last inequality yields

$$Q_r < Q_s \iff B \left( \frac{F}{L} \right) \equiv \left[ \frac{(1 - \alpha)(1 - \rho)}{2} \right]^{\frac{1}{1-r}} \left[ \frac{\rho}{(1 - \alpha)\rho - F/L} \frac{C}{1-r} \right]^{\frac{\rho}{1-r}}$$

$\cdot \left[ \frac{2}{(1 - \alpha)(1 - \rho)} \frac{F}{L} - 1 + \sqrt{\frac{4\rho F}{(1 - \alpha)(1 - \rho)^2 L} + 1} \right] < \frac{f}{L}$.

**Appendix C**

1. Since the function $N\Pi(Q)$ is moved upward when $N$ increases while $\Gamma(Q)$ remains the same, it must be that $Q^*$ increases when $N$ rises (see Figure 2). Using (20), (24), (25) and the oligopolistic firms’ first order condition (23), we obtain

$$(P^*)^{\frac{\rho}{1-r}} = 1 - \frac{C}{\gamma} (Q^*)^{1-r}.$$ 

It follows from this expression that $P^*$ and $Q^*$ move in opposite directions.

2. Substituting (24) and (25) into (20) leads to

$$(P^*)^{-\frac{\rho}{1-r}} = M^* \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-r}} + N \left[ \frac{1 - \rho}{f} \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-r}} Q^* \right]^\rho.$$ 

while substituting (24) into (19) yields

$$(P^*)^{\frac{\rho}{1-r}} = \left( \frac{c}{\rho} \right)^{\frac{\rho}{1-r}} (1 - \alpha)(1 - \rho) \frac{L - NF + \left( \frac{c}{\rho} \right)^{\rho} \left( \frac{f}{1-r} \right)^{1-r} N (Q^*)^\rho - CNQ^*}{f}.$$ 

(C.2)
Equating (C.1) and (C.2) yields the equilibrium mass of MC-firms for a given $N$:

$$M^* = \frac{(1 - \alpha)(1 - \rho)}{f} \left\{ L - N \left[ F + \frac{\alpha}{1 - \alpha} \left( \frac{c}{\rho} \right)^{\rho} \left( \frac{f}{1 - \rho} \right)^{1 - \rho} (Q^*)^\rho + CQ^* \right] \right\}$$

which decreases because the large firms’ output $Q^*$ increases. Note that the resource constraint $M^* < (L - NF)/f$ is always satisfied at (C.3).

**Appendix D**

We show the continuity of the market equilibrium at $N_O$, which is the unique solution to $M^*(N) = 0$. Clearly, $J(0; N) \leq 0$ implies that $\pi^*(H; N) < 0$ for all $H > 0$. Hence, when $M^*(N) = 0$, the values (B.3) in which $H^*(N) = 0$ describe the pure oligopolistic market outcome for $N \geq N_O$:

$$\begin{align*}
    P_O &= \frac{CN^{2\rho - 1}}{\rho N - 1} \\
    Y_O &= \frac{N(L - NF)}{\alpha N + (1 - \alpha)\rho(N - 1)} \\
    Q_O &= \frac{(1 - \alpha)\rho(N - 1)(L - NF)}{CN[\alpha N + (1 - \alpha)\rho(N - 1)]} \\
    \Pi_O &= \frac{[(1 - \rho)N + \rho][1 - \alpha)(L - NF)]}{N[\alpha N + (1 - \alpha)\rho(N - 1)]} - F \\
    W_O &= \alpha^\alpha \left[ \frac{\rho(1 - \alpha)}{C} \right]^{1 - \alpha} \frac{N^{(1 - \alpha)(1 - \rho)} + \alpha(N - 1)^{1 - \alpha}(L - NF)}{\alpha N + (1 - \alpha)\rho(N - 1)}.
\end{align*}$$

This means that the market equilibrium values are continuous functions of $N \geq 2$.

**References**


FIGURE 1: The emergence of mixed market

\[ \frac{f}{L} = \mathcal{S}\left(N, \frac{F}{L}\right) \]

as \( N \) increases.
FIGURE 2: The equilibrium profits and outputs of oligopolistic firms with $N_1 < N_2$.
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