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CORE

DISCUSSION PAPER

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Abstract

Financial derivatives are important hedging tool for asset's manager. Electricity is by its very nature the most volatile commodity, which creates big incentive to share the risk among the market participants through financial contracts. But, even if volume of derivatives contracts traded on Power Exchanges has been growing since the beginning of the restructuring of the sector, electricity markets continue to be considerably less liquid than other commodities. This paper tries to quantify the effect of this insufficient liquidity on power exchange, by introducing a pricing equilibrium model for power derivatives where agents can not hedge up to their desired level. Mathematically, the problem is a two stage stochastic Generalized Nash Equilibrium and its solution is not unique. Computing a large panel of solutions, we show how the risk premium and player's profit are affected by the illiquidity.

Keywords: illiquidity, electricity, power exchange, artitrage, generalized Nash Equilibrium, equilibrium based model, coherent risk valuation.

JEL Classification: C61, G13

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1. Introduction

Liquidity plays a crucial role in financial markets. It enlarges the capacity of the market to accommodate order flows and guarantees the ability to quickly buy/sell sufficient quantities of an asset without significantly impacting the price. Liquidity is among the most important characteristics for asset managers who want to be sure that their portfolio can easily be converted into cash. Insufficient liquidity, on the other hand, creates new risks and frictions. The literature provides ample empirical evidence that liquidity is an important state variable for asset pricing and that investors demand a higher return from less liquid securities (see Pàstor and Stambaugh (2002); Amihud (2002) and the cited literature).

Two important issues arise from insufficient liquidity. The first one is the empirical proxy that is used to measure it. Liquidity is an unobserved variable that embeds several dimensions, as volume, depth, resiliency and tightness[1]. The simplest proxies are the volume of exchange and the bid-ask spread; but it is now recognized that those measures are not fully appropriate. Many other measures, which relate the size of the trade to the size of the price movement, have been proposed and explored. Hasbrouck (2005) provides a comprehensive discussion of some interesting measures. The second issue concerns the effect of insufficient liquidity on the pricing of financial contracts. Indeed, most pricing models rely on the assumption of absence of arbitrage. This assumption is only sustainable in a very liquid market where arbitrageurs can instantaneously exploit all possible mispricings. This does not hold for illiquid markets and hence such models might not be applicable.

Power derivatives are important in restructured electricity markets because they permit agents (producers, distributors, retailers) to hedge their strategy in a quite volatile environment. There was substantial evidence of insufficient liquidity in the early days of the restructured electricity markets(Newbery et al. (2003) and Newbery (2004)). Since then, the volume of spot and derivatives contracts increased significantly but electricity

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[1]for definitions of those concepts, see among O'Hara (1997); Kyle (1985)

still remains considerably less liquid than other energy commodities (Table 8). Nowadays, market operators or power exchanges regularly publish technical reports on the trade volumes and number of active participants (e.g. Market Surveillance of EEX (2009)). But, to our knowledge, no empirical study has really focused on the effects of a possible insufficient liquidity on the derivatives contracts. PJM (2007) recognizes that mature energy markets will require increased forward trading in order to reduce risk and provide clear price signals to support investment and hedging opportunities.

The pricing of financial power derivatives remains a challenging topic, even regardless of liquidity problems. It is well recognized that the non-storability of electricity creates non-hedgeable risks[2]. Also the time series of the underlying spot prices exhibit unusual behaviors due to the idiosyncrasies of electricity. The demand of electricity is variable, stochastic and price inelastic in the short term. These properties combined with the finite capacity and technical characteristics of generators implies a particular spot price dynamics that spikes to extremely high values. These jumps usually occur within a very short period of time. Also, time series of power prices exhibit substantial mean reversion and seasonality. Last but not least, the market is impacted by a very wide set of parameters, such as the fuel prices, power plants availability and network capacity.

Because of this complexity, researchers have developed so-called equilibrium-based model with the goal of understanding the fundamentals of the power derivatives market. The pioneering paper of Bessembinder and Lemmon (2002) analyzes the forward market by assuming that the prices are determined by an economic equilibrium among market participants (producers and retailers) rather than by speculation mechanisms. The authors assume that market agents are risk-averse and hedge their stochastic profit by optimizing their positions in financial contracts. This equilibrium model derives the optimal strategies of these agents on the basis of their incentives to hedge. This leads to the necessary volume of power derivatives at the equilibrium. Bessembinder and Lemmon (2002) find optimal hedge ratio[3] roughly ranging from 0.8 to 1.2, depending on

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[2]One cannot buy power on the spot (day-ahead) market, store it and re-sell it later.
[3]the hedge ratio is the ratio between the volume of future contracts and the expected production

the market parameters. Producers and retailers massively buy/sell financial contracts in order to minimize their risks. All studies based on this type of methodology conclude to similar quantitative results. Such level of trades have never been observed on any power exchange. Due to lack of liquidity, agents can not hedge their production and demand at those levels.

The goal of this paper is to illustrate and quantify the effect of illiquidity in the power exchange on power derivatives. To our knowledge this problem has not been explored before. We construct a two stage stochastic equilibrium model of power derivatives in a perfect competition market (agents are price takers) except for insufficient liquidity. We define liquidity on the basis of the volume exchanged and study its impact by restricting the volume of available derivative contracts. In this set up agents can not hedge up to their desired level because their strategy sets are restricted by the action of the others players. Mathematically, the problem is a Generalized Nash Equilibrium Problem (GNEP) and its solution is not anymore unique. We illustrate the model on a 6-node example taken from Chao and Peck (1998) and quantify the effect of illiquidity by computing a large panel of equilibria and discussing their significance. The model involves both generation and transmission markets. Empirical studies (Siddiqui et al. (2005); Adamson and Englander (2005)) have yet pointed that the risk premia on transmission contracts are unreasonably important and that the low liquidity of Financial Transmission Rights (FTRs) markets is one principal explanation. Shijie et al. (2005) shows that certain market design, as the simultaneous feasibility rules, are also a plausible explanation. This paper studies the impact of illiquidity in one market (here transmission rights) on the other market (here energy).

The paper is organized as follow. We select a market design in section 2 and present the corresponding model of the spot market. In section 3, we focus on equilibrium pricing model in a perfect liquid market and cast the notion of absence of arbitrage in this context. We show that a sufficient condition for eliminating arbitrage at equilibrium in a perfect liquid market is to model the risk aversion by coherent (in the sense of Artzner et al. (1999)) and "equivalent" valuation function. In section 4, we quantify the impact

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of insufficient liquidity in power market. We present our model and address specifically three topics arising from illiquidity. We first show that illiquidity allows for remaining arbitrage possibilities at equilibrium. Secondly, we analyze a range of equilibria with the view of quantifying the effect of illiquidity on the agents hedging strategies, profits volatility and risk-premia. Finally, we focus our attention on illiquidity in the market of financial transmission rights (FTRs) and show that illiquidity in one market (in this case transmission) can drastically decrease the incentive to hedge in an other market (here energy). We conclude in section 5.

2. Spot market equilibrium

2.1. Selecting a Market Design

The restructuring of electricity market has lead to many, sometimes quite different, market designs. Following several authors, we here focus on a particular design where a day-ahead market trades power for physical delivery on a spot market taking place the next day. The spot market is based on an hourly auction with bids for purchase and sale. This market is segmented geographically, and divided in several nodes of supply or demand connected by transmission lines. Congestion management is a key element in restructured electricity markets. The prices are defined at each node of the network reflecting that only feasible bids, i.e. bids that comply with the limited capacity of the network, can be accepted at different nodes. The nodal prices are called Locational Based Marginal Prices (LBMP) and are calculated for each generation and load zone by the System Operator (SO). In such system, the buyers pay the LBMP calculated at the node in which they take delivery of electricity (or point of withdrawal :PoW) and sellers receive the LBMP at the bus to which they supply (or point of injection :PoI).

This organization can be seen as an extremely stylized view of restructured US power markets. The market design in EU countries is different where the most advanced realization is still based on a separation between energy and transmission markets. Prices are not defined at each bus but within a zone, which usually corresponds to the country/market. Congestion management is treated after the clearing of the energy market.

The Power Exchanges that clear the energy market may first accept bids that are not feasible for the transmission network. It is then up to the Transmission System Operators to correct the situation by redispatching and counter-trading operations. Usually an asymmetric reward scheme is applied. The production units that are constrained off still receive the remuneration from the energy market at the calculated spot price. The production units that are constrained on obtain a price equal to their bids in the spot market. While this procedure permits a price that is defined at a national level, it introduces several inefficiencies. The adjustment creates extra cost that are supported by the SO and pass through to the consumer after socialization. There is evidence (Ehrenmann and Smeers (2004); Furió and Lucia (2009)) that it creates undesired incentives to game the system and changes the trading strategies of market participants[4].

2.2. The spot market model

We develop our analysis on an extremely stylized US like market because the integration of energy and transmission makes congestion management more transparent. This integration of the energy and transmission operations also facilitates the quantification of liquidity constraints on financial transmission contracts [5]. We assume a perfectly competitive spot electricity market. Stylized examples are widely used in the literature since Hogan (1992)'s famous three nodes network. We follow suit and construct our arguments on Chao and Peck (1998)'s six nodes example (Figure 1). We adopt both the model and its numerical assumptions (see the original article for more discussion of that example).

Our description of the market is now standard. The power grid contains N buses and L transmission lines. Each line $\ell \in L$ is characterized by its impedance and has a thermal capacity K_ℓ. Using the DC approximation of the AC load flow equations, every MW injected (retrieved) at a generating (load) bus n is responsible for a power flow $\mathtt{PTDF}_{n,\ell}$

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[4]Furió and Lucia (2009) show on the Spanish market that buyers respond by abandoning the daily market in favor on the intraday as far as possible. In the seller's part, their paper concludes that some strategical power plants have incentives to submit their sales bids at high prices in order to not be matched in the spot market but finally are required to produce to solve the transmission constraints.

[5]In Europe, there exist no FTRs defined at a national level, but on some cross-border interconnectors (obtained by the so-called explicit auctions). Those explicit auctions are less important for the hedging and also have the bad property that they do not ensure that the ow always goes from low price area to the high price area (see Kristiansen (2007))
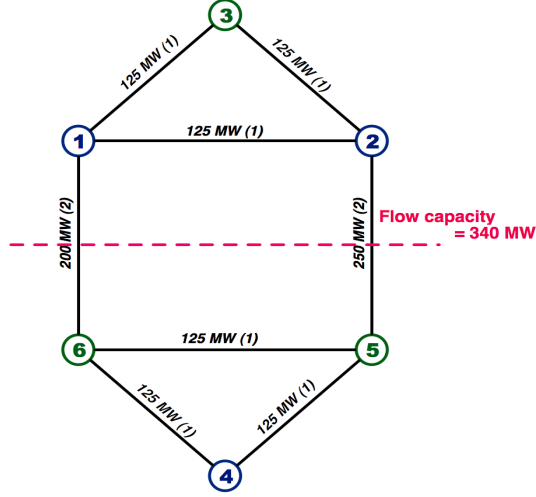
Figure 1: 6-nodes network (Chao and Peck (1998))

on the line $\ell \in L$. The electrical network is controlled by a SO which is responsible for its reliability. At each bus of the network, there is a single economic agent $\nu \in N$ which can be a producer ($\nu \in N_p$) or a retailer ($\nu \in N_r$). Producer have unlimited capacities; they bid their marginal cost of supply C_ν in the spot market. We assume furthermore that the producer has no fixed cost and its total cost function C_ν^T takes the following form[6].

$$C_\nu^T(q_\nu) = a_\nu q_\nu + b_\nu \frac{q_\nu^2}{2} \quad ; \quad C_\nu(q_\nu) = a_\nu + b_\nu q_\nu \tag{1}$$

Each retailer $\nu \in N_r$ serves the final consumers at its bus. It sells power at a fixed retail price P_ν^r. It bids its inverse demand function in the spot market which is also assumed to be linear[7].

$$P_\nu(q_\nu) = a_\nu - b_\nu q_\nu \tag{2}$$

Table (1) reports the bids of the different economic agents.

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[6]The model can easily be extended to more complex production function and production set restricted by a limited capacity.

[7]One can easily extend the model to inelastic demand, which is probably a better representation of actual markets. The computation of the spot equilibrium by maximizing the welfare becomes then the minimization of the total cost for meeting the inelastic demand.

Bus-ID	Supply bids: $C_\nu(q_\nu)$	Bus-ID	Load bids: $P_\nu(q_\nu)$	P_ν^r
1	10 + 0.05 q	3	37.5 - 0.05 q	33.6
2	15 + 0.05 q	5	75 - 0.1 q	61.7
4	42.5 + 0.025 q	6	80 - 0.1 q	62.6

Table 1: The 6-nodes example

The System Operator collects the bids in the spot market and maximizes the total welfare leading to the following mathematical programming problem.

$$\max_{q_\nu \in \mathbb{R}_+^N} \left[\sum_{\nu \in N_r} \int_0^{q_\nu} P_\nu(\xi_\nu)d\xi_\nu - \sum_{\nu \in N_p} \int_0^{q_\nu} C_\nu(\xi_\nu)d\xi_\nu \right]$$

$$\text{s.t.} \quad \sum_{\nu \in N} q_\nu = 0$$

$$-K_\ell \leq \sum_{\nu \in N} \texttt{PTDF}_{\nu,\ell} \ q_\nu \leq K_\ell$$

(3)

The spot price P_ν^s at each node ν is given by the marginal cost at a generating bus, or by the inverse demand function at a load bus. The SO earns a spot profit π_{so}^{spot} (the "merchandising surplus") by collecting the transmission rents:

$$\pi_{so}^{spot} = \sum_{\ell \in L} tr_\ell (P_{\ell_2}^s - P_{\ell_1}^s)$$

(4)

In this expression, tr_ℓ is the power flow on the line ℓ. It can be derived from the injections/withdrawals q_ν and the power distribution factor of the line. $(P_{\ell_2}^s - P_{\ell_1}^s)$ is the difference of prices at the end nodes of line ℓ. The profits of producers/retailers on the spot market are given by:

$$\pi_\nu^{spot} = \begin{cases} q_\nu(P_\nu^s - C_\nu^T(q_\nu)) & \text{if } \nu \in N_p \\ q_\nu(P_\nu^r - P_\nu^s) & \text{if } \nu \in N_r \end{cases}$$

(5)

2.3. Uncertainty and spot scenarios

We consider two types of uncertainties on the spot market. One is the final consumer demand which is particularly sensitive to weather variation. We model this uncertainty by a set of independent scenarios for the parameters a_ν of the load buses. Specifically the load parameters a_ν varies from -25% to +25% (by step of 12.5%). So, a retailer faces the stochastic demand of its final consumers. In the short term, the retail price P_ν^r is fixed. It is here set to 120% of the expected node price at which the retailer serves its clients. The network availability is the second source of uncertainty. Transmission line outages can seriously impact the spot equilibrium. We consider 2 contingencies: the no default case occurs with probability of 90%. The outage of the line linking nodes 1 and 6 occurs with a probability of 10%. This leads to a total of 250 scenarios. We let ω and p_ω denoting a scenario and its probability. Solving the equilibrium of the spot market for the different scenarios we obtain the distribution of results summarized in Table 2.

Bus-ID:	1	2	3	4	5	6
Nodal prices:						
- $\mathbb{E}[P_\nu^s]$	24.72	26.2	28.13	45.98	51.46	53.05
- $\mathbb{V}\text{ar}[P_\nu^s]$	3.08	4.56	17.96	1.71	43.31	76.69
Market Agents:						
- $\mathbb{E}[\pi_\nu^{spot}]$	2197	1300	652	345	1646	1979
- $\text{vol}(\pi_\nu^{spot})$	11%	37%	75%	56%	76%	87%
- $\text{CVaR}_{75\%}(\pi_\nu^{spot})$	1517	702	-83	86	78	-309

Table 2: Nodal price and agents profit statistics

$\mathbb{E}$ and $\mathbb{V}ar$ in the table respectively denote expectation and variance. The statistic "vol" measures the volatility of the profit; it is the ratio between the expectation and the standard deviation. The conditional value at risk CVaR_α is another risk measure that represents the expected profit computed over the $(1 - \alpha)$ worst scenarios. While less extensively used than the Value at Risk (VaR), the CVaR is more and more commonly encountered in the literature. Figure (2) where the shaded area measures the CVaR illustrates the concept.
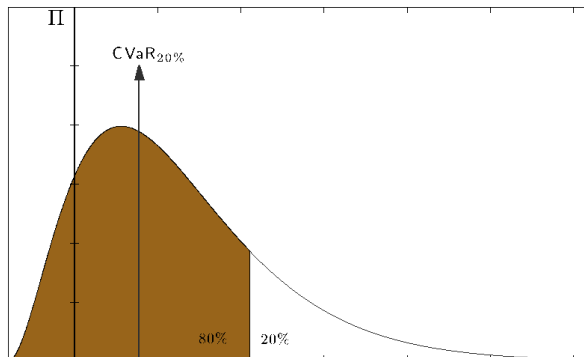
Figure 2: Illustration of the CVaR

Table 2 shows that retailer profits are much more volatile than those of the producers. More importantly, their $CVaR_{75\%}$ (i.e. the conditional expectation of their profit in the 63 worst scenarios) are very low compared to the expected profits; they are even negative for retailers at bus 3 and 6. The system operator collects the merchandising surplus on transmission lines (see Table 9 in appendix A). The revenue accruing from operating a line depends on the price difference at its two extremity buses. In our numerical simulation, the line prices are more volatile than the nodal prices, reflecting the fact that the demand in the three load buses are independent. Table 9 also shows the first and the second moments of those prices. Not surprisingly, the transmission price between the node 1 and node 6 is the most expensive and the most volatile, because those two nodes are linked by a transmission line subject to outages.

3. Equilibrium pricing for financial contracts

The literature offers two main methodologies for pricing derivative products. One approach resorts to risk neutral valuation and constitutes the most common approach in financial mathematics. It is based on stochastic process models that capture the spot price dynamics and serve to value power contingent claims. The other stream of the literature relies on economic models of power production and consumption. Bessembinder and Lemmon (2002) were the first to introduce a two stage equilibrium model of the power future market where market participants want to hedge their profit by contracting a certain amount of futures before bidding in the spot market. Their methodology

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has subsequently been used by several authors. Cavallo and Termini (2005) study the benefits of introducing a market for standardized derivatives. Notably, they showed that this market increases the share of the electricity purchased through the spot market and diminishes the share of the bilateral contracts. Willems and Morbee (2008) quantify how the introduction of power derivatives affects welfare and investment incentives . Their computational results indicate that aggregate welfare in the market increases with the number of derivatives offered and that investment decisions improve with increasing market completeness because of decoupling of investment and speculation. Bülher and Müller-Merbach (2008) extend the initial model of Bessembinder and Lemmon (2002) to a dynamic equilibrium and derive an endogenous term structure of electricity futures prices. Our model is part of this latter stream of literature.

By participating in the financial market, producers and retailers trade the risks incurred because of fluctuating spot prices, demand shocks and network congestion. They have the opportunity to contract different financial derivatives (noted c), whose pay-off is the difference between the derivative price P_c^f and the corresponding realized pay-off $P_{c,\omega}^s$ (which is a known function of the spot market price). Letting x_c^ν be the position of agent ν in contract c, the profit formula (equations 4 and 5) of agents (retailer, generators and SO) become :

$$
\Pi_{\nu,\omega} = \begin{cases} \sum_c x_c^\nu (P_c^f - P_{c,\omega}^s) + q_{\nu,\omega}(P_{\nu,\omega}^s - C_\nu^T(q_{\nu,\omega})) & \text{if } \nu \in N_p \\ \\ \sum_c x_c^\nu (P_c^f - P_{c,\omega}^s) + q_{\nu,\omega}(P_\nu^r - P_{\nu,\omega}^s) & \text{if } \nu \in N_r \end{cases} \tag{6}
$$

$$
\Pi_{so,\omega} = \sum_c x_c^{so}(P_c^f - P_{c,\omega}^s) + \sum_{\ell \in L} tr_\ell (P_{\ell_2}^s - P_{\ell_1}^s)
$$

Agents are price takers in a perfectly competitive market. They can influence neither the spot price, nor the price of the financial contract $P_{c,\omega}^s$ in order to earn extra profit from the trading. The outcome of the spot market is thus independent of the financial portfolios of agents. One can first solve the equilibrium of the spot market and then solve the equilibrium of the forward market on the basis of the obtained spot prices equi-

libria. Notice that this simplification does not hold in a non-competitive environment because forward decisions can influence the outcome of the spot market. Zhang et al. (2009) propose a stochastic equilibrium model with equilibrium constraints (SEPEC) to characterize the interaction between the two markets in a Cournot game.

Agents trade financial contracts because they are risk averse and want to hedge the random profit earned in the spot market. We model this risk aversion by using the modern approach of *risk valuation* (see Artzner et al. (1999) for the original risk measure concepts and Shapiro et al. (2009) for their inclusion in a mathematical programming framework). The original presentation is in terms of risk function[8] ; because of the context we conduct the discussion in terms of risk valuation. A risk valuation is a function ρ which maps the space of risky payoffs (i.e. the set of all possible real-valued functions on Ω) into the extended real line $\bar{\mathbb{R}}$[9]. Investor ν values his total portfolio according to the risk-valuation of its outcomes distribution, let $\rho_\nu(\Pi_\nu)$[10]. The problem of investor (producer and retailer) ν can be formulated as:

$$\mathcal{P}^\nu \equiv \max_{x_c^\nu} \ \rho_\nu(\Pi_\nu) \tag{7}$$

where $\Pi_{\nu,\omega}$ is given by equation 6.

The SO problem is slightly more complicated and is presented latter.

3.1. Risk valuation and arbitrage

Equilibrium pricing models are not based on the assumption of absence of arbitrage but rather suppose that prices are determined by the economic equilibrium resulting from the simultaneous maximization of agents risk valuation. If this equilibrium contains arbitrage opportunities, outside speculators in a perfect liquid market will massively enter the market and trade them away. We do not model these speculators (as explained above, we exclude them from the transmission market) and hence cannot ex ante guarantee an arbitrage free equilibrium. It is however possible to guarantee the absence of

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[8]in the context of minimization of loss.

[9]$\bar{\mathbb{R}} = \mathbb{R} \cup \{+\infty\} \cup \{-\infty\}$

[10]Mean-risk models are an important part of that framework. Agent maximizes his profit's expectation $\mathbb{E}[\Pi]$ accounting a measure $\mathbb{D}[\Pi]$ of the outcomes dispersion: $\rho(\Pi) = \mathbb{E}[\Pi] - \kappa\mathbb{D}[\Pi]$

arbitrage at equilibrium by assuming that agents risk aversion is modeled by "coherent" and "equivalent" risk valuation. By definition "coherence" means that the risk valuation satisfies axioms respectively noted concavity, monotonicity, translation equivariance and positive homogeneity (see definitions in appendix B). A "representation theorem" (see Artzner et al. (1999)) states that every coherent risk valuation ρ can be represented as an expectation taken with respect to the probability measure ζdP, where ζ belongs to the subdifferential of ρ at 0.

$$\rho_\nu(\Pi_\nu) = \inf_{\zeta_\nu \in \partial\rho(0)} \mathbb{E}_{\zeta_\nu}[\Pi_\nu] \tag{8}$$

"Equivalence" means that the probability measure associated with the risk measure (i.e. of density ζdP) is equivalent to the measure dP, that is, that both have the same set of zero measure events.

When the market is perfect (i.e. no transaction cost or portfolio restriction), $\mathcal{P}^\nu$ optimality condition implies that there exists a (at least one) probability measure, defined by $\zeta_\nu^* dP$, under which prices are discounted martingales. In case of futures contracts, payments are due at maturity and hence satisfy:

$$P_c^f = \mathbb{E}_{\zeta_\nu^*}[P_{c,k}^s] \tag{9}$$

According to the first theorem of finance, if this probability measure $\zeta_\nu^* dP$ is equivalent to the true one, then the market is arbitrage free. We call the probability measure $\zeta_\nu^* dP$ the agent's risk neutral measure; it depends on the assumed risk valuation function and the profit distribution of the agent. The probability measures $\zeta_\nu^* dP$ are identical among all players when the market is complete. Also, financially speaking, the density ζ_ν^* can be interpreted as the "agent's state prices" at equilibrium, i.e. how much a particular agent values one unit of extra profit which only occurs for a particular state.

An equilibrium model where agents risk aversion is modeled with a non coherent risk valuation might lead to solution with arbitrage opportunities. For example, mean-variance, as used in Bessembinder and Lemmon (2002) or Willems and Morbee (2008), is not coherent because it violates the positive homogeneity and monotonicity axioms. The

absence of arbitrage in the equilibrium solution can be checked ex-post, by verifying if the prices of the derivatives are strictly inside the convex hull of the spot payoffs of these derivatives (in which case they can be expressed as expectations of the spot pay-offs in some risk-neutral probability measures). The CVaR, which is growing in popularity in the literature on electricity restructuring[11] , is a coherent risk valuation but does not lead to equivalent measures. Indeed, CVaR_α is an expectation under a probability measure which disregards profits greater than some threshold level that is only exceeded α of the time. So it can also lead to equilibrium solution with arbitrage opportunities.

We model the agents risk aversion by a $\mathsf{E\text{-}CVaR}_{\alpha,\beta}$, which is a weighted sum of the expectation of the profit and a CVaR_α.

$$\mathsf{E\text{-}CVaR}_{\alpha,\beta}(\Pi_\nu) = (1-\beta)\,\mathbb{E}\big[\Pi_\nu\big] + \beta\,\mathsf{CVaR}_\alpha\big(\Pi_\nu\big) \tag{10}$$

This function satisfies the four axioms of coherence and is also equivalent. Indeed, the associated agent's state prices are :

$$\partial(\mathsf{E\text{-}CVaR})(\Pi_\nu) = \begin{cases} & \zeta_{\nu,\omega} = (1-\beta) + \beta\alpha^{-1} & \text{if} \quad \Pi_{\nu,\omega} < \mathsf{VaR}_\alpha(\Pi_\nu) \\ \zeta_\nu : \mathbb{E}\big[\zeta_\nu\big] = 1, & \zeta_{\nu,\omega} = (1-\beta) & \text{if} \quad \Pi_{\nu,\omega} > \mathsf{VaR}_\alpha(\Pi_\nu) \\ & \zeta_{\nu,\omega} = (1-\beta) + [0, \beta\alpha^{-1}] & \text{if} \quad \Pi_{\nu,\omega} = \mathsf{VaR}_\alpha(\Pi_\nu) \end{cases}$$

One can see that the probability measure of density $\zeta_\nu^* dP$ is equivalent to the true one and defines the risk-neutral measure for the agent. For our computations, we set the parameters β to 0.9 and α to 20%.

3.2. The model

We consider two types of financial derivatives. Energy futures are the most important ones in terms of volume. An agent holding a long futures position receives the difference between the energy spot price at maturity and the futures price. Forward energy contracts do not exist for all nodes. Indeed, in order to enhance liquidity, energy futures markets are restricted to a few busses (the hubs). In order to allow agents located at different nodes to mitigate congestion charges between their home node and

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[11]see among Bartelj et al. (2010); Resta and Santini (2008); Gonzalez et al. (2007)

the hub, properly designed electricity markets implement periodic auctions of financial transmission right (FTRs). We therefore complement the 6-nodes spot market by assuming forward energy and transmission trading where agents can trade energy futures and point to hub FTRs contracts. We suppose that there is only one future energy contract traded at node 6, which is the sole hub of the network. This situation is representative of many U.S. market, where energy futures exist only for some hubs and not at each node of the network. Considering node to hub FTRs contracts allows one to span all node to node transmission risks.

The problem of producers and retailers was stated in (7). The description of the behavior of the SO is slightly different. The SO is the ultimate counter party in the transmission market. The SO initially auctions FTR contracts but wants to restrict their set to a volume that is adequate and reliable for its congestion management. It limits the total amount of auctioned FTR so that the corresponding flow on the lines satisfies the N-1 rule[12]. Also, we assume that it does not take any futures energy position on the hub (here node 6). The SO therefore sells FTR so as to maximize the risk valuation of these FTR subject to the constraints N-1:

$$
\begin{aligned}
\mathcal{P}^{so} \equiv \quad & \max_{x_c^{so}} \quad \rho_{so}(\Pi_{so}) \\
& -K_\ell \leq \sum_c \texttt{PTDF}_{\text{ftr},\ell}^{\text{N-1}} \, x_{\text{ftr}}^{so} \leq K_\ell \\
& x_{\text{energy}}^{so} = 0
\end{aligned}
\tag{11}
$$

Finding an equilibrium in the financial market means finding a tuple $(\mathbf{x}_c^\nu, x_c^{so}, \mathbf{P}_c^f)$ such that, for each agent $\mathbf{x}_c^\nu$ solves $\mathcal{P}^\nu$ (relation (7)), x_c^{so} solves $\mathcal{P}^{so}$ (relation (11)) for given price of the derivatives contract $\mathbf{P}_c^f$, and such that the market clearing condition holds:

$$
\sum_\nu x_c^\nu + x_c^{so} = 0
\tag{12}
$$

The complete formulation of the equilibrium model is given in appendix C. It is a Nash Equilibrium Problem (NEP); we show in this appendix how the problem can be solved heuristically by a sequence of linear programming problems.

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[12]That is must satisfy all the thermal lines limit under all singular lines outage.

3.3. Simulation results

Table 3 reports the prices and volumes[13] of the different derivatives at equilibrium. P_c^f and "Volume" respectively denote the price and the trade volume of each contract. $P_c^f - \mathbb{E}[P_c^s]$ is the risk premium embedded in the contract price. One first observes the high trade volume. The quantity of energy futures at node 6 amounts to 82% of the expected spot quantities. One also observes that the contracts with the highest risk premium are also the most traded ones on the market. Those contracts are actually the riskiest in term of pay-off and the most effective for agent in order to hedge their profit.

|	P_c^f	$P_c^f - \mathbb{E}[P_c^s]$	Volume (MW)
FUTURE 6	53.5	0.45	564
FTR 1→6	28.9	0.58	462
FTR 2→6	27.4	0.55	696
FTR 3→6	25.53	0.55	359
FTR 4→6	7.59	0.46	242
FTR 5→6	1.57	-0.01	94

Table 3: Equilibrium prices, risk premium and volume of the derivatives contract

The benefits of the derivative contracts for the players can be seen on table 4. The volatility of the profit decreases considerably compared to the situation with the full exposition to the spot market (compare to table 2). The $\mathsf{CVaR}_{75\%}$ are closer to the expected profits. Forward positions dramatically reduce the risk exposure of all agents with only a small change in expected profit. This percussive impact of the derivatives for hedging is illustrated in the figure, by comparing the cumulative distribution function (CDF) of the spot profit $(\pi_{6,\omega}^{spot})$ and the profit after hedging $(\Pi_{6,\omega})$ for the retailer located at the node 6 (the hub).

These figures confirm results precedingly obtained by various authors. The current treatment adds to that literature by the introduction of a transmission market and the

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[13]The volume refers to the total quantity of MW sold/bought on the market.
[14]Cumulative distribution function of the profit : $\mathrm{cdf}(\pi_\nu) = \mathsf{Prob}(\Pi_\nu \leq \pi_\nu)$
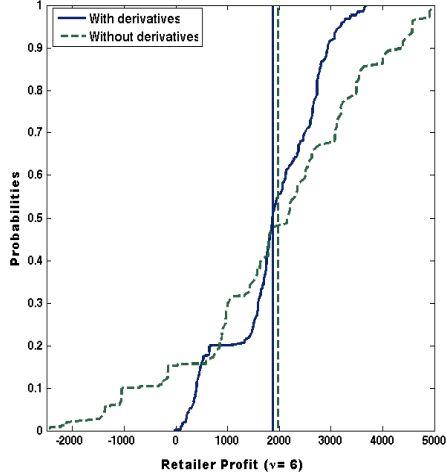
	$\mathbb{E}[\Pi_\nu]$	$\mathrm{vol}(\Pi_\nu)$	$\mathrm{CVaR}_{75\%}(\Pi_\nu)$
1	2198	1.8%	1517
2	1283	4.9%	1209
3	657	33%	329
4	349	17%	277
5	1602	49%	600
6	1890	48%	604
SO	11138	35%	6624

Table 4: Statistics of the market players total profit and cdf[14] for the retailer at node 6

use of a coherent and equivalent risk measure to guarantee the absence of arbitrage. This establishes a link between our approach and the more standard risk neutral valuation.

4. Insufficient Liquidity

To the best of our knowledge, the literature on contingent claim pricing in electricity does not quantitatively discuss the effect of illiquidity on derivative contracts. Similarly pricing models in that literature do not include illiquidity as a state variable. This is surprising as it is indeed sometimes noted that insufficient liquidity can play a crucial role in optimal management of a commodity portfolio (e.g. Geman and Ohana (2008)). Indeed, while prices are commonly taken as exogenous variables in that literature[15], the positions in the portfolio are constrained by bounds that reflect the illiquidity. These models implicitly assume that modifying the bounds to reflect illiquidity does not change prices. Also, and as mentioned before, the literature of equilibrium models commonly predicts high hedge ratio, even when agents are not very risk averse[16]. Our model, even though it uses a different risk function, concludes similarly (Table 5). The problem is

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[15] which assumption only holds surprisingly in very liquid market

[16] As mentioned previously, Bessembinder and Lemmon (2002) find an optimal hedge ratio varying from 0.8 to 1.2, depending on the market parameters. In Willems and Morbee (2008)'s computation, the total optimal number of futures goes up to 68GW when the expected demand is only 60 GW.

that volumes such as those predicted by equilibrium models have never been observed on any power exchange.

	1	2	3	4	5	6
Hedge ratio	0.92	0.82	0.26	0.63	0.6	1.4

Table 5: Market player's future position divided by his expected spot quantity

Illiquidity constrains the strategies of the agents and makes these hedge positions impossible : agents cannot hedge their profit as they desire. We account for these preoccupations by explicitly introducing constraints on hedging possibilities with the view of assessing their impact on prices. We do so by imposing an upper bound on the volume of each traded contract.

$$\sum_{\nu} |x_c^{\nu}| \leq L_c \tag{13}$$

These constraints are shared by all players. In game parlance this implies that the set of hedging strategies of a player is restricted by the hedging actions of the others(noted $\mathbf{x}^{-\nu}$). This suggests the following reformulation of the producers and retailers optimization problem as:

$$
\begin{aligned}
\mathcal{P}^{\nu}(\mathbf{x}^{-\nu}) \equiv \quad & \max_{x_c^{\nu}} \rho_{\nu}(\Pi_{\nu}) \\
\text{s.t.} \quad & \Pi_{\nu,\omega} = \sum_c x_c^{\nu}(P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^s \\
& |x_c^{\nu}| + \sum_{-\nu} |x_c^{-\nu}| \leq L_c \qquad (\lambda_c^{\nu})
\end{aligned}
\tag{14}
$$

We assume that only producers and retailers face those liquidity bounds. The SO has a different liquidity problem as it issues financial transmission rights subject to a feasibility constraint (N-1 rule) . The formulation of the SO problem remains as stated in (11).

The inclusion of these additional constraints transforms the problem into a Generalized Nash Equilibrium Problem. The goal is to find a hedge tuple $\mathbf{x}^* = (\mathbf{x}^{*,\nu})$ such that $\mathbf{x}^{*,\nu}$ solves the problem of maximizing the risk valuation for each agent taking the hedging strategies of the others as given. Any such tuple is called a Generalized Nash Equilibrium(GNE). It is well known (e.g. Harker (1991)) that General Nash Equilibrium Problems (GNEP) can be formulated as Quasi Variational Inequality problems(QVI).

These may have multiple or possibly infinitely many solutions. This lack of uniqueness is often interpreted as a serious difficulty that has limited the usefulness of the concept as GNE. For this reason it has often been criticized by economists as a plausible solution concept of a meaningful game. We take a quite different position and note that the multiplicity of solutions reflects a fundamental feature of a market affected by liquidity problems. Illiquidity is a market failure and the indeterminate outcome of the market is a consequence of that market failure. With this remark in mind and given our practical objective of illustrating the impact of the liquidity constraints on the equilibrium, we aim at finding a large set of GNEs in order to assess the type of inefficiency that illiquidity can lead to.

From a mathematical point of view, our model is a GNEP with shared constraints (Rosen (1965); Fukushima (2008)), meaning that the liquidity constraint bears on all market agents. This special class of problems has received increasing attention in recent years. Unlike the NEP, there are only few methods available to compute GNEP. Recently, Nabetani et al. (2008) introduced two algorithms based on parametrized VIs related to the GNEP which, under a mild constraint qualication, allow one to find all solutions of the GNEP. Fukushima (2008) also presents a new solution concept called restricted GNE and proposes a heuristic control penalty algorithm to find them. In this paper, we use the method presented in Nabetani et al. (2008) based on price-directed parametrization. We randomly sample on the players shadow prices in order to obtain different solutions. We compute up to 4000 equilibria for each case. The exact procedure is explained in appendix E.

4.1. Illiquidity and arbitrage

Arbitrage opportunities are more likely to persist over time in illiquid markets. These result from the difficulties confronted by arbitrageurs to exploit mispricing. For example, Deville and Riva (2007) show for the case of option markets that arbitrages are temporary but that the speed of reversion to the no arbitrage situation is critically impacted by liquidity-linked variables. Perfect liquidity and unconstrained portfolio formation are key hypothesis to sustain the fundamental no-arbitrage assumption that most asset pricing

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theories rely upon[17].

This paper suggests that modeling illiquidity by shared constraints on tradable volumes implies that the obtained equilibrium solutions may contain arbitrage opportunities, regardless of the risk valuation used to model agents risk aversion. Indeed, one can show that using a coherent and "equivalent" risk valuation, the optimality condition of an agent restricted by shared constraints on the positions takes the form.

$$P_c^f = \mathbb{E}_{\zeta_\nu^*}[P_c^s] + \lambda_c^\nu \tag{15}$$

The variable λ_c^ν is the shadow price associated with the illiquidity constraint. In a GNEP, this shadow price can differ by agents. These arbitrage opportunities may exist in equilibria when the volume constraints are tight and when all agents are not able to hedge up to the desired level (i.e. $\forall \nu, \lambda_c^\nu \neq 0$). No one can exploit the remaining arbitrage as the volume constraint is tight . This is the market failure induced by illiquidity.

4.2. Market simulations

We impose two liquidity constraints, one on the volume of energy futures, the other on the volume of FTRs. Those liquidity bounds can be justified by several factors. The peculiarities of transmission contracts is certainly an important one when it comes to FTRs. First this market is indeed organized through an auction which, because of the physical feasibility restriction imposed by the SO, limits liquidity[18]. Secondly the number of agents at each node is limited. Siddiqui et al. (2005) Adamson and Englander (2005) pointed the high risk premium of the FTRs and that the low liquidity of FTRs markets is one major explanation. For the case of Europe some power derivatives contracts are not purely financial and hence are by construction physically limited[19] .

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[17]Recently, the theory has been adapted to tackle assets which dynamics changes with the order balance. Çetin et al. (2004) extend the fundamental theorems of finance and show that the no-arbitrage condition still implies the existence of a risk neutral measure, but that the perfect hedging, even for complete market, does not hold anymore. One difficulty of applying the model is to quantify correctly the asset's dynamics with respect to the level of liquidity.

[18]There exist yet a secondary market, not regulated by the SO, where one can trade those contracts through bilateral contract.

[19]For example, PHELIX futures traded on the European Energy Exchange (EEX) a physical delivery. This is clearly a barrier for external speculators.

We set the liquidity constraints on energy futures as a fraction of the expected total power produced on the spot market. For the FTRs, this liquidity constraint is fixed to a share of the total point to hub capacities. For a first scenario (noted $LIQ_{66\%}$), we assume that the total of futures contracted at the hub does not exceed 66% of the expected total production and the FTR's volume is limited by the total capacity of the transmission lines (summed over all lines of the grid). For a second scenario (noted $LIQ_{33\%}$), those bounds are fixed to 33% and 50% respectively.

	$LIQ_{66\%}$		$LIQ_{33\%}$	
	P_c^f	Volume	P_c^f	Volume
FUTURE 6	$[51.2 , 53.7]$	$[33 , 457]$	$[50.1 , 54.3]$	$[17 , 228]$
FTR 1→6	$[26.8 , 29.1]$	$[34 , 307]$	$[25.4 , 29.2]$	$[0 , 28]$
FTR 2→6	$[25.2 , 27.5]$	$[2 , 262]$	$[23.8 , 27.6]$	$[0 , 232]$
FTR 3→6	$[23.3 , 25.8]$	$[21 , 196]$	$[21.9 , 26.2]$	$[0 , 26]$
FTR 4→6	$[5.5 , 7.6]$	$[125 , 140]$	$[4.5 , 7.8]$	$[0 , 179]$
FTR 5→6	$[1.4 , 2.1]$	$[75 , 136]$	$[0.9 , 3.2]$	$[0 , 25]$

Table 6: Computed intervals for equilibria prices and volume of derivatives contracts)

Table (6) shows the range of contract prices and volumes found at equilibrium. These ranges grow with the illiquidity sometimes leading to equilibria where the sign of the risk premium changes compared to the perfect liquid case. Tables (10) and (11) in appendix C show important statistics of the profit in the different equilibria. The agents profit distribution can be severely impacted by insufficient liquidity. As can be seen from the two scenarios studied, it may happen that each agent is excluded from the financial trading. This corresponds to the worst hedging situation. Also, as the liquidity falls, the intervals of the $\mathsf{CVaR}_{75\%}(\Pi)$ and $\mathsf{vol}(\Pi)$ (and obviously the volume) shrinks to a value closer to the spot. Figures of the profit's cumulative distribution function in table 12 indicates how the profit of retailer at the hub is impacted by the illiquidity.

4.3. Interdependence between transmission and energy markets

Surprisingly, the liquidity of the energy futures market is not always binding. Indeed, because of the limited number of FTRs, players can not really hedge profits, which essentially depend on the spot price at their home node. If they can not purchase those

FTRs, they have less incentive to take futures positions at the hub. The incredibly low volume of $17MW$ is achieved when all Northern players are unable to buy any FTRs.

This interdependence between the energy and transmission markets is revealed in a

Volume FTRs	Volume Futures	P_c^f (FUTURE 6)
960	570 (92%)[20]	$\left[52.9, 53.6\right]$
720	478 (69%)	$\left[52.9, 53.7\right]$
480	320 (56%)	$\left[52.9, 53.8\right]$
240	269 (39%)	$\left[53.5, 54.0\right]$
0	152 (22%)	54.8

Table 7: Induced energy futures volume for a given liquidity bounds on FTRs

striking way in Table 7. It shows the maximum (over all computed GNEs) volume of energy futures as a function of the bound (illiquidity) of FTRs. One clearly sees the reduced incentive of agents to enter energy futures position when the illiquidity of the FTR market increases. In the extreme situation, when no transmission market exists, the volume of energy futures drops to 152 MW. One also notes that the maximal energy futures price tends to be higher. This reflects the fact that producers, not located at the hub, demand higher expected returns on the energy future, as they can less perfectly hedge their profit (which depends highly on the congestion costs because of the lack of FTRs).

5. Conclusions

In this paper, we have studied the impact of insufficient liquidity on the pricing through an equilibrium based model. The problem is formulated as a Generalized Nash Equilibrium problem and the solution is not unique. Computing a large panel of solutions, we show that insufficient liquidity can dramatically impact the agents profit distribution and that the risk premium may be more important in illiquid market. We show that equilibrium models without speculation may have residual arbitrage opportunities and identify two reasons why this is so. One is related to the notion of coherent risk valuation.

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[20]The percentage of volume with respect to the expected quantities contracted in the spot market

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Modeling risk aversion through non coherent and equivalent risk valuation may lead to arbitrage opportunities. These can be eliminated by allowing speculators in the market. The second one is intrinsic to illiquidity and corresponds a market failure. Lack of liquidity may create arbitrage opportunities. Through this paper, we rely on a definition of illiquidity based on the volume. We leave to future research to explore the impact of other measures such as bid-ask spread.

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A. Complementary figures and tables

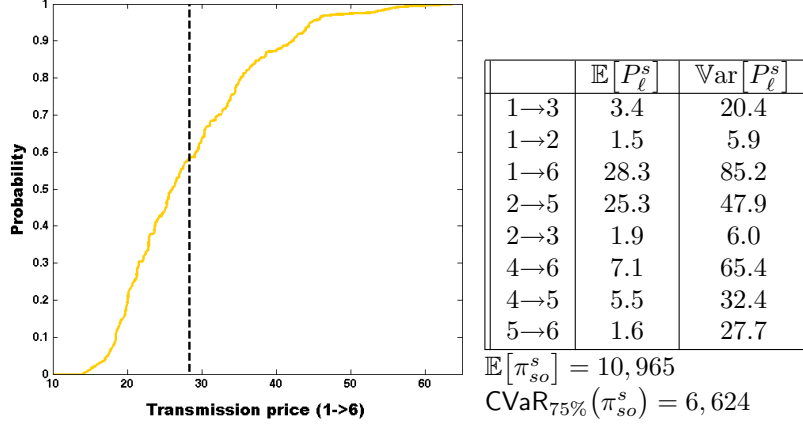| Category	Oil	Natural Gas	Coal	Electricity
Physical Suppliers	7730	7730	1990	1197
Physical Wholesale Buyers	4836 + 3216[21]	2376	313	72 + 7325[20]
Volume (Physical)	7.6B barrels ($22B)	22B MMBtu ($152B)	1.1B short tons ($22 B)	3.8B MWh ($152 B)
Price Volatility	11%	29%	6%	66%

Table 8: Commodity Comparison (from PJM (2007))

---

[21]non-bulk data

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	$\mathbb{E}\left[P_\ell^s\right]$	$\mathbb{V}\mathrm{ar}\left[P_\ell^s\right]$
1→3	3.4	20.4
1→2	1.5	5.9
1→6	28.3	85.2
2→5	25.3	47.9
2→3	1.9	6.0
4→6	7.1	65.4
4→5	5.5	32.4
5→6	1.6	27.7

$\mathbb{E}\left[\pi_{so}^s\right] = 10,965$

$\mathsf{CVaR}_{75\%}\left(\pi_{so}^s\right) = 6,624$

Table 9: Statistics of the transmission spot prices and cumulative distribution function for 1→6

	$\mathbb{E}\left[\Pi_\nu\right]$	$\mathsf{vol}\left(\Pi_\nu\right)$	$\mathsf{CVaR}_{75\%}\left(\Pi_\nu\right)$	Volume
1	[2116 , 2226]	[1.8% , 22%]	[1509 , 2160]	[1 , 712]
2	[1220 , 1315]	[3.1% , 37%]	[697 , 1260]	[0 , 551]
3	[628 , 743]	[27% , 75%]	[−83 , 444]	[0 , 656]
4	[334 , 441]	[14% , 79%]	[−33 , 316]	[17, 363]
5	[1583 , 1814]	[44% , 76%]	[78 , 711]	[0, 590]
6	[1860 , 2502]	[41% , 48%]	[599 , 1003]	[113, 677]

Table 10: Producers and retailers profits for LIQ$_{66\%}$

	$\mathbb{E}\left[\Pi_\nu\right]$	$\mathsf{vol}\left(\Pi_\nu\right)$	$\mathsf{CVaR}_{75\%}\left(\Pi_\nu\right)$	Volume
1	[2197 , 2960]	[5% , 24%]	[1480 , 2775]	[0 , 546]
2	[1300 , 1973]	[2% , 48%]	[614 , 1905]	[0 , 485]
3	[652 , 1173]	[37% , 76%]	[−83 , 418]	[0 , 198]
4	[332 , 921]	[13% , 250%]	[−145 , 370]	[0, 426]
5	[1577 , 2210]	[42% , 76%]	[78 , 838]	[0, 571]
6	[1853 , 2655]	[38% , 87%]	[−308 , 1254]	[0, 541]

Table 11: Producers and retailers profits for LIQ$_{33\%}$
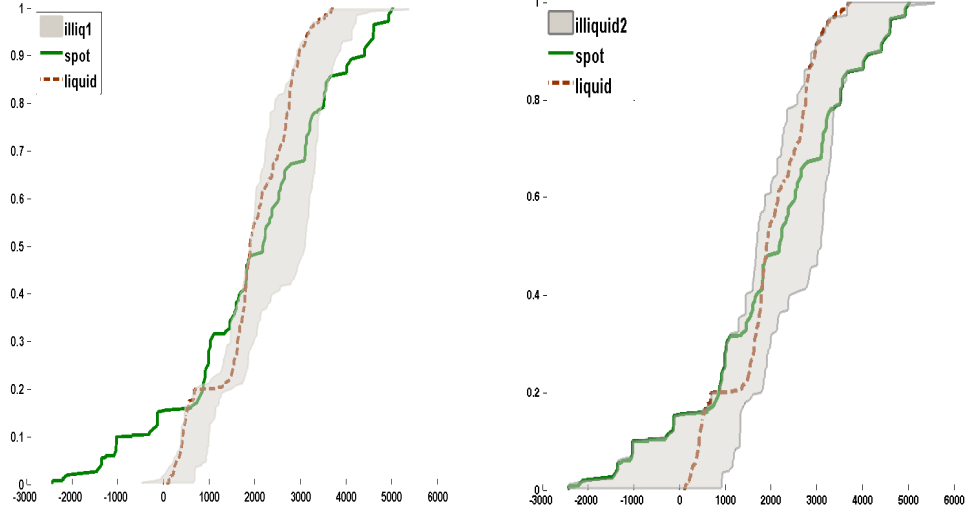
Table 12: Range for the cumulative distribution function of the profit of for the retailer located at the hub (left : $\text{LIQ}_{66\%}$, right: $\text{LIQ}_{33\%}$)

B. Risk-valuation and coherence

Let $(\Omega, \mathcal{F}, P)$ be a probability space (equipped with a sigma algebra $\mathcal{F}$ and a probability measure P), $\mathcal{Z} := \mathcal{L}_p(\Omega, \mathcal{F}, P)$ be the set of all $\mathcal{F}$-measurable functions Z such that $\int_\Omega |Z(\omega)|^p dP(\omega) < \infty$. A risk valuation is a function $\rho(Z)$ which maps $\mathcal{Z}$ into $\bar{\mathbb{R}}$. In the context of maximization of a risk measure of an random reward , it is said that a risk valuation is coherent if it satisfies the following axioms.

- *Concavity:* $\rho(tZ_1 + (1-t)Z_2) \geq t\rho(Z_1) + (1-t)\rho(Z_2) \qquad \forall Z_1, Z_2 \in \mathcal{Z}, \forall t \in [0,1]$

- *Monotonicity:* If $Z_1, Z_2 \in \mathcal{Z}$ and $Z_1 \succeq Z_2$, then $\rho(Z_1) \geq \rho(Z_2)$

- *Translation equivariance:* If $a \in \mathbb{R}$ and $Z \in \mathcal{Z}$, then $\rho(Z + a) = \rho(Z) + a$

- *Positive homogeneity:* If $t > 0$ and $Z \in \mathcal{Z}$, then $\rho(tZ) = t\rho(Z)$

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C. Formulation and computation of the future equilibrium without liquidity constraints

Following Rockafellar and Uryasev (2002) who show how the CVaR could be cast in an optimization form, the problem of an agent maximizing a E-CVaR can be formulated as:

$$\mathcal{P}^{\nu} \equiv \max_{t_{\nu}, x_c^{\nu}, \mathbf{U}_{\nu,\omega}} \left\{ \beta_{\nu} t_{\nu} + \sum_{\omega} p_{\omega} \left((1 - \beta_{\nu}) \mathbf{\Pi}_{\nu,\omega} - \beta_{\nu} \alpha_{\nu}^{-1} \mathbf{U}_{\nu,\omega} \right) \right\}$$

$$\tag{16}$$

$$\text{s.t:} \quad \mathbf{U}_{\nu,\omega} \geq 0$$

$$\mathbf{U}_{\nu,\omega} \geq t_{\nu} - \mathbf{\Pi}_{\nu,\omega} \qquad (\phi_{\omega}^{\nu})$$

$$\mathbf{\Pi}_{\nu,\omega} = \sum_c x_c^{\nu} (P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{spot}$$

Applying the standard duality theory, one can convert the utility optimization problem of the agents into the following optimality conditions. Equations 17-19 define the optimal value of the state prices $\zeta_{\nu,\omega}$ for all agents (i.e. retailers, producers and SO).

$$0 \leq \mathbf{U}_{\nu,\omega} \quad \perp \quad \beta_{\nu} \alpha_{\nu}^{-1} + (1 - \beta_{\nu}) - \zeta_{\nu,\omega} \geq 0 \tag{17}$$

$$0 \leq \zeta_{\nu,\omega} - (1 - \beta_{\nu}) \quad \perp \quad \mathbf{U}_{\omega}^{\nu} - t_{\nu} + \sum_c x_c^{\nu} (P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{spot} \geq 0 \tag{18}$$

$$\sum_{\omega} \zeta_{\nu,\omega} = 1 \tag{19}$$

Equations 20-21 impose that the SO's forward strategy satisfies the rule of N-1.

$$0 \leq \mu_{\ell}^+ \quad \perp \quad K_{\ell} - \sum_f \text{PTDF}_{c,\ell}^{N-1} x_c^{so} \geq 0 \tag{20}$$

$$0 \leq \mu_{\ell}^- \quad \perp \quad \sum_f \text{PTDF}_{c,\ell}^{N-1} x_c^{so} - K_{\ell} \geq 0 \tag{21}$$

Finally,the following equations gives the price of the contract at equilibrium and the market clearing condition.

$$P_c^f = \sum_{\omega} (p_{\omega} \zeta_{\nu,\omega}) P_{c,\omega}^s \tag{22}$$

$$P_c^f = \sum_{\omega} (p_{\omega} \zeta_{so,\omega}) P_{c,\omega}^f + \sum_{\ell} (\mu_{\ell}^- - \mu_{\ell}^+) \text{PTDF}_{c,\ell}^{N-1} \tag{23}$$

$$\sum_{\nu} x_c^{\nu} = 0 \tag{24}$$

One can see from complementarity conditions (18) that the resulting Nash Equilibrium Problem is non convex. We propose an heuristic to compute it by a sequential joint maximization method. Suppose that the equilibrium prices $\mathbf{P}_c^f$ are known. One can compute the optimal strategies by solving the following linear problem (where $\mathbf{P}_c^f$ are fixed).

$$
\begin{aligned}
\mathcal{E}_{lp} := \quad \max_{\nu \in \{N_r, N_p, SO\}} & \left\{ \beta_\nu \mathbf{t}_\nu + \sum_\omega p_\omega \left((1 - \beta_\nu)\mathbf{\Pi}_{\nu,\omega} - \beta_\nu \alpha_\nu^{-1} \mathbf{U}_{\nu,\omega} \right) \right\} \\
& \mathbf{U}_{\nu,\omega} \geq 0 \\
& \mathbf{U}_{\nu,\omega} \geq \mathbf{t}_\nu - \mathbf{\Pi}_{\nu,\omega} \\
& \mathbf{\Pi}_{\nu,\omega} = \sum_c x_c^\nu (P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{spot} \\
& -K_\ell \leq \sum_c \mathtt{PTDF}_{c,\ell}^{N-1} \, x_c^{so} \leq K_\ell & (\mu_\ell^-, \mu_\ell^+) \\
& \sum_\nu x_c^\nu = 0 & (\eta_c)
\end{aligned}
$$

The Karush-Kuhn-Tucker optimality conditions of $\mathcal{E}_{lp}$ are similar to the original NEP except that P_c^f is replaced by $\mathbf{P}_c^f$ and that equality (22) and (23) are changed to:

$$
\mathbf{P}_c^f + \eta_c = \sum_\omega (p_\omega \, \zeta_{\nu,\omega}) P_{c,\omega}^s \tag{25}
$$

$$
\mathbf{P}_c^f + \eta_c = \sum_\omega (p_\omega \, \zeta_{so,\omega}) P_{c,\omega}^s + \sum_\ell (\mu_\ell^- - \mu_\ell^+) \mathtt{PTDF}_{c,\ell}^{N-1} \tag{26}
$$

One see that when the dual variables η_c are all equal to zero (i.e. given the derivatives prices, no agent have incentive to modify its portfolio), then the solution of $\mathcal{E}_{lp}$ is also solution of the Nash Equilibrium. This lead to the following heuristic algorithm.

Require: $\delta > 0$, $\mathbf{P}_c^f \in \mathbb{R}^c$
1: **while** $\epsilon \geq \delta$ **do**
2:     Solve $\mathcal{E}_{lp}$ using lp
3:     $\mathbf{P}_c^f \leftarrow \mathbf{P}_c^f + \eta_c$
4:     $\epsilon = ||\eta_c||$
5: **end while**

D. Non-coherent risk measure

While mean-variance have been probably the most used risk function for modeling risk aversion, it is not coherent and, in the context of pricing, it may lead to solution

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with arbitrage opportunities. This is highlighted by the following example. Consider a market with 2 goods. The price of those goods are denoted (p_1^s, p_2^s), which depend on the state s of the world. We consider 3 possible state, each having a probability ϕ_s to occur. There is a financial market where futures contract on those goods are traded. Their futures prices are (P_1^f, P_2^f).

	$s = sc1$	$s = sc2$	$s = sc3$
ϕ_s	0.3	0.3	0.4
p_1^s	1	1	2
p_2^s	1	2	3.5

There are 2 agents yielding a stochastic profit (π_a^s, π_b^s) depending on s. The profit of agent a is positively affected by good 1 and negatively by good 2. The profit of agent b is inversely impacted.

	$s = sc1$	$s = sc2$	$s = sc3$
π_a^s	45	31.67	50
π_b^s	30	110	50

$\mathbb{Cov}(.,.)$	p_1^s	p_2^s
π_a^s	2.8	3.6
π_b^s	-4.8	2.4

Agents are risk averse and their utility is modeled by a mean-variance as risk measure with a parameter $\alpha = 0.1$.

$$\rho(\Pi) = \mathbb{E}[\Pi] - \alpha \mathbb{Var}[\Pi] \tag{27}$$

They trade future in order to maximize this risk measure. Intuitively, a, giving his stochastic profit, have incentive to sell future 1 and buy future 2 and inversely for agent b.

The equilibrium conditions are, as derived in Bessembinder and Lemmon (2002):

$$P^f = \mathbb{E}[p_s] + \frac{\alpha_a \alpha_b}{\alpha_a + \alpha_b} \sum_{i=a,b} \mathbb{Cov}(\pi_i^s, p_s) \tag{28}$$

$$x_i^f = \frac{\Sigma^{-1}}{\alpha_i}(p^f - \mathbb{E}[p_s]) - \Sigma^{-1}\mathbb{Cov}(\pi_i^s, p_s) \tag{29}$$

The equilibrium solution are reported in the next Table. One can see that this solution

contains arbitrage opportunities[22]. Indeed, there exist no equivalent risk-neutral probability measure. Graphically, futures prices do not belong strictly to the convex hull of spot prices.

		Future 1	Future 2
P^f		1.5	2
x_a^f		109.1	-46.6
x_b^f		-109.1	46.6

<hr/>
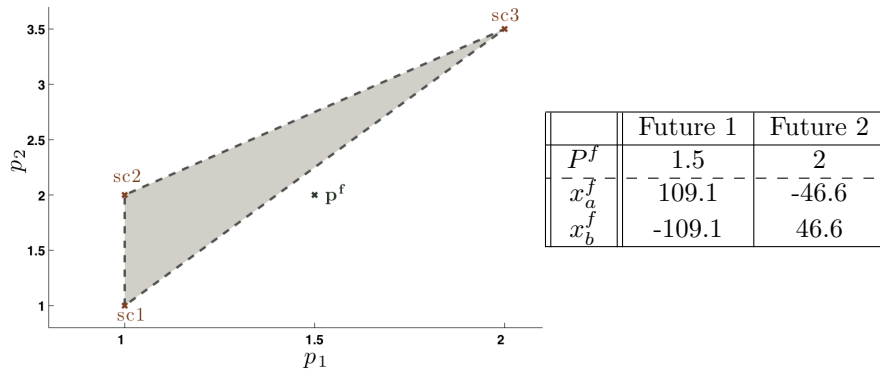
[22]Numerically, the strategy $q^f = (-1, 0.5)$ is an arbitrage opportunity. Notice that, as financial contracts are futures, the payments are due at maturity.

E. Formulation and computation of the future equilibrium with liquidity constraints

Choosing a E-CVaR, the problem of agent's risk measure maximization, with shared volume constraints, becomes:

$$\mathcal{P}^\nu(\mathbf{x}^{-\nu}) \equiv \max_{\mathbf{t}_\nu, x_c^\nu, \mathbf{U}_{\nu,\omega}} \left\{ \beta_\nu \mathbf{t}_\nu + \sum_\omega p_\omega \left((1 - \beta_\nu) \mathbf{\Pi}_{\nu,\omega} - \beta_\nu \alpha_\nu^{-1} \mathbf{U}_{\nu,\omega} \right) \right\}$$

$$
\begin{aligned}
\text{s.t:} \quad & \mathbf{U}_{\nu,\omega} \geq 0 \\
& \mathbf{U}_{\nu,\omega} \geq \mathbf{t}_\nu - \mathbf{\Pi}_{\nu,\omega} && (30) \\
& \mathbf{\Pi}_{\nu,\omega} = \sum_c x_c^\nu (P_c^f - P_{c,\omega}^s) + \pi_{\nu,\omega}^{spot} \\
& x_c^\nu \leq y_c^\nu && (\gamma_{c,\nu}^+) \\
& -x_c^\nu \leq y_c^\nu && (\gamma_{c,\nu}^-) \\
& y_c^\nu + \sum_{-\nu} y_c^{-\nu} \leq L_c && (\lambda_c^\nu)
\end{aligned}
$$

The complementary conditions of this problem are quite similar to the previous problem. Equations 17 - 21 stay the same. One have to add the condition for the volume constraints.

$$0 \leq \gamma_{c,\nu}^+ \quad \perp \quad y_c^\nu - x_c^\nu \geq 0 \tag{31}$$

$$0 \leq \gamma_{c,\nu}^- \quad \perp \quad y_c^\nu + x_c^\nu \geq 0 \tag{32}$$

$$0 \leq \lambda_c^\nu \quad \perp \quad L_c - y_c^\nu - \sum_{-\nu} y_c^{-\nu} \geq 0 \tag{33}$$

$$\gamma_{c,\nu}^+ + \gamma_{c,\nu}^- = \lambda_c^\nu \tag{34}$$

The equilibrium derivative prices are also changed into :

$$P_c^f = \sum_\omega (p_\omega \; \zeta_{\nu,\omega}) P_{c,\omega}^s + \gamma_{c,\nu}^+ - \gamma_{c,\nu}^- \tag{35}$$

As the shared constraints are separable, we compute the different GNE using the parametrized Variational Inequality approaches described in Nabetani et al. (2008). We construct a family of VIs that contains all the equilibria of the initial GNEP. We perturb the initial objective function of agent by penalizing it by its financial volume with a positive weight n^ν:

$$\tilde{\rho}_\nu(x_c^\nu, y_c^\nu) = \rho_\nu(x^\nu) - \sum_c n_c^\nu \, y_c^\nu \tag{36}$$

We then compute the associate NEP, using the heuristic developed in appendix C. According to Nabetani et al. (2008), the equilibrium found is a solution of the initial GNEP if:

$$\sum_{\nu,c} n_c^\nu (L_c - \sum_\nu y_c^\nu) = 0 \tag{37}$$

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