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Product differentiation and vertical integration
in presence of double marginalization

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**Product differentiation and vertical integration
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Abstract

In this paper, we present a model of endogenous vertical integration and horizontal differentiation. There exists two output brands and two versions of the input. The only mean for output differentiation is the input version used in output production. Firms may choose to vertically integrate to produce internally the required input version at marginal cost, rather than to buy it at the market price, if that version is made available. We show that vertical mergers increase the possibility that output goods are differentiated. Moreover, this occurs when the cost to differentiate the input is high. On the other hand, vertical integration is detrimental for brand variety if the cost to differentiate inputs is negligible.

Keywords: horizontal differentiation, vertical agreements, successive Cournot oligopolies.

JEL Classification: D43, L13, L42

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1 Introduction

This paper studies the effect of vertical mergers in a two layer industry with horizontally differentiated input and outputs. The *only* mean through which downstream firms may differentiate their brands is the input version used to produce output (Belleflamme and Toulemonde, 2003 or Gabszewicz and Turrini, 1998). This suggests that the decision of upstream firms on input differentiation affects the decision of downstream firms on output differentiation and vice versa¹. Examples of this occurrence abound in the electronic and computer industries where the choice of software companies (upstream firms) as to what application to produce affects the type of hardware selected by hardware companies (downstream firms). Similarly, economic outlets have recently reported that a majority of economic activities in US is "project oriented" (Pepall and Norman, 2001). These projects combine the work of traditional employees, workers under contract and consultants, working into a team. When the project is realized the team breaks down, and each participant engages in new projects. The team can be viewed as an input to realize a specific objective, which can be reached only if the most appropriate team is constituted. A good example of such projects activities is the TV series productions. Finally, empirical investigations report that the quality of products depends significantly on the degree of availability of skilled workers (Gabszewicz and Turrini, 1998). Indeed, it is easy to imagine a link between the available skills (input specification) and the feasible quality (characteristics) of the final good².

The interdependence between the decision of input supplier and output producers leads to different equilibrium structures characterized by different allocations of firms on different brands (Belleflamme and Toulemonde, 2003, Matsushima 2004).

These examples share a further characteristic: input suppliers have *flexible manufacturing production* technologies. This implies that each input supplier can potentially produce both types of input versions. Eaton and Schmitt (1994) or Norman and Thisse (1999) find that flexible manufacturing promotes concentration through mergers. Similarly, when flexible manufacturing is introduced in a two layer industry, vertical mergers may arise between upstream and downstream firms inducing anticompetitive effects on equilibrium prices (Church and Gandal, 2000, Pepall and Norman, 2001).

In the context described above characterized by *(i)* inputs as the mean of output differentiation and *(ii)* flexible technology for inputs, we investigate the implications of endogenous vertical mergers on horizontal differentiation of output brands. Vertical integration has several effects on profits of merged firms and of the unintegrated ones. Firstly, it allows internalizing the production of input and consequently makes the entity independent from input suppliers deci-

¹For instance, if all input suppliers choose to produce the same input version, this would naturally lead to homogenous output goods. A less extreme distribution of input suppliers among different input versions, opens the door to horizontal output differentiation.

²This is an example of vertical differentiation, while the other ones represent horizontal differentiation.

sion. Secondly, merging can be profitable in presence of double marginalization. For instance, an industry where double marginalization is an issue is the cable television in the US. In this industry, producers charge for programming per subscriber, and this corresponds to a linear pricing. As it is well known, linear pricing in multi layers, non competitive, industries leads to double marginalization. Evidence in the U.S. market is consistent with the presence of double marginalization (Waterman and Weiss, 1996, Belleflamme and Peitz (forthcoming)). Finally, the merged firm may increase the rival input cost by withdrawing their participation in the input market (increasing rival costs' strategy)³.

We find that allowing firms to vertically integrate expands the sets of parameters where output firms decide to differentiate. Furthermore, the effects of vertical agreements on horizontal differentiation depend on the level of input costs. When the cost to differentiate the input is negligible, brands are differentiated if firms operate separately. As the cost to differentiate inputs gets higher, output brands are differentiated if firms can vertically integrate.

The decisions of firms are modelled in a sequential game. In the first stage, two downstream firms choose their brand of output. In the second stage, firms decide what is the structure of their firm: to integrate or not. In the third stage, production of output and input takes place⁴. The timing of the game reflects the irreversibility of decisions. Thus, our model is suitable to situations in which the choice of technology to produce the output is less reversible than the decision to vertically integrate or not.

The literature on product variety generally agrees that variety should be higher under vertical separation (Kühn and Vives (1999)). This is the case when firms have already decided the brand to produce, and vertical integration affects their survival in the market. Indeed, collusive agreements can bring input foreclosure and determine the exit of firms, reducing variety. Instead, in this model vertical integration helps brand differentiation. The two results are not contradictory. Indeed, we find that when the input cost difference is not high, vertical integration does reduce differentiation of brands. But, we also find that vertical mergers facilitate differentiation when the input cost difference is high enough. Moreover, in presence of double marginalization, the set where output differentiation takes place is larger than the corresponding set without vertical mergers. Finally, further works related to our paper are for instance Choi and Yi (2000). Authors study the anticompetitive effects of vertical integration with differentiated outputs. In their model, upstream firms should decide the specification of the input to produce, being able to produce *only* one input version. Choi and Yi (2000) show how vertical integration can be anticompetitive because input specification choice leads clearly to exclusion. Pepall and Norman (2001) analyze successive Bertrand oligopolies considering that downstream firms use Leontief technologies and the corresponding inputs

³In other setups, insightful analysis of equilibrium foreclosure have been proposed by Ordober, Saloner and Salop (1990), and Hart and Tirole (1990).

⁴We use successive oligopolies, because they are a natural setup to study the spill-overs induced by the strategic behavior of firms in the upstream market on the behavior of firms in the downstream market and vice versa.

used are complementary. Our analysis departs from Pepall and Norman (2001) because we make endogenous the choice of the brand for *both* input and output firms.

The paper is organized as follows. The next section presents the model. Section 3 deals with the equilibrium of the game. Section 4 studies the effects of vertical integration of output differentiation. Section 5 concludes.

2 The model

Assume there exist two brands of output a and b . Two downstream firms $D1$ and $D2$ shall decide to produce *one* of the two goods by transforming the input on a one-to-one basis. They can both decide to produce the same good i , $i = a, b$, and face the following output demand

$$p_i = 1 - z_i, \quad i = a, b$$

where z_i is the total amount produced of brand i . Alternatively, firms may decide to differentiate producing one brand a and the other brand b . The output demands faced by the firms in this case are:

$$\begin{aligned} p_a &= 1 - z_a - \gamma z_b \\ p_b &= 1 - \gamma z_a - z_b \end{aligned}$$

These demands are derived from the maximization problem of a representative consumer with taste for variety (Singh and Vives, 1984) endowed with a utility function separable in money m , given by $u(z_a, z_b) = z_a + z_b - \frac{z_a^2}{2} - \frac{z_b^2}{2} - \gamma z_a z_b + m$. The parameter γ measures the degree of product differentiation between the two brands in the downstream market. As γ increases the products become more similar⁵.

Consider also two upstream firms, denoted by $U1$ and $U2$, who produce an homogeneous input with cost c that can be used in the production of good a . Production of brand b requires a different version of the input that can be produced *possibly* by each of the upstream firms sustaining an additional cost t , $t < 1$. For simplicity normalize c to 0. Then, the cost t is the *unit cost for input differentiation*.

In the following, we develop a game of brand differentiation and vertical integration. The game is composed of three stages: in the first stage, firms decide which good to produce; in the second, they decide to vertically integrate or not, and in stage three the production of the goods takes place. A major advantage of using this modeling is that vertical relationships between the upstream and the downstream can be assessed purely in terms of the parameters γ and t .

Consider the type of equilibria that may arise:

⁵When $\gamma = 1$ the downstream products are perfect substitutes in consumption. In contrast, when $\gamma = 0$ the products are completely differentiated. In this case, each firm in the downstream market is a single-product monopolist.

- If each downstream firm chooses a different brand, *(i)* none of the firms may merge; *(ii)* the firms can both decide to integrate with one upstream firm - *full integration*; or *(iii)* only one of the downstream firms merges with one upstream firm while the other couple of firms operates separately.
- If both downstream firms decide to produce the same brand we again may have *(i)*, *(ii)* or *(iii)* as equilibrium of the game.

We solve the game using backward induction.

2.1 Downstream firms differentiate

Assume for the time being that in the first stage, say, $D1$ chooses brand a and $D2$ chooses to produce brand b .

2.1.1 Absence of vertical integration

Downstream Market Game When none of the firms decide to merge in the second stage, the profit of the downstream firms, Π_a and Π_b , respectively, write as

$$\begin{aligned}\Pi_a(z_a, z_b) &= (1 - z_a - \gamma z_b)z_a - \omega_a z_a, \\ \Pi_b(z_a, z_b) &= (1 - z_b - \gamma z_a)z_b - \omega_b z_b,\end{aligned}$$

where ω_a and ω_b denote the input price paid by each firm for each input version. The firms select the output quantities $z_a(\omega_a, \omega_b)$ and $z_b(\omega_a, \omega_b)$ playing *à la* Cournot.

Upstream market game Denote by (x_a, x_b) the amount of input supplied by upstream firm $U1$, where $x_a \geq 0$ is the input supplied to the downstream $D1$ producing brand a , and $x_b \geq 0$ is the input supplied to the downstream $D2$. Analogously, denote by (y_a, y_b) input quantities supplied by upstream firm $U2$, respectively to firm $D1$ and firm $D2$ (see Fig. 1 below). It follows that the market clearing conditions for each good are

$$\begin{aligned}z_a(\omega_a, \omega_b) &= x_a + y_a, \\ z_b(\omega_a, \omega_b) &= x_b + y_b.\end{aligned}$$

Using the above market clearing conditions and the first order conditions of the profit of downstream firms, we obtain the input demand as

$$\begin{aligned}\omega_a &= 1 - 2(x_a + y_a) - \gamma(x_b + y_b) \\ \omega_b &= 1 - \gamma(x_a + y_a) - 2(x_b + y_b)\end{aligned}$$

The upstream firms select input supplies maximizing

$$\begin{aligned}\Gamma_1 &= \omega_a x_a + (\omega_b - t)x_b, \\ \Gamma_2 &= \omega_a y_a + (\omega_b - t)y_b.\end{aligned}$$

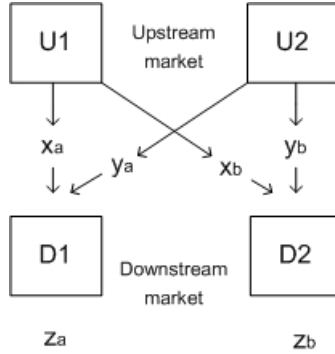


Figure 1: Successive markets

Solving the system of four equations of the input quantities selected by the upstream firms, we get, at a symmetric equilibrium,

$$x_a^* = y_a^* = \frac{2 - \gamma + t\gamma}{3(4 - \gamma^2)}$$

and

$$x_b^* = y_b^* = \frac{2 - 2t - \gamma}{3(4 - \gamma^2)}.$$

The supply of input to downstream firm $D2$ is nonnegative for $t < 1 - \frac{\gamma}{2}$. These equilibrium quantities are substituted into the expression of z_a and z_b , consequently, using z_a^* and z_b^* , equilibrium output prices obtains as

$$p_a^* = \frac{8 - 2\gamma(1 - t) - \gamma^2}{3(\gamma + 2)(2 - \gamma)},$$

$$p_b^* = \frac{2(2t - \gamma + 4) - \gamma^2(2t + 1)}{3(\gamma + 2)(2 - \gamma)}$$

We can check that $\frac{\partial p_b^*}{\partial \gamma} < 0$. Indeed, the higher the degree of differentiation between brands, i.e. the lower γ , the higher the output price. This is the well-known property of output prices of differentiated goods. In contrast, for brand a , there exists a threshold of t , i.e. $t' = \frac{(2-\gamma)^2}{\gamma^2+4}$ ⁶, such that for $t > t'$ the more homogeneous the goods, i.e. the higher γ , the higher the price p_a . The intuition for this is as follows. When the input difference cost t becomes enough large and exceeds threshold t' , differentiation of brands is very costly. Therefore, in that interval of t , the convenience to product differentiate is less strong than the cost to differentiate. It follows that decreasing the rate of differentiation does not reduce the market power of the firm producing brand a . This property of

⁶Notice that $t' < \frac{(\gamma-2)^2}{(\gamma^2+4)}$ is a subset of the admissible set $t < 1 - \frac{\gamma}{2}$.

output prices is the example of how the two characteristics of successive markets interact.

The profits of downstream and upstream firms in this and the following sections are given in the matrixes (3) and (4).

2.1.2 Vertical Integration: D1-U1

Let us now analyze the cases when one downstream firm, say $D1$, decides to integrate say with $U1$, in the second stage game, while the other two firms operate separately. The decision to vertically integrate does not imply a priori that the new entity does not enter the input market. In other words, we do not assume input foreclosure. If it arises, it is an *equilibrium foreclosure* as in Gaudet and Van Long (1996). This assumption is natural in our analysis since we address the endogenous choice of input supplier as to what input specification to produce.

The profit Π_{11} of the entity composed of the downstream $D1$ and the upstream $U1$ writes as

$$\Pi_{11} = (1 - z_a - \gamma z_b)z_a + (\omega_b - t)x_b$$

where x_b is the quantity of input supplied in the input market by the entity. The profit of the nonintegrated downstream obtains as

$$\Pi_2 = (1 - \gamma z_a - z_b)z_b - \omega_b z_b.$$

Solving the system of the best replies, the quantity of output selected by each downstream firm as a function of the input price ω_b is

$$z_a(\omega_b) = \frac{2 - \gamma + \gamma\omega_b}{4 - \gamma^2}$$

and

$$z_b(\omega_b) = \frac{2 - \gamma - 2\omega_b}{4 - \gamma^2}.$$

If the supply of the entity is positive, the market clearing condition in the input market equalizes the input demand derived from the output quantity z_b selected by the downstream firm $D2$, and the supplied quantities of the unintegrated upstream and the entity, namely $z_b = y_b + x_b$. Hence, the input demand function is

$$p_b = \left(\frac{1}{\gamma + 2} - x_b - y_b \right) \left(2 - \frac{1}{2}\gamma^2 \right). \quad (1)$$

Given the above input demand, the unintegrated upstream and the integrated entity choose the quantities y_b^* and x_b^* maximizing their profit Γ_2

$$\Gamma_2 = \left(\frac{1}{\gamma + 2} - x_b - y_b \right) \left(2 - \frac{1}{2}\gamma^2 \right) y_b - t y_b$$

and Π_{11} . It follows that

$$x_b^* = y_b^* = \frac{2 - 2t - \gamma}{12 - 3\gamma^2}.$$

Notice that in the admissible set $t < 1 - \frac{\gamma}{2}$, x_b^* and y_b^* are positive.

2.1.3 Vertical Integration D2-U2

Consider now the possibility that the downstream firm *D2*, which needs the costly input version, integrates with one of the upstream firms, say *U2*, while the other two firms operates separately.

The entity composed of the downstream *D2* and the upstream firm *U2* has as profits Π_{22}

$$\Pi_{22} = (1 - \gamma z_a - z_b)z_b - tz_b + \omega_a y_a$$

The unintegrated downstream firm maximizes its profit Π_1 ,

$$\Pi_1 = (1 - z_a - \gamma z_b)z_a - \omega_a z_a.$$

They select $z_a(\omega_a)$ and $z_b(\omega_a)$. Given the integration between the downstream *D2* and the upstream *U2*, the only firm demanding input in this case is the downstream *D1*, thus the market clearing condition in the input market is $z_a(\omega_a) = y_a(\omega_a) + x_a(\omega_a)$. The inverse demand function for the input is,

$$\omega_a = \left(\frac{\gamma - t\gamma - 2}{\gamma^2 - 4} - y_a - x_a \right) \left(2 - \frac{1}{2}\gamma^2 \right). \quad (2)$$

The unintegrated upstream maximizes its profit Γ_1 taking into account the demand given above, which yields $x_a(y_a)$. By substituting $x_a(y_a)$ in the demand function (2) we obtain the input demand function faced by the integrated entity as

$$\omega_a = \frac{1}{4}(t\gamma - \gamma - 4y_a + \gamma^2 y_a + 2).$$

Now, by substituting the above expressions of z_a , z_b and ω_b in the profit function of the entity we can find the quantity of input y_a^* it selects to supply

$$y_a^* = x_a^* = \frac{t\gamma - \gamma + 2}{12 - 3\gamma^2},$$

where y_a^* and x_a^* are positive in the whole range of admissible set of t .

It easy to check by differentiating profits, that this merger is always profitable for the downstream firm, when the other two firms operate independently, but it will not always be accepted by the upstream firm. The reason stands on the incentive of the upstream to use its market power in the input market rather than be a department of the downstream firm. Indeed, the cost advantage gained by the merger by the elimination of double marginalization does not always offset the cost advantage of downstream firm *D1*.

2.1.4 Full Integration

Assume now that each couple of firms integrated. In the new equilibrium two downstream firms produce two differentiated goods. The profits of each firm writes simply,

$$\begin{aligned}\Pi_{11}^* &= (1 - z_a - \gamma z_b)z_a, \\ \Pi_{22}^* &= (1 - \gamma z_a - z_b)z_b - tz_b.\end{aligned}$$

Solving a classical Cournot, we obtain the equilibrium profits for each entity operating in downstream market.

2.2 Homogenous goods

When both downstream firms decide to produce the same output brand, at the first stage of the game, the model reduces to a classical successive Cournot duopoly with linear demand. Thus, in this case, the only equilibrium of the integration game is full integration. Indeed, in absence of differentiation, competition in the output market is fiercer than in the case of two differentiated brands. Consequently, double marginalization internalization drives both downstream firms to propose a merger. Both upstream firms accept, because the profit of the entity exceeds the standing alone profit.

3 The equilibrium of the game

The equilibrium of the game depends on different forces that affect firms' profits. Firstly, *(i)* downstream firms decide to differentiate to gain market power in the downstream market. And they may vertically integrate *(ii)* to ensure the input specification necessary for their brand and *(iii)* to increase (or to reduce) the cost (dis)advantage with the rival firm through internalizing double marginalization.

On the other hand, the decision of upstream firms whether to accept the merger depends on the size of double marginalization, which in turn depends on degree of output differentiation.

To define the equilibrium of the game, we start with the case when downstream firms differentiate. Comparing the payoffs, from the best replies we obtain that firms fully integrate because integration is a dominant strategy for U1 and D1. The best reply of the two other firms is to vertically integrate, too⁷. Because of the interaction between the degree of differentiation of brands and the difference in the input costs, as far as it concerns the second stage, full integration arises because vertical externality is high enough to make the vertical integration profitable.

Combining the best response of the firms at the second stage and the first stage of the game, the equilibrium of the game when firms decide both whether to integrate and to differentiate is:

⁷See Appendix 1 for the full analytical derivation.

Proposition 1 *Both output brands are supplied to final consumers by two merged firms when the input cost is high enough ($1 > t > \tilde{t}(\gamma)$). Only the cheap output brand is supplied by two merged firms when the input cost is low enough ($0 < t < \tilde{t}(\gamma)$).*

Proof. see Appendix. ■

It is interesting to notice that horizontal differentiation takes place for high input cost differences, when firms fully integrate. While, without vertical agreements, the cost to adapt the input, pushes both upstream firms to concentrate on the basic input version. Moreover, the higher the degree of differentiation between output goods, the higher the incentive to produce both input versions. Thus, if firms in the output and input market produced separately, we would expect that at low t and γ , horizontal differentiation would take place. And for high t and γ , only brand a would be produced. This is the contrary of what we find in Proposition 1. It follows that the change in market structure induced by vertical agreements does not simply reduce or expand the set (γ, t) where differentiation occurs. Vertical agreements also change the incentives of firms at any level of t and γ .

The intuition of this result depends on the size of double marginalization as the cost t increases. Each integrated entity increases the rivals' cost as the difference in input versions increases. This leads to an increase of double marginalization. Thus, when t is high, firms fully integrate to take advantage of the elimination of the vertical externality. Moreover, this is more profitable if output goods are differentiated as compared to the case of homogeneous goods. The contrary holds when the input cost difference t is low.

We further investigate the role of vertical integration on output brand differentiation in the following section.

4 Vertical integration and brand differentiation

Here, we ask what is the effect of equilibrium vertical agreements on brand variety. To disentangle the effect of vertical integration, we first need to know when horizontal brand differentiation occurs if firms do not have the option to vertically integrate. As we said above, we expect that when firms operate separately, then, for low γ and t (high output differentiation and low cost of differentiation), maximum differentiation should occur. To verify this intuition, we may consider the simpler two stage game: in the first stage, firms decide what brand to produce and in the second stage production takes place. The effect of the vertical integration on brand differentiation can be identified comparing the equilibrium differentiation of brands in the latter game with the one described above in this paper.

Solving by backward induction, when in the first stage downstream firms $D1$ and $D2$ choose to produce the same brand A , the profits of the downstream firms are equal to 0.049, and the profit of upstream firms are 0.037. If in the first stage firms choose two different brands, the payoffs of the game are the one presented in the section 2.1.1.

By the direct comparison of the solution of the two games⁸, it follows

Proposition 2 *Vertical integration leads to a larger set of (γ, t) in which output differentiation takes place. Moreover, the sets of parameters where differentiation occurs with and without vertical integration do not intersect.*

Proof. see Appendix 2. ■

Vertical integration allows firms to produce internally the input that is required for the output brand and moreover the corresponding cost is the marginal cost rather than the market input price. Thus, as compared to the case of separated firms, the merged entity can produce with profits at higher input costs.

This analysis leads us to some conclusions that concern the structure of sectors where input suppliers have flexible production and brands differentiation depends on inputs used. We show that vertical agreements effects on horizontal differentiation depend on the level of input costs. When the cost to differentiate the input is negligible, brands are differentiated only if firms operate separately. As input cost differences get higher, output brands are differentiated, if firms can vertically integrate.

5 Conclusion

The framework of successive markets is a natural setup to study the spillovers induced by the strategic behavior of firms in the upstream market on the behavior of firms in the downstream market and vice versa. In this paper, there exist two brands of the output good and two possible versions of the input. Downstream firms select each one output brand out of two possible ones. The input firms decide whether to produce both input versions or just the basic version that is the less costly.

Upstream firms face the effective demand for input coming from the maximization of downstream firm profit. Consequently, the input demand embodies how the degree of differentiation affects the game in the downstream market. It follows that the decision of input suppliers is driven both from the cost to adapt the basic input, supply side, and the degree of differentiation of output goods, demand side. On one hand, the cost to adapt the input pushes both upstream firms to concentrate on producing only the basic input version. While, on the other hand, the other higher the degree of differentiation between output goods, higher is the incentive to produce both input versions.

⁸See appendix 2 for calculations.

References

- [1] Belleflamme P, and Toulemonde E., (2003), "Product Differentiation in Vertical Successive Markets", *Canadian Journal of Economics*, Vol. 36, 523-545.
- [2] Belleflamme P. and Peitz M, "Industrial organization: markets and strategies", Cambridge University Press, forthcoming.
- [3] Choi P. P. and Yi S.S., (2000), " Vertical foreclosure with choice of input specification", *Rand journal of economics*, 31, pp 717-743.
- [4] Church J. and Gandal N., (2000),"Systems competition, vertical mergers and foreclosure", *Journal of economics and management strategy*, 9, pp 25-51.
- [5] Eaton C. and Schmitt N., (1994), "Flexible Manufacturing and Market Structure," *American Economic Review*, , vol. 84(4), pages 875-88, September.
- [6] Gabszewicz J. and Turrini A., (2000), "Workers' skills, product quality and industry equilibrium," *International Journal of Industrial Organization*, Elsevier, vol. 18(4), pages 575-593, May.
- [7] Gaudet G. and Van Long N., (1996), "Vertical Integration, Foreclosure and Profits in the Presence of Double Marginalisation", *Journal of Economics and Management Strategy*, Vol. 5(3), 409-432.
- [8] Hart O. and Tirole J., (1990), "Vertical Integration and Market Foreclosure", *Brookings Papers on Economic Activity*, Special Issue: 205-276.
- [9] Kuhn K.U. and Vives X., (1999), "Excess Entry, Vertical Integration, and Welfare", *Rand Journal of Economics*, Vol. 30, No. 4, pp. 575-603.
- [10] Matsushima N., (2004), *International Journal of Industrial Organization*, Volume 22, Issues 8-9, Pages 1091-1114.
- [11] Norman G. and Thisse, J. F., (1999), "Technology Choice and Market Structure: Strategic Aspects of Flexible Manufacturing," *Journal of Industrial Economics*, Blackwell Publishing, vol. 47(3), pages 345-72, September.
- [12] Pepall L. and Norman G., (2001)," Product differentiation and upstream-downstream relations", *Journal of Economics and Management Strategy*, 10, pp 201-233.
- [13] Ordober, J. and Saloner, G. and Salop, S., (1990), "Equilibrium vertical foreclosure", *American Economic Review*, Vol. 80, 127-142
- [14] Singh, N., and Vives X. , (1984), "Price and Quantity Competition in a Differentiated Duopoly", *Rand Journal of Economics* 15, 546-554.

6 Appendix

6.1 Appendix 1: The equilibrium of the game

Integration game

If in the first stage downstream firms differentiate, the integration game payoffs write as follows:

D2

D1	Int. $\frac{(t\gamma-\gamma+2)^2}{2(\gamma+2)^2(\gamma-2)^2}$; $\frac{(\gamma-2t+t\gamma^2-2)(2t+\gamma-2)}{2(\gamma+2)^2(\gamma-2)^2}$	Int. $\frac{\Pi_{11}}{2}$; $\frac{4}{9} \frac{(2t+\gamma-2)^2}{(\gamma+2)^2(\gamma-2)^2}$
	Not int. $\frac{4}{9} \frac{(-\gamma+t\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2}$; $\frac{\Pi_{22}}{2}$	Not int. $\frac{4}{9} \frac{(t\gamma-\gamma+2)^2}{(\gamma+2)^2(\gamma-2)^2}$; $\frac{4}{9} \frac{(2t+\gamma-2)^2}{(\gamma+2)^2(\gamma-2)^2}$

(3)

$$\frac{\Pi_{22}}{2} = \frac{(144-16\gamma^2-\gamma^4)t^2+2(\gamma-2)(4\gamma-6\gamma^2+\gamma^3+72)t+(12\gamma-\gamma^2+44)(\gamma-2)^2}{72(\gamma-2)^2(\gamma+2)^2} \text{ and}$$

$$\frac{\Pi_{11}}{2} = \frac{8(\gamma^2+4)t^2-16(\gamma-2)(3\gamma+\gamma^2-2)t+(12\gamma-\gamma^2+44)(\gamma-2)^2}{72(\gamma+2)^2(\gamma-2)^2}.$$

U2

U1	Int. $\frac{(t\gamma-\gamma+2)^2}{2(\gamma+2)^2(\gamma-2)^2}$; $\frac{(\gamma-2t+t\gamma^2-2)(2t+\gamma-2)}{2(\gamma+2)^2(\gamma-2)^2}$	Int. $\frac{\Pi_{11}}{2}$; $\frac{(2t+\gamma-2)^2}{18(2-\gamma)(\gamma+2)}$
	Not Int. $\frac{(-\gamma+t\gamma+2)^2}{18(2-\gamma)(\gamma+2)}$; $\frac{\Pi_{22}}{2}$	Not Int. $\frac{2}{9} \frac{(t\gamma-\gamma-2t+t^2+2)}{(\gamma+2)(2-\gamma)}$; $\frac{2}{9} \frac{(t\gamma-\gamma-2t+t^2+2)}{(\gamma+2)(2-\gamma)}$

(4)

We use the above matrixes of payoffs to identify the best replies and the equilibria of the game.

Proof. Proposition 1

(i) Proposing a merger is a dominant strategy for the downstream D1 since $\frac{(t\gamma-\gamma+2)^2}{2(\gamma+2)^2(\gamma-2)^2} > \frac{4}{9} \frac{(-\gamma+t\gamma+2)^2}{(\gamma-2)^2(\gamma+2)^2}$ and $\frac{\Pi_{11}}{2} > \frac{4}{9} \frac{(t\gamma-\gamma+2)^2}{(\gamma+2)^2(\gamma-2)^2}$. Accepting such a merger is a dominant strategy for the upstream firm too, because $\frac{(t\gamma-\gamma+2)^2}{2(\gamma+2)^2(\gamma-2)^2} > \frac{(-\gamma+t\gamma+2)^2}{18(2-\gamma)(\gamma+2)}$ and $\frac{\Pi_{11}}{2} > \frac{2}{9} \frac{(t\gamma-\gamma-2t+t^2+2)}{(\gamma+2)(2-\gamma)}$ hold true. When D1 and U1 integrate, D2 always proposes the merger, $\frac{(\gamma-2t+t\gamma^2-2)(2t+\gamma-2)}{2(\gamma+2)^2(\gamma-2)^2} > \frac{4}{9} \frac{(2t+\gamma-2)^2}{(\gamma+2)^2(\gamma-2)^2}$, and U2 accepts since $\frac{(\gamma-2t+t\gamma^2-2)(2t+\gamma-2)}{2(\gamma+2)^2(\gamma-2)^2} > \frac{(2t+\gamma-2)^2}{18(2-\gamma)(\gamma+2)}$. So, the equilibrium of when

downstream firms differentiate is full integration.

(ii) When in the first stage of the game, downstream firms decide to produce the same less costly output brand A, the matrix of payoffs of the integration game is

	<i>D2</i>	
<i>D1</i>	Int. 0.11; 0.11	Int. 0.17; 0.027
	Not Int. 0.027; 0.17	Not Int. 0.049; 0.049

At the first stage of the game, the integrated firms are better off in full integration with differentiation rather than producing both the same good A if $\frac{(\gamma-2t+t\gamma^2-2)(2t+\gamma-2)}{2(\gamma+2)^2(\gamma-2)^2} > \frac{0.11}{2}$. This is true if

$$1 - \frac{\gamma}{2} > t > \frac{1}{4}(2 - \gamma) \frac{2(\gamma + 2) \sqrt{-\gamma + 0.47\gamma^2 + 0.56} + \gamma^2}{2 - \gamma^2} = \tilde{t}(\gamma)$$

Hence, for $1 - \frac{\gamma}{2} > t > \tilde{t}(\gamma)$ the equilibrium of the game is full integration and both brands are produced, while for $t < \tilde{t}(\gamma)$ the equilibrium of the game is full integration but only brand A is produced. This is represented in the graph below, where $\tilde{t}(\gamma)$ is the dash curve, while the line defines the admissible set $t < 1 - \frac{\gamma}{2}$.

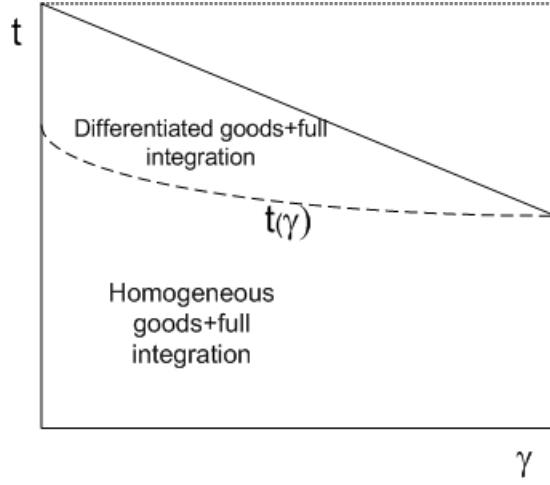


Figure 2: Below the dashed curve, the equilibrium is full integration with homogenous goods. While above the dash curve, in the admissible set, the equilibrium of the game is full integration with differentiated brands.

■

6.2 Appendix 2 : Two-stage game

The game in absence of vertical integration follows two stages. In the first stage, firms decide what brand to produce. In the second stage, production takes place.

Then, if in the second stage the firm do choose two different goods, the profit of the downstream and upstream firms are as above $\Pi_1^* = \frac{4}{9} \frac{(t\gamma - \gamma + 2)^2}{(\gamma + 2)^2(\gamma - 2)^2}$, $\Pi_2^* = \frac{4}{9} \frac{(2t + \gamma - 2)^2}{(\gamma + 2)^2(\gamma - 2)^2}$ and $\Gamma_1^* = \Gamma_2^* = \frac{2}{9} \frac{(t\gamma - \gamma - 2t + t^2 + 2)}{(\gamma + 2)(2 - \gamma)}$. If instead, in the first stage of the game, both firms produce the same brand, the profit of the downstreams are $\Pi_1^* = \Pi_2^* = 0.049$, while $\Gamma_1 = \Gamma_2 = 0.037$.

Comparing the profits with differentiation and without differentiation, $\frac{4}{9} \frac{(2t + \gamma - 2)^2}{(\gamma + 2)^2(\gamma - 2)^2} < 0.049$ if $t > 0.16\gamma^2 - 0.5\gamma + 0.33 = \bar{t}$, while it is always the case that $\frac{2}{9} \frac{(t\gamma - \gamma - 2t + t^2 + 2)}{(\gamma + 2)(2 - \gamma)} > 0.037$. Thus, for $t > \bar{t}$, downstream firms both produce the same good A, even though the upstream firms would supply the input version necessary to produce output brand B.

Proof. *Proposition 2*

The equilibrium of the game is full integration with differentiated brands if $t > \tilde{t}$. While if firms cannot integrate both brands are differentiated if $t < \tilde{t}$.

Finally we can evaluate the area of differentiation under each game and we find that

$$\int_0^1 \left(\left(1 - \frac{1}{2}\gamma\right) - \tilde{t} \right) d\gamma = 0.3603 > \int_0^1 \bar{t} d\gamma = 0.13889$$

■

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