

2010/14



Currency substitution in the economies of Central Asia:  
How much does it cost?

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**CORE**

DISCUSSION PAPER

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and Econometrics

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**Currency substitution in the economies of Central Asia:  
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Asel ISAKOVA<sup>1</sup>

April 2010

**Abstract**

Underdeveloped financial markets and periods of high inflation have stimulated dollarization and currency substitution in the economies of Central Asia. Some authors argue that the latter can pose serious obstacles for the effective conduct of monetary policy and can affect households' welfare. This study uses a model with money-in-the-utility function to estimate the elasticity of substitution between domestic and foreign currencies in three economies of Central Asia - Kazakhstan, the Kyrgyz Republic and Tajikistan. Utility derived from holding money balances is represented by a CES function with money holdings denominated in two currencies. The residents are assumed to diversify their monetary holdings due to instability of the domestic currency. The steady state analysis reveals that though currency substitution decreases governments' seigniorage revenue, holding foreign money can be welfare generating if domestic currency depreciates vis-à-vis the currencies in which households' foreign balances holdings are denominated. De-dollarization can only be achieved through further macroeconomic stabilization that will bring price and exchange rate stability. Financial sector development will also decrease currency substitution through the provision of reliable financial instruments and the gaining of public confidence.

**Keywords:** currency substitution, dollarization, monetary policy, seigniorage, welfare, transition economies.

**JEL Classification:** E58, P2, E41

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I am grateful to Jan Hanousek, Michal Kejak, Rafael Wouters and Céline Poilly for their very helpful comments and considerate advice and suggestions. I am also grateful to the Center for Operations Research and Econometrics where this research work was conducted with the support of the Marie-Curie Doctoral Fellowship. A shorter version of this article will be published in autumn 2010 by Edward Elgar in "Euro and Economic Stability: Focus on Central, Eastern and South-eastern Europe" (edited by Nowotny, E., P. Mooslechner and D. Ritzberger-Gr nwald).

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the author.

# 1 Introduction

Currency substitution, or the use of foreign currency to finance transactions, by domestic residents has been a widespread phenomenon in emerging market and transition economies. During the 1990s, currency substitution and dollarization started to increase rapidly in former centrally planned economies, and remained an important characteristic of these economies for most of the 1990s and 2000s.<sup>1</sup> This study investigates the importance of currency substitution in a group of transition economies in Central Asia and estimates the degree of substitutability between domestic currency and foreign currency in these economies. This empirical analysis contributes to an understanding of the economic importance of currency substitution in three economies - Kazakhstan, the Kyrgyz Republic and Tajikistan. Moreover, the study examines the implications of currency substitution for seigniorage revenues of the government and its welfare cost.

The countries of Central Asia have experienced important structural socio-economic and political transformation related to the demolition of old administrative systems and building new institutions of the free market.<sup>2</sup> Building a market economy required economic liberalization, including price liberalization and gradual capital markets decontrol. Price liberalization resulted in an accelerated pace of inflation and rapid depreciation of newly introduced national currencies. The weak positions of domestic legal tenders and their decreasing purchasing power led to a flight from national money and an increase in foreign currency holdings by residents. Currency substitution was a result of the general economic instability and undermined the credibility of the domestic money.<sup>3</sup> Moreover, the rudimentary financial sector institutions were not able to provide households with reliable financial instruments for saving in domestic currency. Holding foreign currency (mostly U.S. dollars) thus became a way to hedge against the risk of inflation and depreciation of the local currency.

Macroeconomic stabilization in Central Asian economies at the end of the 1990s brought down inflation rates and thus helped local currencies regain credibility. This has not, however, reversed the process of dollarization. There is no estimated measure of the cash holdings denominated in U.S. dollars in Central Asia. The level of foreign currency denominated deposits is thus used to reflect the importance of currency substitution. Several factors have influenced the population's decision to hold foreign currency.<sup>4</sup> Among them are the memory of past inflation and instability, uncertainty about future economic developments, underdeveloped financial markets and weak confidence towards local commercial banks. For example, only 5 percent of the population in the Kyrgyz Republic hold deposits

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<sup>1</sup>See, for example, Baliño, Bennett and Borensztein (1999), Feige (2003)

<sup>2</sup>See, Gürgen, et al. (1999)

<sup>3</sup>Currency substitution and dollarization are faced by most developing and transition economies. For example, Sahay and Végh (1995), Savastano (1996), Feige (2003), Havrylyshyn and Beddies (2003) studied groups of countries in Latin America, Central and Eastern Europe, and former Soviet republics and provided evidence that macroeconomic instability and high rates of inflation are major reasons for currency substitution and dollarization in these economies.

<sup>4</sup>The factors that motivate holding a foreign currency are based on evidence from surveys done by the Austrian National Bank in CESEE countries (see Ritzberger-Grünwald and Stix, 2007, Dvorsky et al., 2008, Stix, 2008) and the study by Zoryan (2005) on dollarization in Armenia.

in banks.<sup>5</sup> The income and wealth of the population is also an important factor affecting dollarization. Tajikistan and Kyrgyz Republic remain the poorest countries in the transition region. The data on foreign currency denominated assets, foreign capital flows, and inflows of remittances from abroad indicate that there is a significant inflow of foreign currency in the economies of Central Asia.<sup>6</sup> The inflows of remittances constitute not only a source of foreign currency but also an important source of finance. Households use foreign money as a savings instrument, keeping certain amounts in cash. Very often the foreign currency (in particular, the U.S. dollar) is a currency of denomination for prices on big ticket items (eg. real estate, cars, equipment). The issue of currency substitution and its policy implications thus remain important.

In the present study the elasticity of substitution between domestic and foreign currencies in Central Asian economies is estimated. For this purpose an empirical estimation of an optimizing model with money-in-the-utility function is performed. The nonlinear Euler equations that characterize the first-order-conditions of optimization by a representative consumer are estimated using the General Method of Moments (GMM) procedure as proposed by Hansen (1982). After the key parameters are estimated, they are used for further comparison of steady states with different degrees of dollarization and different inflation rates in order to examine the implications for seigniorage revenues and the welfare loss incurred by households due to holding foreign money balances.

The paper is organized as follows. Section 2 discusses the literature on currency substitution. Section 3 briefly presents the economic background and recent developments in the economies of Central Asia. Section 4 presents the theoretical model. Section 5 discusses the data used in the study. Section 6 presents the empirical results. Sections 7 and 8 examine the seigniorage losses and welfare implications of currency substitution in Central Asia, and Section 9 concludes.

## **2 Currency Substitution: Theoretical Background and Empirical Evidence**

The problem of currency substitution and dollarization has been extensively studied in the economic literature. Many developing economies have experienced high levels of dollarization following periods of macroeconomic instability.<sup>7</sup> In this study, no formal distinction is made between dollarization and currency substitution and the two terms are used interchangeably.<sup>8</sup>

Dollarization in transition economies is an important issue to address for several reasons. First,

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<sup>5</sup>EBRD Transition Report, 2008

<sup>6</sup>See Table B 1 in Appendix B.

<sup>7</sup>See, for example, Rennhack and Nozaki (2006) who discuss the magnitude and trends in dollarization in different developing regions.

<sup>8</sup>Calvo and Végh (1992) report that currency substitution is very often referred to as dollarization in economies with high inflation episodes.

dollarization might pose obstacles for an effective monetary policy by influencing the monetary transmission mechanism. Sahay and Végh (1995), Baliño, Bennett, and Borensztein (1999), Havrylyshyn and Beddies (2003) and other authors argue that dollarization makes the conduct of monetary policy more challenging as it influences the stability of the money demand and makes exchange rates more volatile. Horváth and Maino (2006) study the transmission mechanism of monetary policy in Belarus, and discuss the ways in which dollarization affects different channels of monetary policy transmission. On the one hand, a high level of dollarization brings more volatile exchange rates and a stronger pass-through from exchange rates to prices. On the other hand, the interest rate channel might become weaker as holding foreign currency denominated assets makes local economic agents less sensitive to changes in interest rates on domestic currency assets.

Furthermore, dollarization affects the ability of governments to earn revenue from seigniorage. Bufman and Leiderman (1992) study dollarization in Israel, and find that dollarization and currency substitution may affect the ability of a government to finance its budget deficit. They show that small increases in dollarization have resulted in large seigniorage losses in Israel. Harrison and Vymyatnina (2007) argue that currency substitution can also preclude a government from using an inflationary tax to finance its expenditure programs, as the spending power is limited by the willingness of domestic residents to hold domestic currency. They claim that foreign currency cash transactions can encourage tax evasion and shift the economy to underground activities.

Finally, some authors argue that currency substitution might affect the ability of central banks to provide accurate macroeconomic forecasts. Thus, in the context of dollarization and currency substitution effective implementation of an inflation targeting regime might be affected as well. Though Leiderman, Maino and Parrado (2006) find that in Latin American economies dollarization can still allow an inflation targeting regime to be implemented, the latter might still be an important argument against dollarization in the economies of Central Asia due to their underdeveloped financial sectors and weak monetary transmission channels. The effect of dollarization on monetary stability and monetary policy depends on its size and substitutability between foreign and local currencies, and the development of financial sectors.

Numerous studies have examined currency substitution in developing and transition economies. Some authors base their studies on the so-called portfolio balance model, where agents allocate their wealth in domestic and foreign money, and domestic and foreign bonds. A linear demand for money and foreign money is then estimated using a simple Ordinary Least Squares (OLS) regression or other appropriate empirical methodology. The demand for foreign currency as a measure of currency substitution is represented as a function of interest rates on domestic and foreign bonds and other variables. Komárek and Melecký (2003) apply this approach to the case of the Czech economy. Mongardini and Mueller (1999) examine currency substitution in the Kyrgyz economy. More recently, Harrison and Vymyatnina (2007) have used this methodology to study currency substitution in Russia.

Other authors employ a dynamic optimization framework with a money-in-the-utility model with

two currencies. In this literature, estimation of the structural parameters is based on estimating the Euler equations derived from the optimality conditions. This approach allows for explicit estimation of the main parameters of the model such as the level of dollarization, the elasticity of substitution between the domestic and foreign currency, as well as the magnitude of relative risk aversion and intertemporal substitution. Estimation of the non-linear equations is performed using a Generalized Method of Moments (GMM) framework. This approach was employed by İmrohoroğlu (1994), who examines currency substitution in Canada, and Bufman and Leiderman (1992), who use a model of the same type to investigate currency substitution in Israel. In the case of transition economies, a similar framework can be found in Friedman and Verbetsky (2001), who study the economy of Russia, and Selçuk (2003), who investigates currency substitution in some economies of Central and Eastern Europe – the Czech Republic, Hungary, Poland, and the Slovak Republic.

In the present study, the second methodology is used. The value added of this approach is that by explicitly estimating the parameters of the model, the implications of dollarization for seigniorage revenues and households' welfare can be analyzed. Bufman and Leiderman (1992) examine how changes in the level of dollarization affect the seigniorage revenue of the Israeli government, while Friedman and Verbetsky (2001) examine seigniorage loss and changes in economic welfare due to changes in dollarization in the Russian economy. In the present study, this approach will be used to examine three Central Asian economies.

### **3 Institutional Framework and Currency Substitution in Central Asia**

The dissolution of the Soviet Union at the beginning of the 1990s led to a deep socio-economic crisis in Central Asia: a severe output decline, general macroeconomic instability, and hyperinflation. Economic relations with other republics in the FSU were demolished. This had a negative impact on living standards and caused a deep recession in the economies of the region.<sup>9</sup>

Although the beginning of the transformation process appeared to be a painful experience for the countries of Central Asia, they managed to restore positive economic growth in the late 1990s and have even demonstrated impressive growth rates in the 2000s. High prices for hydrocarbons, rapid structural reforms, large inflows of foreign investments, and political stability have spurred the economy of Kazakhstan and improved considerably the living standards in this country in recent years. High energy prices and increasing investments in the oil and gas sectors were the main factors that drove economic growth in Kazakhstan.<sup>10</sup> Two other economies have experienced relatively modest developments in comparison to their big neighbor. Kyrgyz Republic's growth was driven mainly by gold production and investments in the gold sector, while the economy of Tajikistan could only start to recover from its recession at the end of the last decade due to the civil war that persisted even after

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<sup>9</sup>See Pomfret (2006)

<sup>10</sup>IMF Staff estimates that in Kazakhstan oil accounts for more than 50 percent of exports and 40 percent of government revenues. (IMF Country Report, 2009)

the peace accord was signed in 1997. Real economic recovery could only start in 2000.<sup>11</sup>

A period of hyperinflation in the first half of the 1990s was a consequence of price liberalization and overall economic decontrol in Central Asian countries. Moreover, newly established central banks were heavily financing state enterprises' losses and government deficits. A rapid growth in money supply contributed to high levels of inflation in all countries. With the introduction of national currencies, the central banks in the region gradually took control of prices and could achieve price stability by the end of the 1990s. Economic developments of the 2000s stimulated rapid developments in the financial markets in Central Asian states. Large inflows of capital and foreign exchange into these economies in the form of export receipts, remittances, foreign direct investment, and external borrowing by banks have supported economic growth and financial markets developments. Remittances have recently developed into an important source of foreign exchange for the Kyrgyz Republic and Tajikistan.<sup>12</sup> They have contributed to growth and poverty reduction, but also have turned into a policy challenge. One of the issues arising from large inflows of remittances is that the latter contribute to the growing inflow of foreign currency in the Central Asian economies that is beyond the control of monetary authorities.<sup>13</sup>

Central banks in the region have recently gained more importance and control and the framework of monetary policy has evolved over the period of transition in these economies. In the earlier period, central banks' policies were characterized by targeting money growth by means of conducting a tight monetary policy to take control of inflation through managing the money supply. As local currencies continued depreciating in the late 1990s, the countries' policy makers became concerned with the external balances and the stability of local money relative to major currencies (mainly the U.S. dollar). Gradual liberalization of exchange rate regimes and capital account caused higher volatility of the exchange rates. An important means of supporting stable exchange rates proved to be foreign exchange interventions. Thus, the monetary policy framework became more concerned with exchange rate stability. The instruments employed by central bankers together with the monetary policy framework, have been evolving over the last several years, yet the most effective instrument remains interventions in the foreign exchange markets and control over the money supply. At this stage of development, currency substitution might largely impede the effects of the monetary policy in Central Asian economies, as large amounts of foreign currency in circulation increase the part of money supply that is not under the control of central banks. As this affects domestic money demand, exchange rates become more volatile.<sup>14</sup> Such instruments as official interest rates have

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<sup>11</sup>See Pomfret (2006)

<sup>12</sup>IMF Country Report (2007) estimates that Tajikistan has one of the highest remittances to GDP ratio among former FSU economies.

<sup>13</sup>IMF Regional Outlook (September 2006) analyzes remittances inflows in the region of Central Asia. IMF staff states that remittances discourage domestic saving because they are used to finance consumption and housing construction rather than for investing in productive capacity. Moreover, remittances inflows might contribute to exchange rate appreciation and fuel inflation.

<sup>14</sup>One might argue that money growth targeting and exchange rate interventions are not important instruments of monetary policy in developed and advanced transition economies. Today, central bankers can use interest rate setting and inflation

limited efficiency due to thin financial sectors and underdeveloped financial intermediation.

It is worth reiterating the motives to hold foreign currency in the three economies. First of all, a memory of macroeconomic instability and high inflation explains people's concern about the stability of local currencies. This is particularly true for Tajikistan, where actual macroeconomic stabilization started only in the 2000s. In Kyrgyz Republic, some political unrest in 2005 followed by a revolution undermined the building sentiment of credit to national policies and added to the feeling of uncertainty about future economic developments. Second, Kyrgyz Republic and Tajikistan remain among the countries with the lowest income per capita. Labor emigration is a widespread phenomenon in these countries. Thus, remittances constitute an important source of foreign currency in circulation. Furthermore, underdeveloped financial markets and a lack of confidence in local banking institutions hinders households from taking their foreign cash holdings to a bank. In Kyrgyz Republic, for example, only 5 percent of the population have a bank account. A similar situation can be attributed to Tajikistan. In Tajikistan, the situation is aggravated by the considerable size of the shadow economy (due to drug trafficking), where monetary transactions are most probably performed in a foreign currency.<sup>15</sup>

The situation in Kazakhstan differs as this country has a lot more developed financial markets and a high income per capita. Integration into world financial markets, presence of foreign banks and oil dependence do however constitute the factors that drive dollarisation though in a slightly different form, that is financial dollarisation. This situation reflects importance of foreign currency in economic transactions in Kazakhstan.

## 4 A Model of Currency Substitution

The model presented in this section is based on a standard money-in-the-utility function model with two currencies. This framework has been employed by several other studies which examine substitution between domestic and foreign currencies in different countries.<sup>16</sup> The model represents a situation in which residents hold foreign currency as a simple and natural hedge against local inflation due to the motives and contexts described in the previous section. The foreign currency is thus assumed to be stable and trustworthy. In fact, in Central Asian economies, foreign currency is easily disposable and very often held as a store of value, i.e. the foreign currency yields utility in terms of households' confidence towards the latter. Local and foreign currencies can be easily exchanged in the market at the market exchange rate. The model is rather standardized and simplified. There is no production activity in the economy. Agents receive an endowment every period that constitutes their

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targeting frameworks to achieve their goals. The practice, however, shows that the monetary authorities in the economies of Central Asia continue to rely heavily on foreign exchange interventions to provide price and exchange rate stability.

<sup>15</sup>Pomfret (2006) estimates trade in drugs and weapons to account for around 30-50% of all economic activity in Tajikistan.

<sup>16</sup>See İmrohoroglu (1994), Bufman and Leiderman (1993), Friedman and Verbetsky (2001), Selçuk (2006)



wealth together with holdings of real balances that are unspent in the previous period, the interest rate earned on the bond, and a lump-sum transfer from the government.

The economy consists of a continuum of infinitely lived identical individuals with total measure one. A representative agent is assumed to derive utility from the consumption of a single good and from the liquidity services provided by holdings of domestic and foreign money. Thus, an agent maximizes the expected value of the discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, x_t), \quad (1)$$

where  $\beta$  is the discount factor and  $c$  is consumption, and  $x$  denotes liquidity holdings. Money services are produced by using a combination of domestic and foreign real balances in a CES production function:

$$x = [(1 - \alpha)m^{-\rho} + \alpha m^{*-\rho}]^{-\frac{1}{\rho}} \quad (2)$$

where  $m$  denotes domestic real money balances and  $m^*$  denotes foreign money balances. Coefficient  $\alpha$  is a share of foreign money balances in producing money services. Parameter  $\rho$  is used to compute the elasticity of substitution between domestic and foreign currency, and represents the substitutability between the two currencies. The money services part of the utility function reflects the willingness of residents to diversify their money holdings portfolio to lower the risk of losing their monetary assets due to economic instability and inflation in the home country. The foreign country is assumed to be more stable in an economic sense, i.e. its inflation rate is zero or lower than in the domestic economy. The budget constraint of a representative household is as follows:

$$c_t + m_t + m_t^* + b_t = y_t + \tau_t + \frac{m_{t-1}}{(1 + \pi_t)} + \frac{m_{t-1}^*(1 + \epsilon_t)}{(1 + \pi_t)} + \frac{b_{t-1}(1 + r_{t-1})}{(1 + \pi_t)}, \quad (3)$$

where  $r_t$  is a nominal interest rate on one period bonds between period  $t - 1$  and  $t$ . Variables  $\pi_t$  and  $\epsilon_t$  represent the inflation rate and rate of depreciation of the national currency, respectively. The nominal exchange rate is the ratio between the domestic price level and foreign price level:  $E_t = \frac{P_t}{P_t^*}$ . The residents care about the stability of the exchange rate and the relative value of the domestic currency to foreign currency. As they assume that the foreign currency is more stable, holding it gives them a certain confidence about conserving the value of their monetary assets. Each period every individual receives an endowment  $y$ , and a lump-sum transfer from the government  $\tau$ . Moreover, agents hold financial assets  $b$ , that give the nominal interest rate  $r_t$  between period  $t$  and  $t + 1$ .

Rearranging the first order conditions we obtain the following Euler equations:

$$\beta E_t \left[ \frac{u_{c_{t+1}}}{u_{c_t}} \frac{1 + r_t}{(1 + \pi_{t+1})} \right] = 1, \quad (4)$$

$$\frac{u_{m_t}}{u_{c_t}} = 1 - \beta E_t \left[ \frac{u_{c_{t+1}}}{u_{c_t}} \frac{1}{(1 + \pi_{t+1})} \right], \quad (5)$$

$$\frac{u_{m_t^*}}{u_{c_t}} = 1 - \beta E_t \left[ \frac{u_{c_{t+1}}}{u_{c_t}} \frac{1 + \epsilon_{t+1}}{(1 + \pi_{t+1})} \right] \quad (6)$$

Euler equation (4) is the standard condition for optimal allocation of consumption between periods  $t$  and  $t + 1$ . It equates the marginal utility cost of giving up one unit of consumption in period  $t$  to the expected utility gain from shifting that unit to consumption in the next period. Equations (5) and (6) equate the expected utility costs and benefits of reducing consumption in the current period by one unit and allocating that unit to money holdings and then to consumption in the next period.

To estimate the model and analyze the implications for seigniorage revenue and welfare cost of dollarization, the following utility function specification is used:

$$U(c_t, x_t) = \frac{(c_t^{1-\gamma} x_t^\gamma)^{1-\sigma} - 1}{1 - \sigma}, \quad (7)$$

where  $x_t$  is represented by equation (2).

It is assumed that the coefficient  $\gamma$  lies in the interval between 0 and 1, and reflects the transaction requirement of money, and parameter  $\sigma$  represents the coefficient of relative risk aversion (RRA) and should be positive. The situation  $\sigma = 1$  is considered as a logarithmic specification of the utility function. The parameter  $\rho$  measures the degree of currency substitution and should be more than  $-1$ . Then the elasticity of substitution between domestic and foreign money is computed as  $1/(1 + \rho)$ .

Using the specified utility function, the following optimality conditions are derived:

$$\beta E_t \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{x_{t+1}}{x_t} \right)^{\gamma(1-\sigma)} \frac{1 + r_t}{1 + \pi_{t+1}} = 1 \quad (8)$$

$$(1 - \alpha) \frac{\gamma}{1 - \gamma} \frac{c_t}{x_t} \times [(1 - \alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times m_t^{-\rho-1} + \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{x_{t+1}}{x_t} \right)^{\gamma(1-\sigma)} \frac{1}{(1 + \pi_{t+1})} \right\} - 1 = 0, \quad (9)$$

$$\alpha \frac{\gamma}{(1 - \gamma)} \frac{c_t}{x_t} \times [(1 - \alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} m_t^{*- \rho-1} + \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{x_{t+1}}{x_t} \right)^{\gamma(1-\sigma)} \frac{(1 + \epsilon_{t+1})}{(1 + \pi_{t+1})} \right\} - 1 = 0. \quad (10)$$

The optimality conditions are transformed into the following estimation equations:

$$d_{1,t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \left[ \frac{1 - \alpha + \alpha \left( \frac{m_{t+1}^*}{m_{t+1}} \right)^{-\rho}}{1 - \alpha + \alpha \left( \frac{m_t^*}{m_t} \right)^{-\rho}} \right]^{\gamma(\sigma-1)/\rho} \frac{1 + r_t}{1 + \pi_{t+1}} - 1, \quad (11)$$

$$d_{2,t+1} = \frac{\gamma(1-\alpha) \left( \frac{c_t}{m_t} \right)}{1 - \alpha + \alpha \left( \frac{m_t^*}{m_t} \right)^{-\rho}} - (1-\gamma) \times \left\{ \left[ 1 - \beta \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \right] \times \left[ \frac{1 - \alpha + \alpha \left( \frac{m_{t+1}^*}{m_{t+1}} \right)^{-\rho}}{1 - \alpha + \alpha \left( \frac{m_t^*}{m_t} \right)^{-\rho}} \right]^{\gamma(\sigma-1)/\rho} \times \frac{1}{1 + \pi_{t+1}} \right\}, \quad (12)$$

$$d_{3,t+1} = (1-\alpha) \left( \frac{m_t^*}{m_t} \right)^{1+\rho} - \alpha - \beta \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \left[ \frac{1 - \alpha + \alpha \left( \frac{m_{t+1}^*}{m_{t+1}} \right)^{-\rho}}{1 - \alpha + \alpha \left( \frac{m_t^*}{m_t} \right)^{-\rho}} \right]^{\gamma(\sigma-1)/\rho} \times \left( (1-\alpha) \left( \frac{m_t^*}{m_t} \right)^{1+\rho} (1 + \epsilon_{t+1}) - \alpha \right) \times \frac{1}{(1 + \pi_{t+1})}. \quad (13)$$

The utility function in equation (7) can be considered a special case of the utility function with habit formation in consumption:

$$U(c_t, x_t) = \frac{[(c_t - \delta c_{t-1})^{1-\gamma} x_t^\gamma]^{1-\sigma} - 1}{1 - \sigma}, \quad (14)$$

where an introduced parameter  $\delta$  measures the intensity of habit persistence in the consumption decision of the agent. If  $\delta > 0$ , the model exhibits habit formation in a traditional sense. In this case, the larger the  $\delta$ , the less pleasure from a given amount of consumption, and the larger must be the purchases to generate the same utility. In the case when  $\delta < 0$ , the household's decisions are subject to durability in the sense that not only current, but also past consumption generates utility.  $c_{t-1}$  is the "habit stock", i.e. the reference level to which the consumer compares her current consumption level.

When  $\delta = 0$ , habits do not play any role and the consumer cares only about her present consumption level. If  $\delta = 1$ , habits are very strong and consumer derives utility only from consumption growth rate. Under habit persistence, an increase in current consumption lowers the marginal utility of consumption in the current period and increases it in the next period. The derived estimation equations for the case when  $\delta \neq 0$  are presented in Appendix C. In this study, habit formation in consumption is introduced to improve the fit of the theoretical model.

## 5 Data and Estimation Procedure

The GMM procedure is applied to estimate the system of equations derived in the previous section. This procedure was developed by Hansen (1982) who formulated the estimation problem as follows.<sup>17</sup> Let  $w_t$  be an  $(h \times 1)$  vector of variables that are observed at date  $t$ , let  $\theta$  denote an unknown  $(a \times 1)$  vector of coefficients, and let  $h(\theta, w_t)$  be an  $(r \times 1)$  vector-valued function,  $h: (\mathbb{R}^a \times \mathbb{R}^h) \rightarrow \mathbb{R}^r$ . Since  $w_t$  is a random variable, so is  $h(\theta, w_t)$ . Let  $\theta_0$  denote the true value of  $\theta$ , and the true value is characterized by the property that

$$E \{h(\theta_0, w_t)\} = 0. \quad (15)$$

Further, denote  $Y_T \equiv (w'_T, w'_{T-1}, \dots, w'_1)'$  be a  $(Th \times 1)$  vector containing all the observations in a sample of size  $T$ , and suppose that  $g(\theta; Y_T)$  is the sample average of  $h(\theta, w_t)$  :

$$g_T(\theta; Y_T) \equiv \frac{1}{T} \sum_{t=1}^T h(\theta, w_t). \quad (16)$$

The idea of the GMM is to choose  $\theta$  so as to make the sample moment  $g(\theta, Y_T)$  as close as possible to the population moment of zero. Thus, the GMM estimator  $\hat{\theta}_T$  is the value of  $\theta$  that minimizes the scalar:

$$Q(\theta; Y_T) = [g(\theta; Y_T)]' W_T [g(\theta; Y_T)], \quad (17)$$

where  $\{W_T\}_{T=1}^{\infty}$  is a sequence of  $(r \times r)$  positive definite weighting matrices which may be a function of  $Y_T$ . Hansen (1982) describes this procedure for obtaining a consistent and efficient estimator for  $W_T$ .

In the present study, the unknown parameters to be estimated are denoted as  $\theta_1 = (\alpha, \beta, \gamma, \sigma, \rho)'$  or  $\theta_2 = (\alpha, \beta, \gamma, \sigma, \delta, \rho)'$ . To account for endogeneity it is necessary to use instruments. The instruments used are the lagged values of the variables in the estimated equations<sup>18</sup>:

<sup>17</sup>Procedure description follows Hamilton (1994).

<sup>18</sup>Lagged values of the explanatory variables represent good instruments to use in GMM estimation. This type of instruments are usually used by authors who perform GMM estimation in the optimization framework. For example, Hansen

$$I_t = \left\{ 1, \frac{m_{t-p+1}}{m_{t-p}}, \frac{m_{t-p}^*}{m_{t-p}}, \frac{c_{t-p+1}}{c_{t-p}}, \frac{c_{t-p}}{m_{t-p}}, 1 + r_{t-p} \right\}.$$

The monthly data used for estimation span from 2000 to 2008 in the case of Kazakhstan and the Kyrgyz Republic, and from 2002 to 2008 for Tajikistan.

The variables employed include consumer price indices (CPI), nominal exchange rates of national currencies to the U.S. dollar, interest rates, industrial production volume or average real wages as a proxy for consumption, and data on deposits in the second-tier banks. The data on deposits include deposits denominated in foreign and local currencies and is used as a proxy for foreign and domestic money balances respectively.

The data on interest rates include the official rates of central banks, deposit rates, lending rates, money market rates, and a Federal Funds rate of the Fed. The main sources of the data are central banks and statistical offices of these countries, and the IMF International Financial Statistics database.<sup>19</sup>

## 6 Empirical Results

The estimation results for each country are reported in Table 1. Results are reported for different interest rates. In each case, the minimal value of the objective function  $J_T$  is presented in the table as well. This is a chi-square test statistic for the validity of the model's overidentifying restrictions.

Table 1 presents results for the case of utility function without habit formation in consumption. The parameter estimates for  $\beta$  are economically meaningful and are below unity for every country. Thus, households in these economies value future consumption less than consumption in the present period. In some cases, the value of  $\beta$  is less than the value of 0.98 usually assumed in the economic literature. In Tajikistan,  $\beta$  is less than 0.9 for those cases when refinance and interbank rates are used in the estimation. This result might suggest that the residents in these economies are very "present-oriented" and put less value on future consumption. This might also be due to the data on interest rate dynamics. The values of the estimates for  $\gamma$  vary from 0.01 to 0.18 among countries depending on the choice of interest rate. Thus, the share of money in providing utility is significantly lower than the share of consumption. This result is in line with other similar studies. The share of foreign money holdings in providing monetary services  $\alpha$  is estimated between 0.47 and 0.62. This implies quite high efficiency of foreign money and therefore a high level of currency substitution in all three economies.

The elasticity of substitution parameter  $s$  is of particular interest. It is assumed to be positive

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and Singleton (1982), Eckstein and Leiderman (1992), İmrohoroğlu (1994) and others. Fuhrer, Moore and Schuh (1995) analyze the quality of the instruments used in the GMM estimation and state that the GMM estimates are biased in small samples and this bias persists even in large samples due to irrelevance of the instrumental variables. Therefore, the authors propose using lagged values of the variables as instruments and argue that the lags are usually well-correlated with the right-hand side variables and should represent a solution to the problem associated with poor instrument relevance.

<sup>19</sup>A more detailed description of the data can be found in Table A 1 in Appendix A.

from 0 to infinity. If it equals 0, then the two currencies are complements, but if it is more than 0, then there is substitutability between the two currencies. İmrohoroğlu (1994) studied dollarization in Canada and finds that the elasticity of substitution between the U.S. dollar and the Canadian dollar is less than 1. The author explains that this implies little substitution between the two currencies, as the implicit demand for the U.S. dollar does not appear to be responsive to the relative currency price. In the studies by Friedman and Verbetsky (2001) and Selçuk (2006) the elasticity is found to be greater than 1. In the present study the elasticity of substitution between the U.S. dollar and local currencies significantly exceeds 1 several times. In the manner of Imrohoroglu (1994), this implies that holding foreign currency is highly responsive to the relative currency price in Central Asia. In the Kyrgyz Republic and Tajikistan,  $\rho$  was estimated between -0.31 and -0.94, which implies that the elasticity of substitution between domestic and foreign currencies  $s = \frac{1}{(1+\rho)}$  is between 1.45 and 17. The parameter estimates for  $\rho$  could only be estimated in the restricted range of values for Kazakhstan when habit formation in consumption is assumed. These results show that the two currencies are very good substitutes in the Central Asian states, and residents can easily switch from one currency to the other.

Finally, the RRA parameter  $\sigma$  could not be estimated precisely in most cases and its estimates were sometimes negative. The negative parameter of relative risk aversion implies non-convexity of preferences, which poses a difficulty in interpreting the model. The problem of negative values of RRA parameters and imprecision of its estimates has been studied in the economic literature.<sup>20</sup> This discussion is, however, beyond the subject of the present study.

The J-test statistic for testing the overidentifying restrictions of the model indicate that the data provide support for the considered model, or in other words that the overidentifying restrictions are valid.<sup>21</sup> The null hypothesis of validity of overidentifying restrictions could not be rejected in any of the considered cases. Hence, the instruments chosen proved to be valid.

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<sup>20</sup>Negative and sometimes statistically insignificant values of the estimated parameter of relative risk aversion (RRA) and intertemporal elasticity of substitution (IES) have been obtained and discussed to different extents in studies on consumption behavior through estimating the Euler equations by GMM. See, for example, Hansen and Singleton (1982), Hall (1988), Mao (1990), Holman (1998) and others. In his study, Pozzi (2002) proposed an explanation for the imprecision in estimating the RRA parameter and its estimates' negative values.

<sup>21</sup>Since the number of orthogonality conditions exceeds the number of parameters, the validity of overidentifying restrictions should be tested. The test suggested is a  $J_T$  test, where  $J_T$  statistics is a minimized value of the objective function times the number of observations. Under the null hypothesis the overidentifying restrictions are valid, and the  $J_T$ -statistics is asymptotically distributed as  $\chi^2$  with degrees of freedom equal to the number of overidentifying restrictions.

**Table 1. GMM Estimates (No Habit Formation)**

<i>a)Kazakhstan</i>				
	Refinance rate	Treasury bill	Deposit rate	FFR
$\beta$	0.92***(0.00)	0.96***(0.00)	0.97***(0.00)	0.98***(0.00)
$\alpha$	0.51***(0.01)	0.52***(0.01)	0.51***(0.01)	0.53***(0.01)
$\gamma$	0.18***(0.01)	0.11***(0.00)	0.08***(0.01)	0.06***(0.00)
$\sigma$	-0.16* (0.09)	-0.05 (0.06)	-0.13 (0.07)	0.04 (0.03)
$\rho$	-1.13***(0.03)	-1.14*** (0.05)	-1.18***(0.04)	-1.16***(0.07)
<i>J – statistics</i>	8.42 [0.59]	7.11 [0.71]	7.99 [0.63]	7.90 [0.64]
<i>No.obs</i>	105	105	105	105
<i>b)Kyrgyz Republic</i>				
	Repo rate	MMR	Deposit rate	FFR
$\beta$	0.95*** (0.00)	0.96***(0.01)	0.94***(0.00)	0.99***(0.00)
$\alpha$	0.55***(0.01)	0.56***(0.01)	0.56***(0.01)	0.62***(0.03)
$\gamma$	0.05*** (0.00)	0.06***(0.00)	0.06***(0.00)	0.02***(0.00)
$\sigma$	0.02 (0.03)	0.03 (0.08)	0.005 (0.03)	-0.04 (0.01)
$\rho$	-0.76***(0.08)	-0.72*** (0.08)	-0.67***(0.06)	-0.31*(0.17)
<i>J – statistics</i>	5.87 [0.75]	6.60 [0.68]	6.16 [0.72]	5.91 [0.75]
<i>No.obs</i>	94	90	95	95
<i>c)Tajikistan</i>				
	Refinance rate	Interbank rate	Deposit rate	FFR
$\beta$	0.89***(0.00)	0.89***(0.00)	0.92***(0.00)	0.98***(0.00)
$\alpha$	0.54***(0.00)	0.53***(0.00)	0.53***(0.01)	0.47***(0.01)
$\gamma$	0.08***(0.00)	0.07***(0.00)	0.06***(0.00)	0.01***(0.00)
$\sigma$	0.05 (0.04)	0.07** (0.03)	0.01 (0.03)	-0.02 (0.00)
$\rho$	-0.69*** (0.01)	-0.68*** (0.02)	-0.70*** (0.01)	-0.94***(0.03)
<i>J – statistics</i>	5.63 [0.86]	5.26 [0.87]	4.97 [0.89]	5.78 [0.83]
<i>No.obs</i>	71	61	71	71

Notes: a) Standard errors are in parentheses; P-values are in brackets; b)  $\beta$ —discount factor,  $\alpha$ — share of foreign money balances in producing money services;  $s = \frac{1}{(1+\rho)}$ —elasticity of currency substitution

Table C 1 in Appendix C presents the results for the case of Kazakhstan when the utility function assumed exhibits habit formation, or the parameter of habit persistence  $\delta \neq 0$ . In the case of Kazakhstan, this specification helped restore the meaningful values of the parameter  $\rho$ . For the other two countries, the introduction of habit persistence does not change the magnitude of the major parameters in most cases. In Kazakhstan, consumption exhibits a strong persistence with the values of  $\delta = 0.78$ . The values of  $\rho$  vary between -0.70 and -0.87, indicating that substitutability between national and foreign currencies is very high. The estimates of RRA coefficient  $\sigma$  are again negative. The share of dollars in providing money transactions is 0.5. This result confirms the previous specification results on effective foreign currency and high substitutability between domestic and foreign currencies in Kazakhstan. J-statistics values cannot reject the validity of the overidentifying restrictions, i.e. the model provides a good fit for the data.

In the case of the Kyrgyz Republic, the value of the parameter  $\alpha$  was estimated at a lower level than in the first model specification. A strong habit persistence was found in this country as well. The parameter measuring habit formation  $\delta = 0.7$ . The elasticity of substitution between the currencies remains high. In Tajikistan, the habit formation parameter values vary from negative to positive numbers depending on the interest rate chosen.

Empirical results in this section provide strong support for the presence of currency substitution in the economies of Central Asia, and highlight the role of foreign currency as a substitute for domestic money in economic transactions.

## 7 Implications for Seigniorage Revenue

One of the major concerns for policy makers related to currency substitution is its effect on the seigniorage revenue of central banks. Official dollarization, or full replacement of the domestic currency by some foreign currency (for example, the U.S. dollar), thus directly affects the ability of the government to earn revenue from issuing money. Partial, or unofficial dollarization, can affect seigniorage revenue as well. Some have argued that this effect may be of large.<sup>22</sup> In the context of transition and developing economies, then, the loss of seigniorage revenue is an important issue.<sup>23</sup>

In this study, the implications for seigniorage revenue are derived from analyzing a hypothetical steady state of the model. Steady states with different inflation rates ( $\pi$ ) and dollarization ( $\alpha$ ) are

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<sup>22</sup>Bufman and Leiderman (1992) studied currency substitution in Israel and showed that even small increases in dollarization can have significant effect on the seigniorage income of the monetary authorities. Friedman and Verbetsky (2001) examined seigniorage loss for the case of Russia.

<sup>23</sup>Fischer (1982), for example, calculates average seigniorage rates during the 1960s and 1970s for a cross-section of countries and finds that seigniorage accounts for more than 10% of total government revenue in many less developed countries, especially those with high inflation rates. Click (1998) reports average seigniorage as a share of government spending for a set of 90 countries. He finds that the seigniorage revenue share ranges from 5% in Honduras to 62% in Argentina. Lange and Sauer (2005) calculated the seigniorage for the period 1995 till 2000 for 15 Latin American countries, and found that seigniorage accounts for almost 12 % of government revenue in these economies even though inflation rates were reduced in the 1990s.



compared. In the steady state, consumption and real money balances' holdings grow at some constant rate  $\phi > 0$ . The population grows at the rate  $n = 0$ . The real return on the market portfolio,  $R$ , is invariant with respect to both time and inflation rate. Under these conditions, the steady state demand for domestic real money balances can be derived using the optimality conditions from equations 8 to 10. First, the expression for the ratio between foreign and domestic real money balances in terms of model parameters, inflation, and dollarization can be computed following the expression:

$$h = \left( \frac{(1 - \alpha) \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}}}{\alpha} \right)^{-\frac{1}{1+\rho}} \quad (18)$$

Further, the demand for domestic money balances in terms of model parameters is derived as follows:

$$m = \frac{\gamma}{1 - \gamma} \frac{R_b c}{(R_b - R_m) + (R_b - R_{m,f}) \left( \frac{\alpha}{1 - \alpha} \frac{R_b - R_m}{R_b - R_{m,f}} \right)^{1/(1+\rho)}}, \quad (19)$$

where the notations from Friedman and Verbetsky (2001) are used for simplification:  $R_b = (1 + R)/(1 + \pi)$ ,  $1 + R = (1 + \pi)(1 + \phi)^\sigma/\beta$ ,  $R_m = 1/(1 + \pi)$  and  $R_{m,f} = (1 + \epsilon)/(1 + \pi)$ .

In the case with habit formation in consumption there is a slight difference in the money demand equation:

$$m = \frac{\gamma}{1 - \gamma} \frac{(1 + \phi - \delta) R_b c}{(R_b - R_m) + (R_b - R_{m,f}) \left( \frac{\alpha}{1 - \alpha} \frac{R_b - R_m}{R_b - R_{m,f}} \right)^{1/(1+\rho)}}, \quad (20)$$

where  $\delta$  is a parameter of habit persistence.<sup>24</sup>

The money growth rule is assumed as follows:

$$M_t = (1 + \mu) M_{t-1}, \quad (21)$$

where  $\mu$  is the growth rate of the domestic money supply.

For simplicity, the government's budget constraint is assumed to be:

$$M_t = M_{t-1} - T_t, \quad (22)$$

The fiscal policy assumed implies that the government rebates seigniorage revenues to the public through lump-sum transfers  $T_t$ .

To compute the seigniorage, different approaches can be found in the literature. Here the monetary concept of seigniorage computation is used due to the fact that dollarization affects to an

<sup>24</sup>The results of the simulated seigniorage-to-GDP ratio for the case of habit formation are presented in Appendix C.

important extent the ability of the central bank to receive revenue from increasing the money supply. Furthermore, this approach is suitable for calculating seigniorage revenue in the framework of the present model's setup and is simple in computation.<sup>25</sup>

The monetary concept determines seigniorage as follows:

$$\frac{S_t}{Y_t} = \frac{M_t - M_{t-1}}{M_t} \frac{M_t}{P_t Y_t}, \quad (23)$$

where  $M_t$  is the monetary base,  $S_t$  is the seigniorage revenue and  $Y_t$  is GDP. Therefore, the seigniorage ratio to GDP is:

$$\frac{S_t}{Y_t} = \mu \frac{M_t}{P_t Y_t},$$

where  $\mu$  is the growth rate of money supply and

$$\frac{M_t}{P_t Y_t} = \frac{M_t/p_t N_t}{Y_t/N_t} = \frac{m}{y},$$

where  $m$  is the steady state per capita real money balances and  $y$  is the per capita GDP. The seigniorage revenue-to-GDP ratio can be computed as follows:

$$\frac{S}{Y} = \mu \frac{c}{y} \frac{m}{c}. \quad (24)$$

For the calculation of seigniorage loss the values of the parameters are calibrated on the basis of the estimation results in the previous section. The following values are assumed:  $\gamma = 0.07$  and  $\beta = 0.98$ . The parameter  $\alpha$  will be given values from 0.4 to 0.7, and  $\rho$  is assumed to be  $-0.7$ . The ratio of consumption over income is assumed to be 0.8.<sup>26</sup> The RRA parameter  $\sigma$  is assumed to be 0, since it was not estimated precisely and its estimated value was negative in some cases. In the

<sup>25</sup>Though monetary definition of seigniorage is the most widespread concept of seigniorage, other definitions can be found in the recent literature. Fiscal seigniorage refers to the yield on the counterparts of the monetary base after deduction of costs. Lange and Sauer (2005) distinguish an opportunity cost seigniorage that can be computed as follows:

$$S = i \frac{M}{P},$$

where  $i$  is a market interest rate,  $M$  is a monetary base and  $P$  is a price level.

Under rational inflation expectations, the Fischer equation implies that opportunity seigniorage can be calculated as follows:

$$S = (\pi + r) \frac{M}{P},$$

where  $\pi$  is the inflation rate, and  $r$  is a real rate of interest. This definition of seigniorage can be related to monetary seigniorage:

$$S = \frac{dM}{P} = \frac{dM}{M} \frac{M}{P} = \mu \frac{M}{P},$$

where  $\mu$  is the growth rate of base money. Assuming that velocity and the money multiplier are constant, the quantity theory of money implies that money growth equals the sum of the inflation rate ( $\pi$ ) and the real economic growth rate ( $g$ ):

$$S = (\pi + g) \frac{M}{P}.$$

Hochreiter and Rovelli (2002) add to these two concepts a concept of inflation tax seigniorage:

$$S = \pi \frac{M}{P}.$$

<sup>26</sup>This is an average ratio of consumption to GDP or total income across three countries and the period.

steady state, the growth rate of money  $\mu$  is equal to the steady state inflation rate  $\pi$ . The underlying assumption of Purchasing Power Parity (PPP) is used to consider two scenarios of the domestic currency depreciation. In the first scenario, I assume that the foreign inflation rate  $\pi^* = 0$  while the domestic inflation rate changes from 0.2 to 50 percent. In this scenario, the depreciation rate,  $\epsilon_t$  moves together with the domestic inflation rate and  $\epsilon = \pi$ . In the second scenario, the foreign inflation rate is constant but equals 5 percent. This scenario implies that the domestic currency depreciates at a slower rate than does the domestic inflation rate.

Table 2 presents the results of the seigniorage revenue simulation. The simulated values are presented as the ratio to GDP in percent. This ratio was calculated for different values of the domestic inflation rate  $\pi$  and for different values of the share of foreign money balances  $\alpha$ .

The results in Table 2 (a) show that the ratio of seigniorage-to-GDP ratio increases with the rate of inflation, but only to a certain level of the inflation rate. The seigniorage revenue reaches its peak when the inflation rate is 2 percent in the case when the dollarization level  $\alpha = 0.4$ . After that it gradually decreases. This result is similar to those obtained by Friedman and Verbetsky (2001) who found that the government achieves its highest seigniorage revenue at an inflation rate of 1-3 percent depending on the level of dollarization. More important is the relation between dollarization and seigniorage revenue. The results show that the latter is a decreasing function of dollarization. The higher  $\alpha$  is, the lower is the seigniorage revenue-to-GDP ratio.

In scenario 2, if the dollarization level is 0.4, the seigniorage revenue increases until the inflation rate reaches 5 percent. In this scenario, the ratio of government revenue from seigniorage to GDP is higher than in the first scenario for each level of dollarization and each inflation rate. The agents prefer to hold more domestic money when there is inflation abroad. Moreover, they hold more domestic currency when the domestic inflation rate is lower than the foreign inflation rate. From equation 18, it can be seen that if consumption is constant, the demand for domestic money balances will be higher when there is inflation in the foreign country. Results from the second scenario thus support the previous finding that seigniorage revenue is a decreasing function of the level of dollarization. In this scenario, however, the seigniorage-to-GDP ratio is less sensitive to the increases in dollarization.

**Table 2. Simulated Seigniorage/GDP Ratios (%)**a) *Scenario 1:  $\epsilon = \pi(\pi^* = 0\%)$* 

$\pi, \%$	$\alpha$			
	0.4	0.5	0.6	0.7
0.2	0.48	0.28	0.11	0.03
0.5	0.98	0.53	0.19	0.05
1.0	1.42	0.67	0.22	0.05
2.0	1.58	0.61	0.18	0.04
3.0	1.41	0.48	0.14	0.03
4.0	1.19	0.38	0.10	0.02
5.0	0.98	0.30	0.08	0.02
6.0	0.82	0.24	0.06	0.014
8.0	0.59	0.16	0.04	0.010
10	0.44	0.12	0.03	0.007
20	0.16	0.04	0.01	0.003
30	0.09	0.02	0.006	0.001
50	0.05	0.01	0.003	0.0008

b) *Scenario 2:  $\pi^* = 5\%$* 

$\pi, \%$	$\alpha$			
	0.4	0.5	0.6	0.7
0.2	0.62	0.59	0.49	0.28
0.5	1.36	1.27	1.01	0.52
1.0	2.25	2.03	1.48	0.66
2.0	3.30	2.75	1.67	0.60
3.0	3.83	2.91	1.51	0.48
4.0	4.07	2.81	1.28	0.37
5.0	4.15	2.61	1.07	0.29
6.0	4.11	3.37	0.90	0.24
8.0	3.86	1.91	0.65	0.16
10	3.51	1.53	0.48	0.12
20	2.03	0.65	0.18	0.04
30	1.31	0.38	0.10	0.02
50	0.76	0.21	0.06	0.01

Notes:  $\beta = 0.98, \gamma = 0.08, \rho = -0.7, \sigma = 0, c/y = 0.8$

Introduction of a habit formation in consumption changes the numerical results of the seigniorage computation, since the demand for real domestic money balances is represented by equation 21. If the estimated value of  $\delta$  is positive and  $\phi > 0$  but  $\phi < \delta$ , then in the habit formation economy the seigniorage-to-GDP ratio will be lower for every level of dollarization and for each inflation rate than in the economy with no habit formation. Results of the simulated ratio of seigniorage revenue to GDP for the utility function with habit formation are presented in Appendix C.

The findings in this section provide support for the hypothesis that decreasing seigniorage revenue is due to increasing dollarization.

For further analysis, actual seigniorage-to-GDP ratios were calculated using data from the central banks of the countries examined in the study. The actual seigniorage-to-GDP ratio was calculated using data on the monetary base following the monetary seigniorage concept. Results are presented in Table 3. Both simulated and actual ratio of seigniorage to GDP decrease when the dollarization level  $\alpha$  increases. There is, however, no significant variation in actual annual seigniorage revenue over time.

**Table 3. Actual and Simulated Seigniorage/GDP Ratios (in %)**

*a) Kazakhstan*

Period	Inflation rate (in%)	Dollarization level, $\alpha$	Simulated seigniorage/GDP ratio	Actual seigniorage/GDP ratio
2000	13	0.51	0.07	0.05
2001	8	0.64	0.02	0.05
2002	6	0.60	0.06	0.05
2003	6	0.47	0.35	0.06
2004	7	0.43	0.48	0.08
2005	8	0.42	0.46	0.06
2006	9	0.35	0.94	0.096
2007	11	0.32	1.07	0.07

*b) Kyrgyz Republic*

Period	Inflation rate (in%)	Dollarization level, $\alpha$	Simulated seigniorage/GDP ratio	Actual seigniorage/GDP ratio
2000	19	0.58	0.02	0.014
2001	7	0.59	0.06	0.005
2002	2	0.62	0.14	0.002
2003	3	0.64	0.08	0.003
2004	4	0.70	0.02	0.005
2005	4	0.73	0.01	0.005
2006	6	0.66	0.03	0.009
2007	10	0.52	0.09	0.0016

*c) Tajikistan*

Period	Inflation rate (in%)	Dollarization level, $\alpha$	Simulated seigniorage/GDP ratio	Actual seigniorage/GDP ratio
2002	12.3	0.49	0.10	0.01
2003	16.3	0.49	0.07	0.01
2004	7.1	0.60	0.05	0.003
2005	7.1	0.56	0.09	0.003
2006	10	0.70	0.007	0.004
2007	13.1	0.75	0.002	0.015

This implies that actual seigniorage revenues were quite stable over the period considered. Time aggregation in calculating the seigniorage revenue might generate more variability in the seigniorage revenues of the local governments. Nevertheless, the results of the present section support the negative relationship between the revenue that central banks derive from money issuance and the amount of foreign money holdings by the residents. A simple correlation analysis shows that the correlation between dollarization level  $\alpha$  and simulated and actual seigniorage-to-GDP ratios is around -0.82 and -0.71 respectively.

## 8 Economic Welfare and Dollarization

By affecting the seigniorage revenues of the government and thus the amount of lump-sum transfers paid to the public, currency substitution impacts the welfare of households. It is important to note that dollarization itself stems from several factors that have an effect on economic welfare, i.e. high rates of inflation, rapid depreciation of domestic money, etc. These factors stimulate dollarization because economic agents aim to hedge the value of their financial assets and money holdings by switching to foreign currency. Holding foreign money thus becomes a way for households to preserve their wealth.

In this section, the potential implications of dollarization and currency substitution for welfare are discussed. These implications involve the different channels through which holding of foreign money balances by households can affect their welfare. This analysis, though cumbersome, yields some important insights on the economic role of dollarization in transition economies.

The baseline assumption is that the welfare of a household changes if dollarization  $\alpha$  increases, i.e.  $\alpha_0 < \alpha_1$  and  $u(\alpha_0) > u(\alpha_1)$ . To calculate the welfare costs of dollarization in a steady state with a given rate of inflation, one needs to compute the percentage decrease in consumption per capita that would generate the same welfare change as that from moving from the original level of dollarization  $\alpha_0$  to a higher level of dollarization  $\alpha_1$ . Or it is necessary to find such  $\Delta c$  that would return the household to its original level of utility:  $u(c, \alpha_0) = u(c + \Delta c, \alpha_1)$ . For this purpose, the utility function in equation 7 is rewritten in the following way:

$$u(c_t, x_t) = \frac{\left( c_t^{1-\gamma} m^\gamma (1 - \alpha + \alpha h^{-\rho})^{-\frac{\gamma}{\rho}} \right)^{1-\sigma}}{1 - \sigma} - 1, \quad (25)$$

where  $h = \frac{m^*}{m}$ . Plugging in the expression for  $h$  from equation 17, and equating utilities for different levels of dollarization through including consumption compensation,  $\Delta c$ , the following equality is obtained:

$$\begin{aligned} & c^{(1-\gamma)} m_1^\gamma \left( 1 - \alpha_1 + \alpha_1 \left( \frac{1 - \alpha_1}{\alpha_1} \left( \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma}{\rho}} - 1 = \\ & = (c + \Delta c)^{(1-\gamma)} m_2^\gamma \left( 1 - \alpha_2 + \alpha_2 \left( \frac{1 - \alpha_2}{\alpha_2} \left( \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma}{\rho}} - 1, \Leftrightarrow \end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow c \left( \frac{m_1}{c} \right)^\gamma \left( 1 - \alpha_1 + \alpha_1 \left( \frac{1 - \alpha_1}{\alpha_1} \left( \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma}{\rho}} = \quad (26) \\
&= (c + \Delta c) \left( \frac{m_2}{c + \Delta c} \right)^\gamma \left( 1 - \alpha_2 + \alpha_2 \left( \frac{1 - \alpha_2}{\alpha_2} \left( \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma}{\rho}}
\end{aligned}$$

Defining the expression in parentheses as  $f(\alpha)$ , a simpler representation of the previous equality is as follows:

$$\begin{aligned}
c \left( \frac{m_1}{c} \right)^\gamma f(\alpha_1) &= (c + \Delta c) \left( \frac{m_2}{c + \Delta c} \right)^\gamma f(\alpha_2), \Leftrightarrow \quad (27) \\
\Leftrightarrow \left( \frac{c + \Delta c}{c} \right) &= \frac{\left( \frac{m_1}{c} \right)^\gamma f(\alpha_1)}{\left( \frac{m_2}{c + \Delta c} \right)^\gamma f(\alpha_2)}, \Leftrightarrow \\
\Leftrightarrow \frac{\Delta c}{c} &= \frac{\left( \frac{m_1}{c} \right)^\gamma f(\alpha_1)}{\left( \frac{m_2}{c + \Delta c} \right)^\gamma f(\alpha_2)} - 1
\end{aligned}$$

This consumption compensation can be expressed as a ratio of GDP:

$$\frac{\Delta c}{y} = \frac{c}{y} \left[ \frac{\left( \frac{m_1}{c} \right)^\gamma f(\alpha_1)}{\left( \frac{m_2}{c + \Delta c} \right)^\gamma f(\alpha_2)} - 1 \right]. \quad (28)$$

To compute consumption compensation, the money demand expression from equation 18 is plugged into equation 26. Derived consumption compensation in equation 27 consists of two parts: a decrease in the lump-sum transfer from the government due to its loss of seigniorage revenue, and direct changes in households' utility due to holding foreign money.

Using the same parameter values as in the calculation of seigniorage loss in the previous section, the consumption compensation is calculated using equation 27. The welfare loss is computed for a change in dollarization  $\alpha$  from 0.5 to 0.6 for different rates of inflation. Two scenarios of exchange rate determination from the previous section are analyzed. Table 4 presents the results of the simulated changes in welfare represented as percentage in GDP. Negative values of the consumption compensation imply welfare gains, while positive values imply welfare loss.



**Table 4. Consumption Compensation for Increasing Dollarization**

$\pi$ , %	$\Delta C/GDP$ , %	
	Scenario 1: $\epsilon = \pi$	Scenario 2: $\pi^* = 5\%$
0.2	-0.56	1.56
0.5	-0.77	1.42
1	-1.01	1.17
2	-1.28	0.68
3	-1.42	0.24
4	-1.49	-0.11
5	-1.53	-0.40
6	-1.56	-0.62
8	-1.59	-0.93
10	-1.61	-1.12
20	-1.639	-1.47
30	-1.644	-1.56
50	-1.647	-1.61

Notes:  $\alpha$  changes from 0.5 to 0.6,  $\beta = 0.98$ ,  $\rho = -0.7$ ,  $\gamma = 0.08$ ,  $c/y = 0.8$

The results in Table 4 reveal that dollarization in fact brings gains in welfare that can be as large as 1.65 percent of GDP if the domestic inflation rate reaches 50 percent. The welfare gain is an increasing function of inflation. This finding can be explained by the fact that an increase in foreign money holdings hedges households from incurring loss due to depreciating domestic money. The higher the inflation rate, the higher the gain from an increase in foreign currency holdings. This also implies that the loss in seigniorage revenue due to increasing dollarization is exceeded by the gains from holding foreign currency. In the second scenario, holding foreign money brings welfare loss if the domestic inflation rate is lower than the inflation rate in the foreign economy. In this scenario, holding dollars is not optimal since the domestic currency is stronger when foreign inflation exceeds inflation at home. Thus, the welfare loss occurs due to uncertainty about the foreign inflation rate and about the exchange rate between local and foreign currencies. The seigniorage loss is greater than the gain in household consumption as a result of switching to foreign currency. Hence, dollarization in an inflationary environment with depreciating local currency vis-à-vis foreign currency becomes welfare generating. Currency substitution is thus a transitory phenomenon that might result in negative as well as positive changes in welfare. The welfare cost depends on the ability of resident households to diversify their money holdings in such a way as to avoid the risk of sudden depreciation of either currency.

## 9 Conclusion

In the present study currency substitution in three transition countries of Central Asia was examined. Findings show that foreign and domestic currencies are good substitutes in all three economies - Kazakhstan, the Kyrgyz Republic and Tajikistan. The elasticity of substitution between the two currencies is more than unity in all cases. The share of foreign currency in providing money services exceeds 0.5 for all three economies. Currency substitution and dollarization are shown to be of significant magnitude and importance in these transition countries. The study was conducted using a simple dynamic model of money-in-the-utility function with two currencies, where holding money balances denominated in different currencies serves as a hedge against domestic instability and inflation. The steady state implications for seigniorage revenues of the government and household welfare were analyzed. Seigniorage revenue was found to be a decreasing function of dollarization. An increase in dollarization index from 0.4 to 0.5, decreases seigniorage revenue to GDP ratio by almost half. Seigniorage revenues will however depend on the inflation rate abroad. The higher the inflation rate abroad, the higher is the seigniorage ratio in the domestic economy due to increasing local demand for domestic real money balances. Increasing dollarization still results in loss of seigniorage revenue for each dollarization level and inflation rate.

The welfare analysis comprises the loss of seigniorage and a change in welfare due to switching to a foreign currency. The findings of the welfare analysis are sensitive to the scenario of domestic currency depreciation. If foreign inflation is zero, then switching to holding dollars is a welfare generating decision. Though the government loses its revenues from money issuance, the overall effect of currency substitution can be positive. In the second scenario, where the foreign inflation rate was fixed at 5 percent, holding dollars decreases households' wealth if inflation in the home country is lower than inflation abroad. Residents choose to hold foreign currency which in fact has less purchasing power and depreciates at a higher rate than does the domestic currency. Once the domestic inflation rate outpaces foreign inflation, switching to dollars starts bringing gains in welfare. Dollarization thus affects household wealth from two sides: decreasing lump-sum transfers from the government and hedging motives against domestic inflation.

Currency substitution and dollarization thus constitute transitory phenomena that do not necessarily bring welfare loss. Governments willing to dedollarize local economies should be concerned with the stability of local currencies rather than with restricting foreign money holdings.

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## Appendix A Data Description

**Table A 1. Data Sources**

Series	Time span	Source
<i>a) Kazakhstan</i>		
Consumer Price Index	2000:1 - 2008:12	IMF
Deposits denominated in domestic and foreign currency (in mln tenge)	2000:1 - 2008:12	NBK
Industrial Production (in mln. tenge)	2000:1 - 2008:12	Statistical Agency of Kazakhstan
Refinancing Rate of the National Bank (in percent)	2000:1 - 2008:12	IMF
Treasury bill rate (in percent)	2000:1 - 2008:12	IMF
Average deposit rate (in percent)	2000:1 - 2008:12	NBK
Nominal exchange rate (tenge to US dollar) (in tenge)	2000:1 - 2008:12	IMF
<i>b) Kyrgyz Republic</i>		
Consumer Price Index	2000:1 - 2008:3	IMF
Deposits denominated in domestic and foreign currency (in mln som)	2000:1 - 2008:3	NBKR
Industrial Production (in mln. som)	2000:1 - 2008:3	Statistical Agency of Kyrgyz Republic
Rate on repo operations of the NBKR	2000:1 - 2008:3	NBKR
Money market rate	2000:1 - 2008:3	IMF
Deposit rate	2000:1 - 2008:3	IMF
Nominal exchange rate (som to US dollar) (in som)	2000:1 - 2008:3	IMF

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*c) Tajikistan*

Consumer Price Index	2002:1 – 2008:2	IMF
Deposits denominated in domestic and foreign currency (in mln somoni)	2002:1 – 2008:2	NBT
Industrial Production (in mln. somoni)	2002:1 – 2008:2	NBT
Wages (in somoni)	2002:1 – 2008:2	NBT
Official Rate of the NBT (in percent)	2002:1 – 2008:2	NBT
Interbank Rate (in percent)	2002:1 – 2007:3	NBT
Deposit Rate (in percent)	2002:1 – 2008:2	IMF
Nominal exchange rate (somon to US dollar) (in somoni)	2002:1 – 2008:2	IMF

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Notes: NBK -National Bank of Kazakhstan, NBKR - National Bank of the Kyrgyz Republic,  
NBT - National Bank of Tajikistan

## Appendix B Inflation and Dollarization

**Table B 1. Capital Inflows in Central Asia**

	2000	2001	2002	2003	2004	2005	2006	2007	2008
<b>Remittances* (mln. USD)</b>									
Kazakhstan	122	171	205	147	166	178	187	223	192
Kyrgyz Republic	9	11	37	78	189	322	481	715	1232
Tajikistan	...	...	79	146	252	467	1019	1691	2544
<b>FDI** (mln. USD)</b>									
Kazakhstan	1278	2861	2164	2213	5436	2123	6630	6900	10732
Kyrgyz Republic	-7	-1	5	46	132	43	182	208	265
Tajikistan	24	10	36	32	272	55	66	160	190

Source: \* World Bank Migration and Remittances Factbook, 2008.

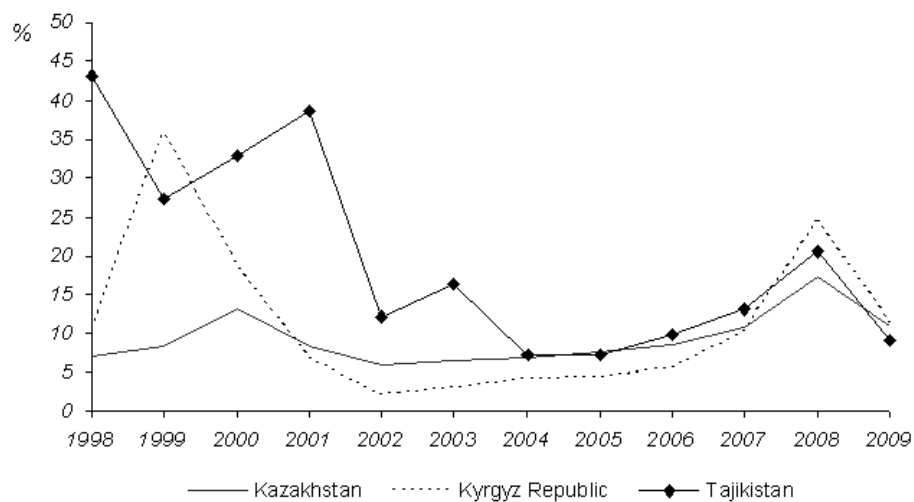
Data retrieved from the website of the World Bank.

\*\* EBRD Transition Report, 2008

Notes: Data for FDI in 2008 are estimates

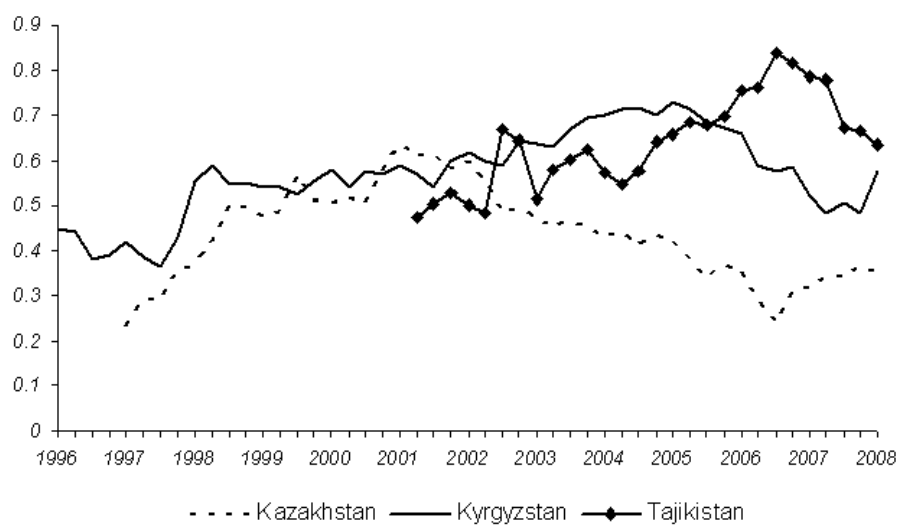


**Figure B 1. Inflation in Central Asia**



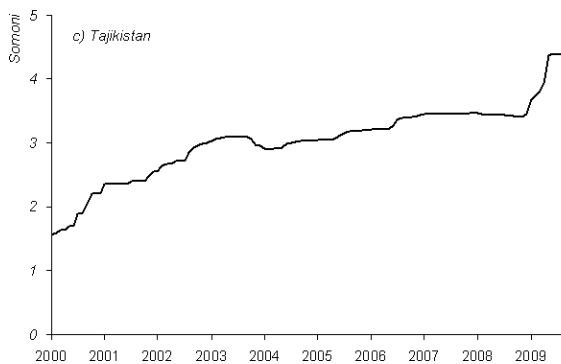
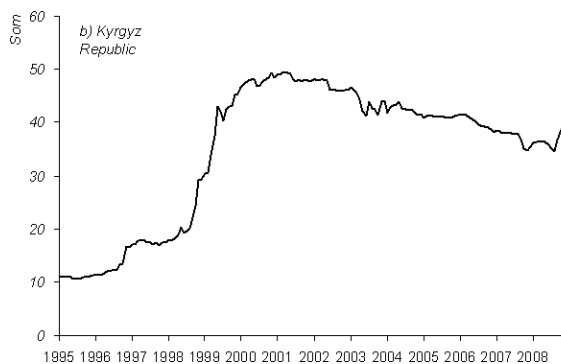
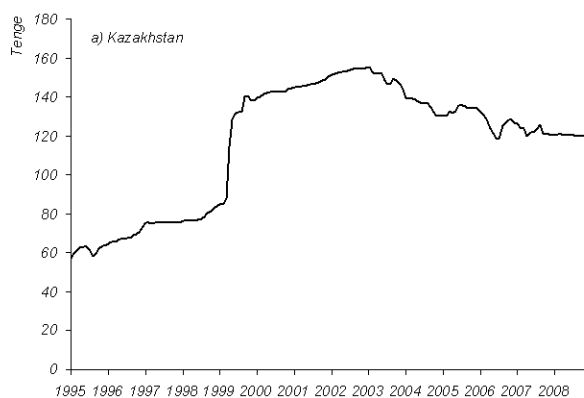
Note: Figures for 2009 are estimates Source: EBRD Transition Report, 2008

**Figure B 2. Dollarization in Central Asia**



Note: Dollarization Index is computed as a ratio of foreign currency denominated deposits to total deposits

**Figure B 3. Exchange Rates: Units of Local Currency to U.S. Dollar**



Source: IMF International Financial Statistics

## Appendix C Habit Formation in Consumption

Derived estimation equation for the case with habit formation are as follows:

$$\begin{aligned}
 d_{1,t+1} = & \beta \left\{ C_t^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+1}-\delta}{C_t-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+1}^{\gamma(1-\sigma)} \right. \\
 & \left. \left( \frac{1-\alpha+\alpha H_{t+1}^{-\rho}}{1-\alpha+\alpha H_t^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \left( \frac{1+r_t}{1+\pi_{t+1}} + \delta \right) \right\} - \\
 & -\delta\beta^2 \left\{ C_t^{\sigma(\gamma-1)-\gamma} C_{t+1}^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+2}-\delta}{C_t-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+1}^{\gamma(1-\sigma)} \right\} \times \\
 & \times \left\{ M_{t+2}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+2}^{-\rho}}{1-\alpha+\alpha H_t^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \left( \frac{1+r_t}{1+\pi_{t+1}} \right) \right\} - 1,
 \end{aligned} \tag{eq. B1}$$

$$\text{where } C_t = \frac{c_t}{c_{t-1}}, M_t = \frac{m_t}{m_{t-1}}, H_t = \frac{m_t^*}{m_t}.$$

$$\begin{aligned}
 d_{2,t+1} = & \left( \frac{\frac{\gamma}{1-\gamma}(1-\alpha)\frac{c_{t-1}}{m_t}(C_t-\delta)}{(1-\alpha+\alpha H_t^{-\rho})} \right) + \\
 & +\beta \left\{ C_t^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+1}-\delta}{C_t-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+1}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+1}^{-\rho}}{1-\alpha+\alpha H_t^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \times \right. \\
 & \times \left( 1 - \delta\beta C_{t+1}^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+2}-\delta}{C_{t+1}-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+2}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+2}^{-\rho}}{1-\alpha+\alpha H_{t+1}^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \right) \frac{1}{1+\pi_{t+1}} - \\
 & \left. - \left( 1 - \delta\beta C_t^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+1}-\delta}{C_t-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+1}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+1}^{-\rho}}{1-\alpha+\alpha H_t^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \right) \right\},
 \end{aligned} \tag{eq. B2}$$

$$\begin{aligned}
 d_{3,t+1} = & (1-\alpha)H_t^{1+\rho} - \alpha - \\
 & -\beta \left\{ C_t^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+1}-\delta}{C_t-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+1}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+1}^{-\rho}}{1-\alpha+\alpha H_t^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \times \right. \\
 & \times \left( 1 - \delta\beta C_{t+1}^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+2}-\delta}{C_{t+1}-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+2}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+2}^{-\rho}}{1-\alpha+\alpha H_{t+1}^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \right) \frac{1}{1+\pi_{t+1}} \times \\
 & \times \left( 1 - \delta\beta C_t^{\sigma(\gamma-1)-\gamma} \left( \frac{C_{t+1}-\delta}{C_t-\delta} \right)^{\sigma(\gamma-1)-\gamma} M_{t+1}^{\gamma(1-\sigma)} \left( \frac{1-\alpha+\alpha H_{t+1}^{-\rho}}{1-\alpha+\alpha H_t^{-\rho}} \right)^{\frac{\gamma}{\rho}(\sigma-1)} \right) \times \\
 & \left. \times \left( (1-\alpha)H_t^{1+\rho}(1+\epsilon_{t+1}) - \alpha \right) \right\}.
 \end{aligned} \tag{eq. B3}$$

**Table C 1. GMM Estimates (Utility with Habit Formation in Consumption) for Kazakhstan**

	Refinance rate	Treasury Bill	Deposit Rate	FFR
$\beta$	0.92***(0.00)	0.95***(0.00)	0.96***(0.00)	0.98***(0.00)
$\alpha$	0.49***(0.00)	0.50***(0.00)	0.50***(0.00)	0.50***(0.00)
$\gamma$	0.10***(0.00)	0.12***(0.01)	0.10***(0.01)	0.07***(0.01)
$\sigma$	-0.12***(0.01)	-0.13***(0.01)	-0.11***(0.01)	-0.08***(0.01)
$\delta$	0.79***(0.00)	0.78***(0.01)	0.78***(0.03)	0.78***(0.01)
$\rho$	-0.70***(0.06)	-0.87***(0.02)	-0.85***(0.02)	-0.87***(0.02)
<i>J – statistics</i>	8.49 [0.75]	7.28 [0.84]	7.28 [0.84]	7.85 [0.80]
<i>No.obs</i>	103	103	103	103

Notes: a) Standard errors are in parentheses; P-values are in brackets; b)  $\beta$ —discount factor,  $\alpha$ — share of foreign money balances in producing money services;  $s = \frac{1}{1+\rho}$  —elasticity of currency substitution

**Table C 2. Simulated Seigniorage/GDP Ratios (%)**Scenario 1:  $\epsilon = \pi(\pi^* = 0\%)$ 

$\pi, \%$	$\alpha$		
	0.4	0.5	0.6
0.2	0.14	0.08	0.03
0.5	0.29	0.16	0.06
1.0	0.43	0.20	0.07
2.0	0.47	0.18	0.05
3.0	0.42	0.14	0.04
4.0	0.36	0.11	0.03
5.0	0.30	0.09	0.02
6.0	0.25	0.07	0.019
8.0	0.18	0.05	0.012
10	0.13	0.04	0.009
20	0.05	0.01	0.003
30	0.03	0.007	0.002
50	0.01	0.004	0.001

b) Scenario 2:  $\pi^* = 5\%$ 

$\pi, \%$	$\alpha$		
	0.4	0.5	0.6
0.2	0.19	0.18	0.15
0.5	0.41	0.38	0.30
1.0	0.68	0.61	0.44
2.0	0.99	0.83	0.50
3.0	1.15	0.87	0.45
4.0	1.22	0.84	0.39
5.0	1.24	0.78	0.32
6.0	1.23	0.71	0.27
8.0	1.16	0.57	0.19
10	1.05	0.46	0.15
20	0.61	0.20	0.05
30	0.39	0.12	0.03
50	0.23	0.06	0.02

## Appendix D Derivation of the Estimated Equations

The specification of the utility function:

$$u(c_t, s_t) = \frac{(c_t^{1-\gamma} x_t^\gamma)^{1-\sigma} - 1}{1-\sigma} \quad (\text{eq. A1})$$

where

$$x_t = [(1-\alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}} \quad (\text{eq. A2})$$

We maximize

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, x_t)$$

s.t. the budget constraint

$$c_t + m_t + m_t^* + b_t = y_t + \tau_t + \frac{m_{t-1}}{1+\pi_t} + \frac{m_{t-1}^*(1+\epsilon_t)}{1+\pi_t} + \frac{b_{t-1}(1+r_{t-1})}{1+\pi_t}, \quad (\text{eq.A3})$$

F.O.C.:

[ $c_t$ ]:

$$(1-\gamma)(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma - \lambda_t = 0, \quad (\text{eq. A4})$$

[ $m_t$ ]:

$$\begin{aligned} & \gamma(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{1-\gamma} x_t^{\gamma-1} \times [(1-\alpha)m_t^{1-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times \\ & \times (1-\alpha)m_t^{-\rho-1} + \beta E_t \lambda_{t+1} \frac{1}{1+\pi_{t+1}} - \lambda_t = 0, \end{aligned} \quad (\text{eq. A5})$$

[ $m_t^*$ ]:

$$\begin{aligned} & \gamma(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{1-\gamma} x_t^{\gamma-1} \times [(1-\alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times \\ & \times \alpha m_t^{*- \rho-1} + \beta E_t \lambda_{t+1} \frac{1+\epsilon_{t+1}}{1+\pi_{t+1}} - \lambda_t = 0, \end{aligned} \quad (\text{eq. A6})$$

[ $b_t$ ]:

$$\beta E_t \lambda_{t+1} \frac{1+r_t}{1+\pi_{t+1}} - \lambda_t = 0, \quad (\text{eq. A7})$$

Using eq. A4 and eq. A7, we get:

$$(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma = \beta E_t \left\{ (c_{t+1}^{1-\gamma} x_{t+1}^\gamma)^{-\sigma} c_{t+1}^{-\gamma} x_{t+1}^\gamma \frac{1+r_t}{1+\pi_{t+1}} \right\}, \quad \Leftrightarrow$$

$$\begin{aligned}
&\Leftrightarrow \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma(1-\gamma)} \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} \left( \frac{x_{t+1}}{x_t} \right)^{-\sigma\gamma} \left( \frac{x_{t+1}}{x_t} \right)^\gamma \frac{1+r_t}{1+\pi_{t+1}} \right\} = 1, &\Leftrightarrow \\
&\Leftrightarrow \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{x_{t+1}}{x_t} \right)^{\gamma(1-\sigma)} \frac{1+r_t}{1+\pi_{t+1}} = 1 \right\}, &\text{(eq. A8)}
\end{aligned}$$

We plug definition of  $x$  in eq. A2 into eq. A8, and we obtain the following expression:

$$\beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{[(1-\alpha)m_{t+1}^{-\rho} + \alpha m_{t+1}^{*- \rho}]^{-\frac{1}{\rho}}}{[(1-\alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}}} \right)^{\gamma(1-\sigma)} \frac{1+r_t}{1+\pi_{t+1}} \right\} - 1 = 0, \quad \Leftrightarrow$$

Take out  $m_t$  and  $m_{t+1}$  from the parentheses to get:

$$\Leftrightarrow \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{[1-\alpha + \alpha \left( \frac{m_{t+1}^*}{m_{t+1}} \right)^{-\rho}]^{-\frac{1}{\rho}} \times m_{t+1}}{[1-\alpha + \alpha \left( \frac{m_t^*}{m_t} \right)^{-\rho}]^{-\frac{1}{\rho}} \times m_t} \right)^{\gamma(1-\sigma)} \frac{1+r_t}{1+\pi_{t+1}} \right\} - 1 = 0, \quad \Leftrightarrow$$

Define  $h = \frac{m^*}{m}$  and we have the first equations to be estimated:

$$\Leftrightarrow \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \left( \frac{1-\alpha + \alpha h_{t+1}^{-\rho}}{1-\alpha + \alpha h_t^{-\rho}} \right)^{-\frac{\gamma(1-\sigma)}{\rho}} \frac{1+r_t}{1+\pi_{t+1}} \right\} - 1 = 0. \quad \text{(eq. A9)}$$

Further, divide eq. A5 by the definition of  $\lambda$  from equation A4.:

$$\begin{aligned}
&\frac{(c_t^{1-\gamma} x_t^\gamma)^{-\sigma\gamma} c_t^{1-\gamma} x_t^{\gamma-1} (1-\alpha)[(1-\alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times m_t^{-\rho-1}}{(1-\gamma)(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma} + \\
&+ \beta E_t \left\{ \frac{(1-\gamma)(c_{t+1}^{1-\gamma} x_{t+1}^\gamma)^{-\sigma} c_{t+1}^{-\gamma} x_{t+1}^\gamma}{(1-\gamma)(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma} \frac{1}{(1+\pi_{t+1})} \right\} - 1 = 0, \quad \Leftrightarrow \\
&\Leftrightarrow \frac{\gamma}{(1-\gamma)} (1-\alpha) c_t x_t^{-1} [(1-\alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times m_t^{-\rho-1} + \\
&+ \beta E_t \left\{ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{x_{t+1}}{x_t} \right)^{\gamma(1-\sigma)} \frac{1}{(1+\pi_{t+1})} \right\} - 1 = 0 \quad \Leftrightarrow
\end{aligned}$$

Plug the definition of  $x$  into the equation, and take  $m$  out of the parentheses:

$$\Leftrightarrow \gamma(1-\alpha)c_t \frac{[1-\alpha+\alpha(\frac{m_t^*}{m_t})^{-\rho}]^{-\frac{1}{\rho}-1} \times m_t^{1+\rho} m_t^{-\rho-1}}{[1-\alpha+\alpha(\frac{m_t^*}{m_t})^{-\rho}]^{-\frac{1}{\rho}} \times m_t} -$$

$$-(1-\gamma) \left( 1 - \beta E_t \left\{ \begin{array}{l} \left(\frac{c_{t+1}}{c_t}\right)^{\sigma(\gamma-1)-\gamma} \left(\frac{m_{t+1}}{m_t}\right)^{\gamma(1-\sigma)} \times \\ \times \left(\frac{1-\alpha+\alpha(\frac{m_{t+1}^*}{m_{t+1}})^{-\rho}}{1-\alpha+\alpha(\frac{m_t^*}{m_t})^{-\rho}}\right)^{\frac{\gamma(\sigma-1)}{\rho}} \times \frac{1}{(1+\pi_{t+1})} \end{array} \right\} \right) = 0, \quad \Leftrightarrow$$

As  $h = \frac{m^*}{m}$ , we obtain the next equation to be estimated:

$$\gamma(1-\alpha) \frac{c_t}{m_t} \frac{1}{(1-\alpha+\alpha h_t^{-\rho})} = (1-\gamma) \times$$

$$\times \left( 1 - \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{\sigma(\gamma-1)-\gamma} \left(\frac{m_{t+1}}{m_t}\right)^{\gamma(1-\sigma)} \left(\frac{1-\alpha+\alpha h_{t+1}^{-\rho}}{1-\alpha+\alpha h_t^{-\rho}}\right)^{\frac{\gamma(\sigma-1)}{\rho}} \times \frac{1}{(1+\pi_{t+1})} \right\} \right)$$

(eq. A10)

Now, to derive the third estimation equation, equation A6 has to be divided by  $\lambda$ :

$$\frac{(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} \gamma c_t^{1-\gamma} x_t^{\gamma-1} [(1-\alpha)m_t^{-\rho} + \alpha m_t^{*\rho}]^{-\frac{1}{\rho}-1} \times \alpha m_t^{*\rho-1}}{(1-\gamma)(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma} +$$

$$+\beta E_t \left\{ \frac{(1-\gamma)(c_{t+1}^{1-\gamma} x_{t+1}^\gamma)^{-\sigma} c_{t+1}^{-\gamma} x_{t+1}^\gamma (1+\epsilon_{t+1})}{(1-\gamma)(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma (1+\pi_{t+1})} \right\} - 1 = 0, \quad \Leftrightarrow$$

Following the same manipulation as in equation A8, we obtain the following equation:

$$\Leftrightarrow \frac{\gamma}{(1-\gamma)} \alpha c_t x_t^{-1} [1-\alpha+\alpha h_t^{-\rho}]^{-\frac{1}{\rho}-1} \times m_t^{1+\rho} m_t^{*\rho-1} +$$

$$+\beta E_t \left\{ \left(\frac{c_{t+1}}{c_t}\right)^{\sigma(\gamma-1)-\gamma} \left(\frac{x_{t+1}}{x_t}\right)^{\gamma(1-\sigma)} \frac{(1+\epsilon_{t+1})}{(1+\pi_{t+1})} \right\} - 1 = 0; \quad \Leftrightarrow$$



$$\begin{aligned}
& \alpha \gamma \frac{c_t}{m_t^*} \frac{1}{((1-\alpha)h_t^{*\rho} + \alpha)} = \\
= (1-\gamma) & \left( 1 - \beta E_t \left\{ \begin{aligned} & \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \\ & \times \left( \frac{1-\alpha+\alpha h_{t+1}^{-\rho}}{1-\alpha+\alpha h_t^{-\rho}} \right)^{\gamma(\sigma-1)/\rho} \times \frac{(1+\epsilon_{t+1})}{(1+\pi_{t+1})} \end{aligned} \right\} \right) \quad (\text{eq. A11})
\end{aligned}$$

Dividing eq. A11 by eq. A10, we first consider the LHS. We manipulate the expression in the denominator taking  $\left(\frac{m_t^*}{m_t}\right)^{-\rho}$  out of the parentheses:

$$\begin{aligned}
\frac{(1-\alpha) m_t^*}{\alpha m_t} \frac{\left( (1-\alpha) \left( \frac{m_t^*}{m_t} \right)^\rho + \alpha \right)}{\left( 1 - \alpha + \alpha \left( \frac{m_t^*}{m_t} \right)^{-\rho} \right)} &= \frac{(1-\alpha) m_t^*}{\alpha m_t} \frac{\left( (1-\alpha) \left( \frac{m_t^*}{m_t} \right)^\rho + \alpha \right)}{\left( \left( \frac{m_t^*}{m_t} \right)^{-\rho} \left[ (1-\alpha) \left( \frac{m_t^*}{m_t} \right)^\rho + \alpha \right] \right)} = \\
&= \frac{(1-\alpha)}{\alpha} \left( \frac{m_t^*}{m_t} \right)^{1+\rho}, \quad (\text{eq. A12})
\end{aligned}$$

In the RHS, denote:

$$Z = \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \left( \frac{1-\alpha+\alpha h_{t+1}^{-\rho}}{1-\alpha+\alpha h_t^{-\rho}} \right)^{\gamma(\sigma-1)/\rho} \times \frac{1}{(1+\pi_{t+1})}$$

This notation simplifies the representation while deriving the next estimation equation. Thus, dividing the RHS term of eq. A11 by the RHS term of eq. A10 and equating to the expression from equation A12, we obtain:

$$\begin{aligned}
\frac{(1-\alpha)}{\alpha} h_t^{1+\rho} &= \frac{1 - \beta E_t [Z]}{1 - \beta E_t [Z] (1 + \epsilon_{t+1})}, \quad \Leftrightarrow \\
\Leftrightarrow \frac{(1-\alpha)}{\alpha} h_t^{1+\rho} \times (1 - \beta E_t [Z] (1 + \epsilon_{t+1})) &= 1 - \beta E_t [Z], \quad \Leftrightarrow \\
\Leftrightarrow \frac{(1-\alpha)}{\alpha} h_t^{1+\rho} - \frac{(1-\alpha)}{\alpha} \beta E_t [Z] (1 + \epsilon_{t+1}) h_t^{1+\rho} &= 1 - \beta E_t [Z], \quad \Leftrightarrow \\
\Leftrightarrow \frac{(1-\alpha)}{\alpha} h_t^{1+\rho} = 1 - \beta E_t [Z] \left( 1 + \frac{(1-\alpha)}{\alpha} (1 + \epsilon_{t+1}) h_t^{1+\rho} \right), \quad \Leftrightarrow \\
\Leftrightarrow (1-\alpha) h_t^{1+\rho} - \alpha &= \beta E_t [Z] ((1-\alpha) (1 + \epsilon_{t+1}) h_t^{1+\rho} - \alpha). \quad (\text{eq. A13})
\end{aligned}$$

Thus, eq. A9, eq. A10 and eq. A13 will be estimated by the GMM method in the following form:

$$d_{1,t+1} = \beta E_t \left\{ \begin{array}{l} \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \\ \times \left( \frac{1-\alpha+\alpha h_{t+1}^{-\rho}}{1-\alpha+\alpha h_t^{-\rho}} \right)^{-\frac{\gamma(1-\sigma)}{\rho}} \frac{1+r_t}{1+\pi_{t+1}} \end{array} \right\} - 1, \quad (\text{eq. A14})$$

$$d_{2,t+1} = \frac{c_t}{m_t} \frac{\gamma(1-\alpha)}{(1-\alpha+\alpha h_t^{-\rho})} - (1-\gamma) \times \\ \times \left( 1 - \beta E_t \left\{ \begin{array}{l} \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \\ \times \left( \frac{1-\alpha+\alpha h_{t+1}^{-\rho}}{1-\alpha+\alpha h_t^{-\rho}} \right)^{\frac{\gamma(\alpha-1)}{\rho}} \times \frac{1}{(1+\pi_{t+1})} \end{array} \right\} \right), \quad (\text{eq. A15})$$

$$d_{3,t+1} = (1-\alpha)h_t^{1+\rho} - \alpha - \beta E_t \times \\ \times \left[ \left( \frac{c_{t+1}}{c_t} \right)^{\sigma(\gamma-1)-\gamma} \left( \frac{m_{t+1}}{m_t} \right)^{\gamma(1-\sigma)} \times \left( \frac{1-\alpha+\alpha h_{t+1}^{-\rho}}{1-\alpha+\alpha h_t^{-\rho}} \right)^{\gamma(\sigma-1)/\rho} \right] \times \quad (29)$$

$$\times \frac{(1-\alpha)(1+\epsilon_{t+1})h_t^{1+\rho} - \alpha}{(1+\pi_{t+1})} \quad (\text{eq. A16})$$

### Derivation of the Demand for Domestic Real Money Balances

To derive the money demand for the domestic money balances, we need the F.O.C.:

[ $c_t$ ]:

$$(1-\gamma)(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{-\gamma} x_t^\gamma - \lambda_t = 0, \quad (\text{eq. A 4})$$

[ $m_t$ ]:

$$\gamma(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{1-\gamma} x_t^{\gamma-1} \times [(1-\alpha)m_t^{1-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times (1-\alpha) m_t^{-\rho-1} + \\ + \beta E_t \lambda_{t+1} \frac{1}{1+\pi_{t+1}} - \lambda_t = 0, \quad (\text{eq. A5})$$

[ $m_t^*$ ]:

$$\gamma(c_t^{1-\gamma} x_t^\gamma)^{-\sigma} c_t^{1-\gamma} x_t^{\gamma-1} \times [(1-\alpha)m_t^{-\rho} + \alpha m_t^{*- \rho}]^{-\frac{1}{\rho}-1} \times \alpha m_t^{*- \rho-1} + \beta E_t \lambda_{t+1} \frac{1+\epsilon_{t+1}}{1+\pi_{t+1}} - \lambda_t = 0, \quad (\text{eq. A6})$$

$[b_t]$  :

$$\beta E_t \lambda_{t+1} \frac{1+r_t}{1+\pi_{t+1}} - \lambda_t = 0, \quad (\text{eq. A7})$$

We assume a hypothetical steady state, where real consumption ( $c$ ) and real money holdings ( $x$ ) grow at a constant rate  $\phi$ . Using equations A 4 and A5, we obtain:

$$\begin{aligned} & \frac{\gamma}{(1-\gamma)} \frac{c}{x} [(1-\alpha)m^{-\rho} + \alpha m^{*-\rho}]^{-\frac{1}{\rho}-1} (1-\alpha)m^{-\rho-1} + \\ & + \beta \frac{1}{(1+\pi)} (1+\phi)^{-(1-\gamma)\sigma} (1+\phi)^{-\gamma\sigma} (1-\phi)^{-\gamma} (1-\phi)^\gamma - 1 = 0, \quad \Longleftrightarrow \end{aligned}$$

Plugging the definition of  $x$  we obtain:

$$\begin{aligned} & \frac{\gamma}{(1-\gamma)} c \frac{[(1-\alpha)m^{-\rho} + \alpha m^{*- \rho}]^{-\frac{1}{\rho}-1}}{[(1-\alpha)m^{-\rho} + \alpha m^{*- \rho}]^{-\frac{1}{\rho}}} \times (1-\alpha)m^{-\rho-1} + \\ & + \beta \frac{1}{(1+\pi)} (1+\phi)^{-\sigma} - 1 = 0 \quad \Longleftrightarrow \end{aligned} \quad (\text{eq. A17})$$

Taking the term  $(1-\alpha)m$  out of the parentheses, we obtain:

$$\frac{\gamma}{(1-\gamma)} \frac{c}{m} \frac{1}{\left[1 + \frac{\alpha}{1-\alpha} h^{-\rho}\right]} (1-\alpha)m^{-\rho-1} = 1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}, \quad (\text{eq. A18})$$

where  $h = \frac{m^*}{m}$ . Further, dividing eq. A6 by eq. A4, we get:

$$\begin{aligned} & \frac{\gamma}{(1-\gamma)} c \frac{[(1-\alpha)m^{-\rho} + \alpha m^{*- \rho}]^{-\frac{1}{\rho}-1}}{[(1-\alpha)m^{-\rho} + \alpha m^{*- \rho}]^{-\frac{1}{\rho}}} \times \alpha m^{*- \rho-1} + \\ & + \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)} - 1 = 0 \quad \Longleftrightarrow \end{aligned}$$

$$\frac{\gamma}{1-\gamma} \frac{c}{m} \frac{1}{\left[1 + \frac{\alpha}{1-\alpha} h^{-\rho}\right]} \alpha m^{*- \rho-1} = 1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}. \quad (\text{eq. A19})$$

Dividing equation A19 by equation A18, we get:

$$\frac{\alpha}{(1-\alpha)} h^{-\rho-1} = \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}}$$

Deriving the ratio denoted by  $h$ , we obtain:

$$h = \left( \frac{(1-\alpha) \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}}}{\alpha} \right)^{-\frac{1}{1+\rho}}. \quad (\text{eq. A20})$$

Plugging the derived expression for  $h$  from equation A20, into equation A18, we get:

$$\frac{\gamma}{(1-\gamma)} \frac{c}{m} \frac{1}{\left[ 1 + \frac{\alpha}{1-\alpha} \left( \frac{(1-\alpha) \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}}}{\alpha} \right)^{\frac{\rho}{1+\rho}} \right]} + \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)} - 1 = 0,$$

From this expression, we can derive  $m$ :

$$m = \frac{\gamma}{(1-\gamma)} \frac{c}{\left( 1 - \frac{\beta(1+\phi)^{-\sigma}}{(1+\pi)} \right) \left[ 1 + \frac{\alpha}{1-\alpha} \left( \frac{(1-\alpha) \frac{1 - \beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1 - \beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}}}{\alpha} \right)^{\frac{\rho}{1+\rho}} \right]} \quad (\text{eq. A21})$$

Following Friedman and Verbetsky (2001), we define the following terms:

$$R_b = (1+R)/(1+\pi), \quad 1+R = (1+\pi)(1+n)(1+\phi)^\sigma/\beta, \quad R_m = 1/(1+\pi) \quad \text{and} \\ R_{mf} = (1+\epsilon)/(1+\pi).$$

In this paper, I assume the population growth rate to equal 0. Following Walsh (2003), the steady state inflation  $\pi$  should be equal to the steady state money growth. First plugging the defined terms into the denominator of equation A21, we obtain:

$$\begin{aligned}
& (1 - \beta(1 + \phi)^{-\sigma} R_m) \left( 1 + \frac{\alpha}{(1 - \alpha)} \left( \frac{(1 - \alpha)}{\alpha} \frac{1 - \beta(1 + \phi)^{-\sigma} R_{mf}}{1 - \beta(1 + \phi)^{-\sigma} R_m} \right)^{\frac{\rho}{1+\rho}} \right) = \\
& = (1 - \beta(1 + \phi)^{-\sigma} R_m) \left( 1 + \frac{\alpha}{(1 - \alpha)} \left( \frac{(1 - \alpha)}{\alpha} \frac{1 - \frac{(1+\pi)}{1+R} R_{mf}}{1 - \frac{(1+\pi)}{(1+R)} R_m} \right)^{\frac{\rho}{1+\rho}} \right) = \\
& = (1 - \beta(1 + \phi)^{-\sigma} R_m) \left( 1 + \frac{\alpha}{(1 - \alpha)} \left( \frac{(1 - \alpha)}{\alpha} \frac{1 - \frac{1}{R_b} R_{mf}}{1 - \frac{1}{R_b} R_m} \right)^{\frac{\rho}{1+\rho}} \right) = \\
& = (1 - \frac{R_m}{R_b}) \left( 1 + \frac{\alpha}{(1 - \alpha)} \left( \frac{(1 - \alpha)}{\alpha} \frac{R_b - R_{mf}}{R_b - R_m} \right)^{\frac{\rho}{1+\rho}} \right) = \\
& = \frac{1}{R_b} (R_b - R_m) \left( 1 + \frac{\alpha}{(1 - \alpha)} \left( \frac{(1 - \alpha)}{\alpha} \frac{R_b - R_{mf}}{R_b - R_m} \right)^{\frac{\rho}{1+\rho}} \right) = \\
& = \frac{1}{R_b} (R_b - R_m) \left( 1 + \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\rho}} \left( \frac{R_b - R_{mf}}{R_b - R_m} \right)^{\frac{\rho}{1+\rho}} \right) = \\
& = \frac{1}{R_b} (R_b - R_m) + \frac{1}{R_b} (R_b - R_m) \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\rho}} \left( \frac{R_b - R_{mf}}{R_b - R_m} \right)^{\frac{\rho}{1+\rho}} = \\
& = \frac{1}{R_b} (R_b - R_m) + \frac{1}{R_b} (R_b - R_m) \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1+\rho}} \left( \frac{R_b - R_{mf}}{R_b - R_m} \right)^{\frac{\rho}{1+\rho}} = \\
& = \frac{1}{R_b} (R_b - R_m) + \frac{1}{R_b} (R_b - R_{mf}) \left( \frac{\alpha}{1 - \alpha} \frac{R_b - R_m}{R_b - R_{mf}} \right)^{\frac{1}{1+\rho}}
\end{aligned}$$

Plug the last derived equation into the denominator of eq. A21:

$$m = \frac{\gamma}{(1 - \gamma)} \frac{R_b c}{(R_b - R_m) + (R_b - R_{mf}) \left( \frac{\alpha}{1 - \alpha} \frac{R_b - R_m}{R_b - R_{mf}} \right)^{\frac{1}{1+\rho}}} \quad (\text{eq. A22})$$

To compute seigniorage loss, we need a definition of seigniorage. Following Walsh (2003), the seigniorage revenue  $s$  can be computed as follows:

$$s_t = \frac{M_t - M_{t-1}}{P_t} \quad (\text{eq. A23})$$

Then the ratio of seigniorage-to-GDP can be represented as follows:

$$\frac{s_t}{Y_t} = \frac{M_t - M_{t-1}}{P_t Y_t} = \frac{M_t - M_{t-1}}{P_t Y_t} \frac{M_t}{M_t} \iff$$

Assuming that money supply grows at the following rate:

$$M_t = (1 + \pi)(1 + \phi)(1 + n)M_{t-1} \quad (\text{eq. A24})$$

Therefore, the seigniorage- to-GDP ratio will be calculated in the following way:

$$\frac{s_t}{Y_t} = \frac{M_t}{P_t Y_t} \left( 1 - \frac{1}{(1 + \pi)(1 + \phi)(1 + n)} \right),$$

In per capita terms:

$$\frac{s_t}{Y_t} = \frac{m_t}{y_t} \left( 1 - \frac{1}{(1 + \pi)(1 + \phi)(1 + n)} \right) \quad (\text{eq. A25})$$

Assuming that  $n=0$ , and in steady state  $M_t = (1 + \pi)(1 + n)(1 + \phi)M_{t-1}$ , while  $1 + \pi = 1 + \phi$ , the seigniorage to GDP ratio is:

$$\frac{s}{Y} = \frac{m}{y} \left( 1 - \frac{1}{(1 + \phi)} \right) \quad (\text{eq. A26})$$

Now the seigniorage ratio can be calculated by plugging the expression for  $m$  from eq. A22.

### Derivation of Welfare Loss

$$\begin{aligned} u(c, x) &= \frac{(c^{1-\gamma} ((1 - \alpha) m^{-\rho} + \alpha m^{*-\rho})^{-\frac{\gamma}{\rho}})^{(1-\sigma)} - 1}{(1 - \sigma)} = \\ &= \frac{(c^{1-\gamma} m^\gamma \left( (1 - \alpha) + \alpha \frac{m^{*-\rho}}{m^{-\rho}} \right)^{-\frac{\gamma}{\rho}})^{(1-\sigma)} - 1}{(1 - \sigma)} \end{aligned}$$

Plug the derived expression for the ratio  $h$  from equation A20:

$$u(c, x) = \frac{c^{(1-\gamma)(1-\sigma)} m^{\gamma(1-\sigma)} \left( 1 - \alpha + \alpha \left( \frac{1-\alpha}{\alpha} \left( \frac{1-\beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1-\beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma(1-\sigma)}{\rho}} - 1}{(1 - \sigma)}.$$

To calculate the welfare costs of various steady state levels of inflation, we need to compute the percentage decrease in consumption per capita that would generate the same welfare loss as that from moving from the original level of dollarization  $\alpha_1$  to a higher level of dollarization  $\alpha_2$ .

So assume that  $c_1 \succ c_2$  and  $c_1 = c_2 + \Delta c$ , and  $\alpha_1 \prec \alpha_2$ .

$$\begin{aligned}
& \frac{c_1^{(1-\gamma)(1-\sigma)} m_1^{\gamma(1-\sigma)} \left( 1 - \alpha_1 + \alpha_1 \left( \frac{1-\alpha_1}{\alpha_1} \left( \frac{1-\beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1-\beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma(1-\sigma)}{\rho}} - 1}{(1-\sigma)} = \\
& \frac{(c_1 + \Delta c)^{(1-\gamma)(1-\sigma)} m_2^{\gamma(1-\sigma)} \left( 1 - \alpha_2 + \alpha_2 \left( \frac{1-\alpha_2}{\alpha_2} \left( \frac{1-\beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1-\beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma(1-\sigma)}{\rho}} - 1}{(1-\sigma)} \Leftrightarrow \\
& \Leftrightarrow c_1^{(1-\gamma)(1-\sigma)} m_1^{\gamma(1-\sigma)} \left( 1 - \alpha_1 + \alpha_1 \left( \frac{1-\alpha_1}{\alpha_1} \left( \frac{1-\beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1-\beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma(1-\sigma)}{\rho}} = \\
& = (c_1 + \Delta c)^{(1-\gamma)(1-\sigma)} m_2^{\gamma(1-\sigma)} \left( 1 - \alpha_2 + \alpha_2 \left( \frac{1-\alpha_2}{\alpha_2} \left( \frac{1-\beta \frac{(1+\epsilon)(1+\phi)^{-\sigma}}{(1+\pi)}}{1-\beta \frac{(1+\phi)^{-\sigma}}{(1+\pi)}} \right)^{\frac{\rho}{1+\rho}} \right) \right)^{-\frac{\gamma(1-\sigma)}{\rho}}
\end{aligned}$$

Defining the expression in parentheses as  $f(\alpha)$ , we then obtain:

$$\begin{aligned}
c_1^{(1-\gamma)} m_1^{\gamma} f(\alpha_1) &= (c_1 + \Delta c)^{(1-\gamma)} m_2^{\gamma} f(\alpha_2), \Leftrightarrow \\
\Leftrightarrow \left( \frac{c_1 + \Delta c}{c_1} \right)^{(1-\gamma)} &= \frac{m_2^{\gamma} f(\alpha_2)}{m_1^{\gamma} f(\alpha_1)}, \Leftrightarrow \\
\Leftrightarrow \frac{c_1 + \Delta c}{c_1} &= \left( \frac{m_2^{\gamma} f(\alpha_2)}{m_1^{\gamma} f(\alpha_1)} \right)^{\frac{1}{1-\gamma}}, \Leftrightarrow \\
\Leftrightarrow \frac{\Delta c}{c_1} &= \left( \frac{m_2^{\gamma} f(\alpha_2)}{m_1^{\gamma} f(\alpha_1)} \right)^{\frac{1}{1-\gamma}} - 1
\end{aligned}$$

As a ratio of GDP per capita, the consumption compensation is:

$$\frac{\Delta c}{y} = \frac{c}{y} \left[ \left( \frac{m_2^{\gamma} f(\alpha_2)}{m_1^{\gamma} f(\alpha_1)} \right)^{\frac{1}{1-\gamma}} - 1 \right].$$

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