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Design of a network of reusable logistic containers

Jean-Chales Lange and Pierre Semal



**CORE**

The word "CORE" is written in a bold, black, sans-serif font. A thin, light blue arc curves over the letters, starting from the top left of the 'C' and ending at the top right of the 'E'.

DISCUSSION PAPER

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**Design of a network of reusable logistic containers**

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**Abstract**

In this paper, we consider the management of the return flows of empty logistic containers that accumulate at the customer's sites. These containers must be brought back to the factories in order to sustain future expeditions. We consider a network composed of several factories and several customers in which the return flows are independent of the delivery flows. The models and their solutions aim at finding to which factory the containers have to be brought back and at which frequency. These frequencies directly define the volume of logistic containers to hold in the network. We consider fixed transportation costs depending on the locations of the customers and of the factories and linear holding costs for the inventory of logistic containers. The analysis also provides insight on the benefit of pooling the containers among different customers and/or factories.

**Keywords:** supply chain management, returnable items, reverse logistic, economic order quantity, network design.

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# 1 Introduction

In this paper, we consider the management of the return flows of empty reusable logistic containers, i.e. any kind of items used to store and send goods (e.g. pallets, kegs, barrels, trestles,...) that must be brought back to the producers for further expeditions. This problem is directly inspired by the distribution of glass. In this real case, the glass is shipped to the customer with special trucks and in special containers. At the customer site, the glass is used according to the local demand. Once the customer has emptied a container, this container can be folded. Later on, the folded containers are brought back, with regular trucks, to the factories. In this example, the return flows are managed independently from the delivery flows because of the differences in the timing of the flows and in the transportation modes (truck specificities). In this paper, we also consider the return flows independently from the delivery flows. The management of the return flows raises several questions like

Q1: to which factory should each customer return the containers?

Q2: at which frequency should they be returned ?

Q3: how many containers are needed in the network?

These questions are interdependent. The link between Q3 and Q2 is straightforward. For example, if we have few containers (Q3) then it is clear that they must rotate quickly. This means that the returns will be, at least on short distances, more frequent (Q2). The dependence with Q1 is more complex. But we will show that, given a total inventory in the system (Q3), the optimal allocation of customers to factories varies with the capacity constraint on the return flows, if any. However, we will also show that in some cases the question (Q1) can be solved independently of (Q2) and (Q3).

This problem is, at the same time, a transportation problem (Q1) and an inventory control problem (Q2 and Q3). The objective is to minimize the costs. Here we consider holding costs for the containers and transportation costs for bringing back the containers. The transportation costs are supposed to be independent from the transported quantity. However, a capacity limit may be imposed on the transport. We only consider point-to-point transport and here discard milkruns.

In this paper, the problem is studied in a strategic perspective. Such a perspective will allow us to compare the potential benefits associated with different transportation capacity limits, with different holding cost rates. It

will also allow us to quantify the benefit of merging different networks by pooling the containers among different customers and/or different factories.

The paper is structured as follows. In Section 2, a detailed model of the problem is suggested with a clear identification of the problem variables, constraints and assumptions. In section 3, the relevant literature is reviewed and our contribution is more precisely defined. The solutions of the problem for increasingly complex cases are then detailed in Sections 4 to 7. Section 4 focuses on the “1-factory 1-customer” case while Section 5 deals with the “1-factory C-customer” case. Since none of these cases raises the issue of allocating customers to factories, we can simply focus on the analysis of the optimal inventory of containers and on the optimal return frequencies, that is, Questions Q2 and Q3. Not only can these two questions be analytically solved but the benefit of pooling the containers among all the customers can be quantified. The benefit is similar in the symmetric “F-factory 1-customer” case which is quickly analyzed in Section 6. Finally, Section 7 deals with the solution of the general “F-factory C-customer” case. We show there under which circumstances question Q1 may (or may not) be solved independently of Q2 and Q3. In each of these sections, we also consider the impact of a potential capacity constraint on transportation. Finally, we will conclude the paper by underlining the main contributions of the study and by discussing a plan for future work.

## 2 The Model

Our problem can be modeled in its general form as follows:

$$\min_{(L, \lambda_c^f, Q_c^f)} LHT + \sum_{c \in C, f \in F} \frac{\lambda_c^f TO_c^f}{Q_c^f} \quad (2.1)$$

such that

$$\sum_{c \in C} \lambda_c^f = \Gamma^f \quad \forall f \quad (2.2)$$

$$\sum_{f \in F} \lambda_c^f = \Lambda_c \quad \forall c \quad (2.3)$$

$$L = \sum_{c \in C, f \in F} \frac{Q_c^f \lambda_c^f}{2\Lambda_c} + \sum_{c \in C, f \in F} \frac{Q_c^f \lambda_c^f}{2\Gamma^f} \quad (2.4)$$

$$Q_c^f \leq CAP \quad \forall c, f \quad (2.5)$$

Our decision variables are:  $L$ , the total inventory of containers also called the “Logistic inventory”;  $\lambda_c^f$ , the rate at which customer  $c$  returns the containers to the factory  $f$  and  $Q_c^f$  the lot size used by customer  $c$  to return the containers to the factory  $f$ . The problem with a fixed logistic inventory  $L$  instead of a variable one will also be studied. Note that  $C$  and  $F$  represent the set of customers and of factories, respectively.

The objective is to minimize, over time  $T$ , the sum of the holding cost and the transportation cost. The holding cost over time  $T$  is  $LHT$  where  $H$  is the holding cost rate per container. For the transportation, the customer  $c$  will organize, over time  $T$ ,  $(\lambda_c^f T/Q_c^f)$  transports to the factory  $f$ , each at the fixed cost  $O_c^f$ . This leads to the total transportation cost. Note that since  $T$  defines the period over which the cost is computed, we can set it equal to 1 without loss of generality.

The two first constraints express the demand and the supply. A factory  $f$  requires for its further expeditions  $\Gamma^f$  containers per time unit. Constraint (2.2) expresses that this demand must be exactly met by the sum of the contributions of the customers to this factory  $f$ . Similarly, each customer  $c$  releases containers at the global rate  $\Lambda_c$ . Constraint (2.3) expresses that this global supply must be partitioned among the different factories.

The third constraint (2.4) is more complex. It expresses that the total number of containers in the system  $L$  is given by the sum of the average numbers of containers at each player of the system.

Let us consider a customer  $c$ . Containers are accumulated at the speed  $\Lambda_c$  and then shipped to a given factory  $f$  with a lot size equal to  $Q_c^f$ . The quantity  $(\lambda_c^f/\Lambda_c)$  represents the portion of time customer  $c$  is accumulating containers for factory  $f$ . During this portion of time, the inventory varies from 0 to  $Q_c^f$ . The average inventory during this time is  $Q_c^f/2$ . The average inventory at customer  $c$  is therefore

$$\sum_{f \in F} \frac{Q_c^f \lambda_c^f}{2 \Lambda_c}$$

The first term of the right hand side of Equation (2.4) thus expresses the sum of the average inventory over all the customer sites.

We hold a similar reasoning for the factories. Let us assume a factory  $f$  has no containers left and just receives the lot size  $Q_c^f$  sent by customer  $c$ . It

will consume the received containers at the speed  $\Gamma^f$ . During this time, its average inventory will be  $Q_c^f/2$ . Since, the quantity  $(\lambda_c^f/\Gamma^f)$  represents the portion of the time the factory  $f$  lives from the containers sent by customer  $c$ , the average inventory at factory  $f$  is given by

$$\sum_{c \in C} \frac{Q_c^f \lambda_c^f}{2 \Gamma^f}$$

By summing up all the factories we obtain the total average inventory on the factory sites. This is the last term of Equation (2.4).

The crucial assumption in the model is that we assume “perfect coordination”. That is, a customer  $c$  sends its containers to factory  $f$  when it has reached its lot size  $Q_c^f$ . On the other side, the factory  $f$  receives this lot size  $Q_c^f$  exactly when needed, that is, when its inventory has just dropped to zero. This “perfect coordination” is discussed in more details in Section 5. This assumption leads to the fact that the suggested solutions are optimistic and they are sometimes referred to as lower bounds. At an operational level, a special attention should thus be given to the issue of coordination. However, at the strategic level, since we aim at comparing different solution schemes and at drawing lessons from these comparisons, this assumption presents the advantage of keeping the problem tractable without introducing obvious biases.

Finally, constraint (2.5) expresses the fact that there could be some limit on the transport capacity. In this paper, we systematically address our problem with and without this constraint.

### 3 Literature Review and Detailed Contribution

The “1-factory 1-customer” case is shown to be an obvious EOQ problem. The relevant literature dates back from Harris (1915). In the “1-factory C-customer” case, all the customers send their containers back to the same factory. The problem reduces to lot-sizing decisions based on linear holding cost and fixed transportation costs that are intertwined with a global inventory constraint or variable  $L$  that creates dependence between the different lot sizes. Although this problem still looks simple, the difficulty comes from the pooling of the containers at the factory. Indeed, it appears that if we aim at minimizing the total inventory at the factory, then the arrivals of the

customer shipments must be coordinated. On the other hand, each customer has its own ideal lot size and therefore its own ideal shipment frequency. The problem thus consists in the reconciliation of the customer shipment frequencies with the arrival coordination at the factory. The only related literature we found is that discussing the management of warehouse capacity. In this problem, a warehouse with limited capacity manages an inventory of several items which compete for that capacity. This problem known as the *Single Resource Multi-Item Inventory System* (SRMIS) or *Warehouse Scheduling Problem* (WSP) has some strong similarities with ours: every replenishment has a fixed cost (every transportation in our problem has a fixed cost); the demand faced by the warehouse has to be satisfied (the demand  $\Gamma^f$  at our factory must be satisfied); there is a lot sizing issue for the replenishment orders (we have a similar one for the transportation) and finally the warehouse does not want to receive all the replenishments at the same time for capacity reasons. Although our “1-factory C-customer” problem is different, it leads to similar problems, models and solution methods. For the interested readers, we review this literature in the next subsection.

Since the formulation of our “1-factory C-customer” problem leads to models similar to those analyzed in the (SRMIS) literature, our contribution does not lie in the solution methods. It lies here in the modeling of our problem and in the managerial interpretation of the results. Indeed, the benefit of pooling the containers among the customers (or among factories in the “F-factory 1-customer” case) can be clearly interpreted and quantified.

For the general “F-factory C-customer” case, no directly relevant literature has been found. Here, we develop the models and the solution methods. The problem presents lot sizing and customer allocation decisions. Our solution methods rely on a decomposition into 2 problems. A first problem aim at finding the optimal lot sizes while a second problem tackles the allocation of the customer. We derive the managerial implications of this case and, among others, analyze the impact of pooling in this context.

It is worth mentioning some recent work by Huang et al. (2005) who have also dealt with an extension of EOQ-like models with multiple factories and multiple customers. In this problem, lot sizing and customer allocation decisions must also be taken. The objective function is, however, different. It includes variable transportation costs and the main focus is on production cost. Batch production campaigns have a fixed setup cost so that the allocation of customers to a factory would reduce the amount of fixed cost to be paid. In this problem, all items are produced at the same frequency at a factory

and the setup cost is paid for the production of the batches of all items at the factory. The solution method considers the same type of decomposition with a lot sizing and an allocation problem. The resulting problems are, however different, since the cost structure and the assumptions used are different. For example, the problem of pooling and the perfect coordination assumption have no equivalent in the model.

### 3.1 SRMIS and WSP literature

The problem of deciding on how much and when to replenish a capacitated warehouse with multi-item has been analyzed in a continuous timeline fashion and as a discrete time problem. If the timeline is continuous, the problem is referred to as the *Single Resource Multi-Item Inventory System* (SRMIS). In its discrete time form, it is known as the *Warehouse Scheduling Problem* (WSP).

The continuous problem is an extension to the classical EOQ model dating from Harris (1915) and can be divided into 2 groups of problems. The first group was referred to as the *strategic SRMIS* by Gallego et al. (1996). In the strategic problem, the capacity limit is a decision variable and a third cost, the capacity cost, is linked with this capacity which must be chosen as to accommodate the maximum total stock volume held at the warehouse at any time. The *tactical SRMIS* integrates the capacity limit of the warehouse as a constraint. In both models, the literature has focused on two main elements: deriving lower bounds on the total cost or on the maximum total stock volume and designing heuristics to find feasible solutions.

In both problems, the heuristics basically face the same challenge : there are quantities which are individually optimal for each item with respect to the holding and the transportation costs, but the coordination of the replenishments has an impact on the maximum total inventory volume, and, consequently, on the capacity. It is obvious that, in the worst case, if no effort is made on coordination, all replenishments could arrive at the same time. It could be preferable to design a coordination pattern where the replenishments are spread over time, in order to limit the maximum total cost volume. This coordination has been referred to as the *staggering problem* in the literature , which has been shown to be NP-complete by Gallego et al. (1992). Lower bounds were given by Anily (1991) and Gallego et al. (1996).

Several families of approaches have been proposed. A first family of solution procedures for the problem actually neglect the staggering problem,



and consequently must consider that the maximum total stock volume is the sum of stationary replenishment quantities. These procedures are referred to in the literature as *Independent Solutions* (IS), and were first referenced in Churchman et al. (1957), Holt (1958), Hadley and Whithin (1963) and Parsons (1966).

The policies regrouped under the name *Rotation Cycle Policies* (RC) aims at finding a common replenishment frequency  $t$  to all items. Once  $t$  has been determined, the staggering of the orders is chosen in order to minimize the maximum total stock volume during the cycle. Examples of such approaches can be found in the works of Goyal (1974), Homer (1966), Krone (1964), Maxwell (1964), Parsons (1965), Silver (1976), Zoller (1977) and Hall (1988).

Another family of procedures rely on a stationary frequency for each item, i.e. each item is always replenished in the same quantity. This family is named *Stationary Order Sizes and Intervals* (SOSI). The paper by Gallego et al. (1996) suggests a heuristic for the staggering problem for SOSI policies. Rosenblatt (1981) compares the two approaches. Haksever and Moussourakis (2005) suggest a MIP formulation for finding SOSI policy with one or several constraints.

The WSP is the discrete version of the SRMIS. The literature has also been dominated by heuristics. Many of the heuristics proposed for the SRMIS are easily extended to the WSP. Without entering in details, let us mention some major contributions for the WSP: Gallego et al. (1996), Hariga and Jackson (1996), Günther (1990), Dixon and Poh (1990), Page and Paul (1976) and Axsäter (1980).

## 4 1-Factory and 1-Customer

The simplest case arises when a single factory deals with a single customer. The customer accumulates empty units at a constant rate while the factory consumes empty units at the same rate. Referring to our model notation, we have here:

$$\begin{aligned}\lambda_1^1 &= \Lambda = \Gamma; \\ Q_1^1 &= Q = L\end{aligned}$$

And the problem reduces to a classical economic order quantity problem with the following cost function :

$$\min \frac{Q}{2}H + \frac{Q}{2}H + \frac{\Lambda}{Q}O$$

where the first term is the holding cost linked with the average inventory at the customer, the second term is the holding cost linked with the average inventory at the factory and the third term of the sum is the transportation cost associated with  $Q$ . The holding cost grows linearly with the lot size, while the transportation cost decreases as the lot size increases. Figure 1 shows the classical inventory evolution at the customer and at the factory, while Figure 2 depicts the evolution of the total logistic cost with the lot size. The optimal solution  $Q^*$  is found by setting the first derivative of the

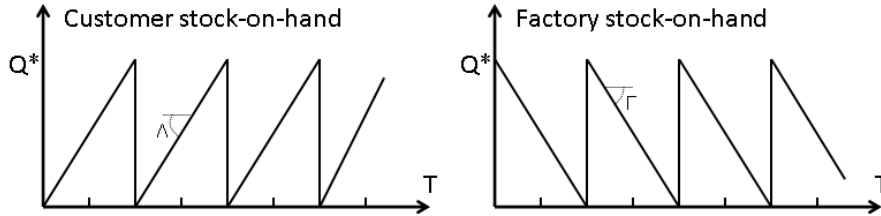


Figure 1: Inventory of empty logistic containers at the customer and at the factory.

objective function to zero:

$$Q^* = \sqrt{\frac{\Lambda O}{H}}$$

This quantity differs from the classical economic order quantity by a factor  $\sqrt{2}$ . This is explained by the fact that we consider holding costs for the inventory at the customer and at the factory. Otherwise, this quantity shares similar characteristics with the classical EOQ, among others, the equality of the holding and transportation costs and the relatively flat slope around the optimal quantity. This leads to a total logistic cost of

$$V(Q^*) = 2\sqrt{\Lambda O H}$$

If there is a constraint on the transported quantities, then the optimal solution becomes

$$Q^* = \min\{\sqrt{\Lambda O / H}, CAP\}$$

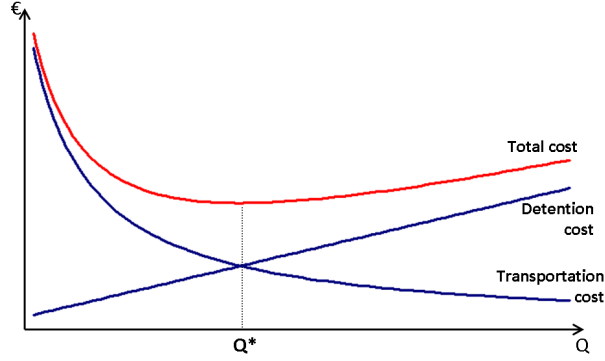


Figure 2: Cost structure and optimal lot size.

## 5 1-Factory and C-Customer

In this section, we consider a single factory having a set of  $C$  customers. Each customer produces empty containers at a rate  $\Lambda_c$  and the factory consumes them at a rate  $\Gamma = \sum_{c \in C} \Lambda_c$ . Referring to our model notation, we have here:

$$\begin{aligned}\lambda_c^1 &= \Lambda_c \\ Q_c^1 &= Q_c\end{aligned}$$

### 5.1 Variable L

#### 5.1.1 Non-pooled containers

Let us first consider that each customer has its own specific type of logistic units. The factory may only send this specific type to the corresponding customer. In this case, it is straightforward to conclude that the problem decomposes into  $C$  independent “1-factory 1-customer” problems. This means that, for each customer, we have the optimal lot size given by

$$Q_c^*(np) = \sqrt{\frac{\Lambda_c O_c}{H}} \quad (5.1)$$

where  $(np)$  just refers to the fact that the containers are not pooled among the customers. The optimal logistic stock is then the sum of the lot sizes

$$L^*(np) = \sum_{c \in C} Q_c^*(np) = \sum_{c \in C} \sqrt{\frac{\Lambda_c O_c}{H}}$$

and the optimal total logistic cost is

$$V(\{Q_c^*(np)\}) = 2\sqrt{H} \sum_{c \in C} \sqrt{\Lambda_c O_c}$$

In this case, the factory must hold  $C$  different inventories, each being consumed at a rate  $\Lambda_c$ , as illustrated on Figure 3 for 2 customers having optimal lot sizes of 20 units.

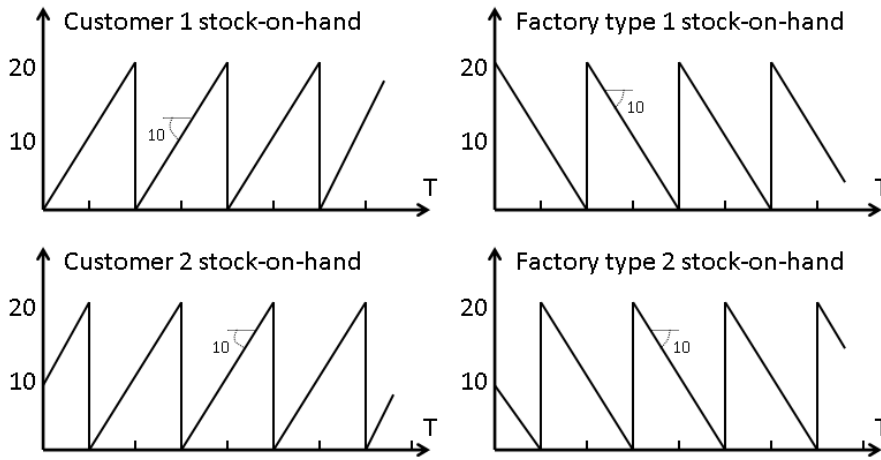


Figure 3: 2-customers non-substitutable containers flows.

### 5.1.2 Substitutable containers

Let us now assume that all the containers in the network are identical and interchangeable. It does not change anything for the customers. However, the factory may now pool all the received containers and consumes a whole lot size  $Q_c$  at rate  $\Gamma$  before requiring another lot size to be supplied. Figure 4 illustrated a case with 2 customers supplying lots of size 20 in a coordinated way. Compared to the non-substitutable container case, this results in a reduction of the average inventory at the factory of 10 units, while keeping unchanged the transportation costs and the inventory costs at the customers.

In our example, each return flow arrives at the factory when its inventory reaches 0. This is what we call *perfect coordination*. In theory, there is no guarantee that such a coordination is achievable in all cases. However, our own experiments have shown that almost perfectly coordinated strategies can generally be found at the operational level. This is in line with the results

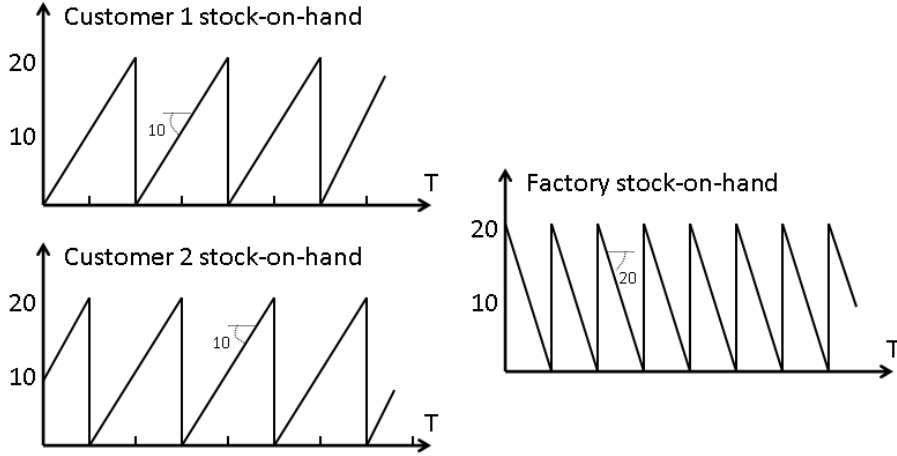


Figure 4: Perfectly coordinated return flows.

in the SRMIS literature that have shown that there generally exist cost solutions which are reasonably close to that of the ideal perfectly coordinated case. We refer the reader to the literature on SRMIS for more insight on the coordination issue.

In order to measure the potential benefit of pooling the containers, let us consider three cases. In the first case, the containers are not substitutable. In this case, a lot  $Q_c$  induces an average inventory of  $Q_c/2$  at the customer site and at the factory site. The total inventory at the customers is thus  $\sum_c Q_c/2$ . The factory also holds  $\sum_c Q_c/2$  containers on the average. In the second case, let us assume that the containers are pooled but the lots are systematically shipped simultaneously to the factory. The inventory at the factory evolves then from  $\sum_c Q_c$  to zero, leading to the same average inventory  $\sum_c Q_c/2$  at the factory. There is therefore no gain in this situation compared to the non-substitutable case. Finally, let us consider perfect coordination. Here, a lot size  $Q_c$  induces still an average inventory of  $Q_c/2$  at the customer site but only an average inventory of  $(Q_c/2)(\lambda_c/\Gamma)$  at the factory site. Indeed,  $(\lambda_c/\Gamma)$  is the fraction of the time it takes the factory to consume the containers from the lot  $Q_c$ . The cost function in this case then becomes

$$V(\{Q_c\}) = \sum_{c \in C} \frac{Q_c}{2} H + \sum_{c \in C} \frac{Q_c}{2} \frac{\Lambda_c}{\Gamma} H + \sum_{c \in C} \frac{\Lambda_c O_c}{Q_c}$$

which leads to the following optimal lot sizes

$$Q_c^*(ppc) = \sqrt{\frac{2\Lambda_c O_c}{H(1 + \frac{\Lambda_c}{\Gamma})}} \quad (5.2)$$

where (*ppc*) refers to the *pooled with perfect coordination* case. This leads to a total inventory of

$$L^*(ppc) = \sum_{c \in C} \frac{Q_c^*(ppc)}{2} \left(1 + \frac{\Lambda_c}{\Gamma}\right)$$

and an optimal cost of

$$V(\{Q_c^*(ppc)\}) = \sum_{c \in C} \sqrt{2\Lambda_c O_c H \left(1 + \frac{\Lambda_c}{\Gamma}\right)}$$

Finally, note that the reasoning remains perfectly valid if some capacity constraint is imposed on the lot sizes. In this case, we would simply have

$$Q_c^*(ppc) = \text{Min} \left\{ CAP, \sqrt{\frac{2\Lambda_c O_c}{H(1 + \frac{\Lambda_c}{\Gamma})}} \right\}$$

### 5.1.3 Comparison between the pooled and non-pooled cases

We have seen that the non-pooled and the pooled cases constitute two references for the total logistic cost in our network. Both cases are decomposable into independent subproblems for each  $c \in C$ . It is the assumption made on the average inventory at the factory which differentiates the two problems.

In the (*np*) case, the optimal lot sizes are  $Q_c^*(np) = \sqrt{\Lambda_c O_c / H}$  leading to the optimal cost of:

$$V(\{Q_c^*(np)\}) = 2 \sum_{c \in C} \sqrt{\Lambda_c O_c H}$$

In the (*ppc*) case, the optimal lot sizes are

$$Q_c^*(ppc) = \sqrt{\frac{2\Lambda_c O_c}{H(1 + \frac{\Lambda_c}{\Gamma})}} \approx \sqrt{\frac{2\Lambda_c O_c}{H(1 + 1/C)}}$$

where the last approximation assumes all customers have equal rates. This leads to a total cost of

$$V(\{Q_c^*(ppc)\}) \approx \sum_{c \in C} \sqrt{2\Lambda_c O_c H (1 + 1/C)}$$

The ratio of the two costs is thus

$$\frac{V^*(ppc)}{V^*(np)} \approx \sqrt{\frac{1 + 1/C}{2}}$$

which is close to 70% if the number of customer  $C$  is not too small.

The managerial lessons are obvious. First, pooling can only be cheaper. Indeed, a factory can delay its calls for containers as long as some are available. This means that the inventory at the factory side will remain very low all the time. Delaying the calls for the lots will allow these lots to be larger on the average, the transportation to be less frequent and the cost to be lower.

In terms of profit, pooling the containers can spare up to 30% of the cost if perfect coordination can be achieved and if the network is not too small. Such a saving is obtained by increasing the lot sizes by a factor of  $\sqrt{2}$ . This increase allows to reduce the transportation frequencies by the same factor  $\sqrt{2}$ . On the inventory side, at the customer the inventory is increased by  $\sqrt{2}$  but this is compensated by the fact that a lot stays almost no time at the factory side. Altogether, the inventory level is also reduced by a factor  $\sqrt{2}$ .

In the case the transportation is constrained by some capacity limit, the profit of pooling could be reduced to the holding cost of the inventory at the factory. Indeed, if all the lots sizes were already at their limit  $CAP$  in the non-pooled case, then the only saving is the inventory at the factory.

## 5.2 Fixed $L$

### 5.2.1 Uncapacitated transportation

Let us consider the case where the total logistic inventory  $L$  is no more a variable. Our problem reduces to:

$$\begin{aligned} \min_{Q_c} \quad & \sum_{c \in C} \frac{\Lambda_c O_c}{Q_c} \\ \text{such that} \quad & \begin{cases} L = \sum_{c \in C} Q_c & (5.3) \\ \text{or} \\ L = \sum_{c \in C} \frac{Q_c}{2} + \sum_{c \in C} \frac{Q_c \Lambda_c}{2 \Gamma} & (5.4) \end{cases} \end{aligned}$$

This is now a transportation cost minimization problem. The link between the lot sizes and the logistic inventory  $L$  is determined by the constraint (5.3)

in the non-pooled case and (5.4) in the pooled case with perfect coordination. Intuitively, this problem is not too difficult. The objective function is separable and strictly decreasing with the lot sizes. The optimal lot sizes would be determined independently if  $L$  was not given. Since  $L$  is given, the lot sizes are chosen so that their marginal contributions to the objective function are equal. Let us solve this problem mathematically for constraint (5.3) in order to understand the solution mechanism. The procedure can be repeated for the other case.

We first build the Lagrangian function

$$\mathcal{L} = \sum_{c \in C} \frac{\Lambda_c O_c}{Q_c} + \beta [\sum_{c \in C} Q_c - L]$$

and then derive the first order optimality conditions

$$\begin{cases} \frac{\Lambda_c O_c}{(Q_c)^2} = \beta & \forall c \in C \\ L = \sum_{c \in C} Q_c \end{cases}$$

which leads to the optimal lot sizes

$$Q_c^*(np) = \frac{L}{\sum_{d \in C} \sqrt{O_d \Lambda_d}} \sqrt{O_c \Lambda_c}$$

These optimal lot sizes are now linked together by the total available logistic inventory  $L$ . Since the ratios between the lot sizes are independent of  $L$ ,

$$\frac{Q_c^*(np)}{Q_d^*(np)} = \frac{\sqrt{\Lambda_c O_c}}{\sqrt{\Lambda_d O_d}}$$

it is easy to define a first set of lot sizes on an arbitrary basis, e.g. fixing  $Q_d^*(np) = 1$ . Then, the resulting logistic inventory can be computed. If it is not equal to  $L$ , all the lot sizes are scaled in order to reach the given  $L$  value. The same process is also valid for the value  $L$  equal to  $L^*$ . In this case, the lot sizes are the EOQ of the minimization problem with  $L$  variable given by (5.1).

The fact that the lot sizes remain proportional for different values of  $L$  comes from the optimality conditions that requires that the gradient of the cost functions at the optimal lot sizes  $Q_c^*$

$$-\frac{\Lambda_c O_c}{(Q_c^*)^2} = -\frac{\Lambda_d O_d}{(Q_d^*)^2} \quad \forall c, d \in C$$

are all equal. This is illustrated on Figure 5 for three different customer cost functions.



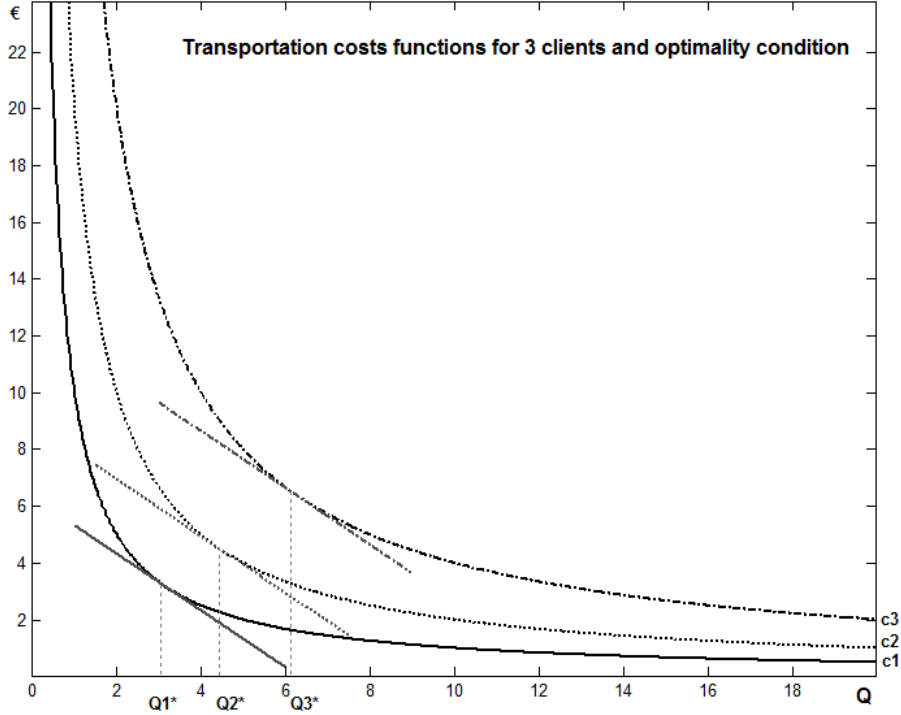


Figure 5: Transportation functions and derivatives.

The case of pooled containers with perfect coordination can be solved by the same procedure. The optimality conditions

$$\frac{\Lambda_c O_c}{(Q_c)^2 (1 + \Lambda_c/\Gamma)} = \frac{\Lambda_d O_d}{(Q_d)^2 (1 + \Lambda_d/\Gamma)} \quad \forall c, d \in C$$

lead to the following optimal lot sizes

$$Q_c^*(ppc) = \frac{2L}{\sum_{d \in C} \sqrt{O_d \Lambda_d (1 + \Lambda_d/\Gamma)}} \frac{\sqrt{O_c \Lambda_c}}{\sqrt{(1 + \Lambda_c/\Gamma)}}$$

The “1-factory C-customer” case thus leads to lot sizes that are easy to compute. In the (np) case, with  $L$  fixed or variable, they are proportional to

$$Q_c^*(np) \propto \sqrt{O_c \Lambda_c} \propto \sqrt{\frac{O_c}{1/\Lambda_c + 1/\Lambda_c}}$$

while in the (ppc) case, they are proportional to

$$Q_c^*(ppc) \propto \sqrt{\frac{O_c}{1/\Lambda_c + 1/\Gamma}}$$

This clearly indicates that, in the (ppc) case, the factory consumes the containers at the speed  $\Gamma$  instead of  $\Lambda_c$ . In both cases, a large  $\Lambda_c$  still favors a large lot size since this lot size rotates quickly. However the advantage is reduced in the (np) case since the advantage of a high rotation takes place at the customer site only.

In practice, since every unit of a lot size rotates almost twice as fast in the (ppc) case than in the (np), the lot sizes will be larger in the (ppc) case. If  $L$  is fixed, they will be almost doubled, leading to a reduction of 50% on the transportation cost.

### 5.2.2 Capacitated transportation

Let us now complete the “1-factory C-customer” problem with the case of a fixed  $L$  and with some capacity constraints on the lots sizes.

The solution of the problem with this additional constraint is more complex analytically. Intuitively, the approach remains the same. The optimal lot sizes are again determined so that their marginal contributions to the objective function are all equal, while satisfying the  $L$  constraint. If now, some capacity constraints are exceeded, then these lot sizes are limited to  $CAP$ . Since this gives an additional available inventory, the other lot sizes can all be increased while keeping the same marginal contributions. The process is reiterated as long as either  $L$  is reached or all the capacity limits have been reached. Another simple method consists in considering the containers of the logistic inventory one by one and in allocating them to the customer with the steepest marginal contribution. If, in this process, a customer reaches the transportation capacity limit, then he is not allowed to receive any additional unit. Again, the process ends when the last container has been allocated or when all the capacity limits have been reached.

Figure 6 represents the typical evolution of the total logistic cost for both cases, pooled or non-pooled, with a small or a large capacity. For low levels of logistic inventory, the situation typically remains unchanged since all optimal lot sizes are smaller than the capacity limit. When the logistic stock becomes larger, the (ppc) curve with capacity constraints separates from the (ppc) curve without capacity constraints as soon as some lot sizes reach the capacity limit. Additional containers allocated to the system still bring a profit but a lower one because those units are allocated to customers with flatter slopes. As the logistic inventory further increases, we reach a point where all lot sizes reach the capacity limit. The capacitated curve becomes a straight line, indicating the linear increase of the holding cost linked with

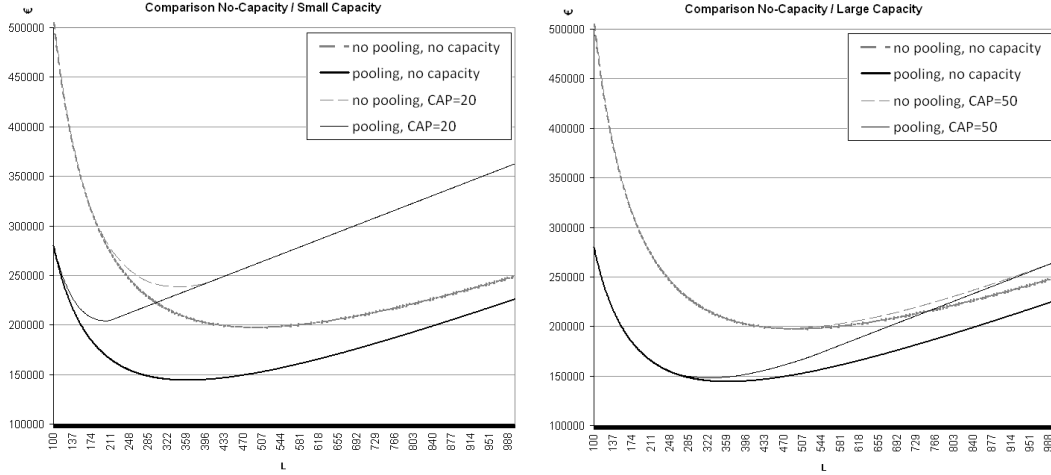


Figure 6: Impact of a small or a large capacity limit.

each additional unit. The same behavior is to be observed in the  $(np)$  case. However, for any given level of the logistic inventory, the  $(ppc)$  optimal quantities are larger than the  $(np)$  optimal quantities. Consequently, the level of  $L$  for which the capacitated curve separates from the uncapacitated curve in the  $(np)$  case is larger than in the  $(ppc)$  case.

## 6 F-Factory and 1-Customer

The case “F-factory 1-customer” with F factories being resupplied by a single customer is completely symmetric to the “1-factory C-customer” case discussed in the previous section. We will quickly analyze this case since it constitutes a kind of summary of the previous case.

Referring to our notation, we have here a single customer and therefore

$$\begin{aligned}
 \lambda_c^f &= \lambda_1^f = \Gamma^f \\
 \sum_f \Gamma^f &= \Lambda \\
 O_c^f &= O^f \\
 Q_c^f &= Q_1^f = Q^f
 \end{aligned}$$

The optimal solution with  $L$  variable is as follows:

$$Q^{f*}(np) = \sqrt{\frac{\Gamma^f O^f}{H}}; L^*(np) = \sum_{f \in F} Q^{f*}(np)$$

and

$$Q^{f*}(ppc) = \sqrt{\frac{2\Gamma^f O^f}{(1 + \Gamma^f/\Lambda)H}};$$

$$L^*(ppc) = \sum_{f \in F} \frac{Q^{f*}(ppc)}{2} (1 + \Gamma^f/\Lambda)$$

If there are some capacity limits, we should just bound the lot sizes accordingly, everything else being equal.

With a given fixed logistic inventory  $L$ , we compute a first set of lot sizes  $Q^f$  using the proportionality rules derived from the above equations. Then, we can scale all these lot sizes in order to reach the target  $L$ . If some capacity bounds are exceeded, we limit these lot sizes to the capacity  $CAP$  and rescale the other  $Q^f$  again. This process will be repeated until we reach the target  $L$  or until all the capacity constraints are reached.

The managerial conclusions are the same as with the “1-factory C-customer” case. Pooling with perfect coordination brings a potential benefit of 30% in the  $L$  variable case. If  $L$  is given, the lot sizes can almost be doubled in the (ppc) bringing a reduction of the transportation costs of almost 50%. Active capacity constraints could limit this lot size increase, reducing the benefit of the pooling.

## 7 F-factory C-customer case

In this section, we consider the general network composed of  $F$  factories and  $C$  customers. If the logistic containers are specific to each factory, the problem degenerates into  $F$  1-factory C-customer problems. Such problems were studied in the section 5. Similarly, if the logistic containers are specific to each customer, the problem degenerates into  $C$  F-factory 1-customer problems as analyzed in Section 6. So, we will assume here that all containers in the network of  $F$  factories and  $C$  customers are identical and can be pooled. We will also assume perfect coordination as in Sections 5 and 6.

The problem has the general formulation discussed in Section 2. Compared to the subproblems analyzed in Sections 4 to 6, the general problem introduces the  $\lambda_c^f$  variables, that is the flow each customer decides to send back to a precise factory  $f$ . The determination of these variables is called the *customer allocation problem*. Along with the customer allocation problem, we must also determine the lot sizes  $Q_c^f$  by which a given customer  $c$  will send back its flow  $\lambda_c^f$  to the factory  $f$ . Finally, the lot sizes will also provide the total inventory of containers  $L$ .

The analysis will show that the general problem is rather easy since it can be decomposed into 2 independent problems: a customer allocation problem that determines which customer sends containers to which factory and a lot size problem that defines how these flows are grouped. However, in some cases, these two problems have to be solved jointly. As in the previous sections, we will consider the cases of fixed and variable  $L$ . The impact of capacity constraints will also be studied.

## 7.1 Uncapacitated problem

### 7.1.1 Decomposition of the problem

Let us first rewrite the general problem as below and solve it for the variables  $Q_c^f$ .

$$\begin{aligned}
& \min_{(Q_c^f)} LH + \sum_{c \in C, f \in F} \frac{\lambda_c^f Q_c^f}{Q_c^f} \\
& \text{s.t.} \\
& \sum_{c \in C} \lambda_c^f = \Gamma^f \quad \forall f \\
& \sum_{f \in F} \lambda_c^f = \Lambda_c \quad \forall c \\
& L = \sum_{c \in C, f \in F} \frac{Q_c^f \lambda_c^f}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)
\end{aligned}$$

Defining the Lagrangian function

$$\begin{aligned} \mathcal{L} = & LH + \sum_{c \in C, f \in F} \frac{\lambda_c^f O_c^f}{Q_c^f} \\ & + \beta \left( \sum_{c \in C, f \in F} \frac{Q_c^f \lambda_c^f}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right) - L \right) \end{aligned}$$

leads to the following first order conditions:

$$\begin{cases} \frac{\lambda_c^f O_c^f}{(Q_c^f)^2} = \beta \frac{\lambda_c^f}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right) & \forall c, f \\ L = \sum_{c \in C, f \in F} \frac{Q_c^f \lambda_c^f}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right) \end{cases}$$

from which we can deduce,  $\forall c, f$ , the optimal quantities

$$Q_c^{f*}(ppc) = k \sqrt{\frac{O_c^f}{\left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)}} \quad (7.1)$$

where  $k$  is a normalization constant

$$k = \frac{2L}{\sum_{d \in C, g \in F} \lambda_d^g \sqrt{O_d^g \left( \frac{1}{\Lambda_d} + \frac{1}{\Gamma^g} \right)}}$$

Reinserting those quantities in the objective function leads to the problem

$$\begin{aligned} \min_{L, \lambda_c^f} \quad & LH + \frac{\left[ \sum_{c \in C, f \in F} \lambda_c^f \sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)} \right]^2}{2L} \\ \text{s.t.} \quad & \end{aligned}$$

$$\sum_{c \in C} \lambda_c^f = \Gamma^f \quad \forall f \quad (1)$$

$$\sum_{f \in F} \lambda_c^f = \Lambda_c \quad \forall c \quad (2)$$

Now, this problem can be solved in two steps. First, we solve the customer

allocation problem

$$\begin{aligned}
\min \quad & \sum_{c \in C, f \in F} \lambda_c^f \sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)} \\
\text{st} \quad & \sum_{c \in C} \lambda_c^f = \Gamma^f \quad \forall f \\
& \sum_{f \in F} \lambda_c^f = \Lambda_c \quad \forall c
\end{aligned}$$

in order to determine the optimal customer allocation  $\lambda_c^{f*}$  values. Secondly, on the basis of these  $\lambda_c^{f*}$  values, the optimal lot sizes  $Q_c^{f*}(ppc)$  can be determined.

### 7.1.2 Customer allocation problem

The customer allocation problem is a simple linear program. It is an instance of the transportation problem with a cost function that exhibits some specificities.

First, the model is not based on the direct transportation cost  $O_c^f$  but on its square root. This comes from the EOQ-like general model. If a customer is more distant than another, than its lot size  $Q_c^f$  will be made larger in order to smooth its impact on the transportation costs. This results in the square root function. Secondly, the transportation cost  $O_c^f$  under the square root is corrected by a term  $(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})$ . Indeed, the faster the inventory along a  $c - f$  relationship rotates, the less costly this inventory is and the larger the lot size can be. This comes from the contribution of  $Q_c^f$  to  $L$

$$L = \sum_{c \in C, f \in F} \frac{Q_c^f \lambda_c^f}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)$$

that decreases as  $\Lambda_c$  and  $\Gamma^f$  increase.

A quick analysis of the linear customer allocation program allows us to identify  $C + F - 1$  linearly independent constraints. It follows that, among the  $CF$  variables,  $C + F - 1$  are basic. Since, each customer must have a least one allocation, we can conclude that there will be, at most,  $F - 1$  customers who are allocated to more than 1 factory.

### 7.1.3 Lot Size calculation

Now, on the basis of these  $\lambda_c^{f*}$  values, the optimal lot sizes  $Q_c^{f*}(ppc)$  can be computed assuming that there is some flow from  $c$  to  $f$ . If  $L$  is fixed, then we simply use the Equation (7.1). If  $L$  is variable, we solve the following EOQ problem

$$\min_L LH + \frac{\left[ \sum_{c \in C, f \in F} \lambda_c^{f*} \sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)} \right]^2}{2L}$$

We obtain

$$L^* = \sum_{c \in C, f \in F} \lambda_c^{f*} \frac{\sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)}}{2H}$$

which leads to the optimal lot sizes

$$Q_c^{f*}(ppc) = \sqrt{\frac{2O_c^f}{H \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)}} \quad (7.2)$$

It is worth noting here that these lot sizes are independent of the flows  $\lambda_c^{f*}(ppc)$ , as long as they are not equal to zero. Without loss of generality, we will assume here below that for the  $c - f$  relationships with  $\lambda_c^{f*}(ppc) = 0$ , the lot sizes are also set to zero.

## 7.2 Capacitated problem with $L$ variable

The problem becomes a bit more complex when some transportation capacity constraints limit the lot sizes. We will consider successively the variable  $L$  and the fixed  $L$  cases.

For the variable  $L$  case, let us first determine the lot sizes  $Q_c^{f*}(ppc)$ . They are defined by Equation (7.2). If some of these lot sizes exceed the capacity threshold, we then limit them as follows

$$Q_c^{f*}(ppc) = \text{Min} \left\{ CAP, \sqrt{\frac{2O_c^f}{H \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)}} \right\} \quad (7.3)$$

In practice, we can compute which lot size will be limited by the capacity  $CAP$  and which ones will not. And this calculation is independent of the



flow  $\lambda_c^{f*}$  from  $c$  to  $f$ . We can then re-introduce these optimal values in the objective function. The cost generated by each lot size  $Q_c^{f*}(ppc)$  is

$$\frac{\lambda_c^f O_c^f}{Q_c^{f*}(ppc)} + \frac{H Q_c^{f*}(ppc) \lambda_c^f}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)$$

which becomes, if  $Q_c^{f*}(ppc)$  is smaller than  $CAP$ ,

$$\lambda_c^f \sqrt{2H} \sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)}$$

and, if it equals  $CAP$

$$\lambda_c^f \left( \frac{O_c^f}{CAP} + \frac{CAP H}{2} \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right) \right)$$

We can now formulate the allocation problem for the capacitated variable  $L$  problem as follows

$$\begin{aligned} & \text{minimize} \\ & \lambda_c^f \\ & \sum_{c,f:Q_c^{f*} \leq CAP} \lambda_c^f \sqrt{O_c^f H \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma^f} \right)} + \\ & \sum_{c,f:Q_c^{f*} > CAP} \lambda_c^f \left( \frac{O_c^f}{CAP} + \frac{CAP H}{2} \frac{(\Lambda_c + \Gamma^f)}{\Lambda_c \Gamma^f} \right) \\ & \text{such that} \\ & \sum_{c \in C} \lambda_c^f = \Gamma^f \quad \forall f \\ & \sum_{f \in F} \lambda_c^f = \Lambda_c \quad \forall c \end{aligned}$$

which is an LP problem that can be easily solved.

### 7.3 Capacitated problem with $L$ fixed

When  $L$  is fixed and some lot sizes are constrained, we have the toughest version of our problem. Indeed, the capacity constraints prevent us from deriving analytical expressions for  $Q_c^{f*}(ppc)$  such as in Equation (7.1). Also, the variable  $L$  approach that allowed us to derive the flow independent expressions (7.3) cannot be used. Indeed, each lot size is now dependent, through

the total logistic inventory  $L$  on the other lot sizes. In other words, in this problem  $Q_c^f(ppc)$  depends on  $\lambda_c^f(ppc)$  and vice versa. Breaking this dependence is the key point. In order to illustrate the complexity of the problem, let us consider different ranges of  $L$ .

If we consider extremely small values for  $L$ , the capacity constraints do not play any role. In this case, our problem is identical to the uncapacitated problem. And the  $\lambda_c^f(ppc)$  could be determined by minimizing

$$\min \sum_{c \in C, f \in F} \lambda_c^f \sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma_f} \right)}$$

In this setting, a more distant customer will receive of larger share of the inventory, reducing the impact of their distance in comparison with the other relations. But, in the capacitated case, this is true only up to a certain lot size equals to  $CAP$ . A distant customer having reached this maximal lot size will see the impact of its distance becoming more important.

Indeed, if we consider very large values for  $L$ , then the capacity limits will be reached and all the lot sizes will be equal to  $CAP$ . In this case, we just aim at minimizing the transportation costs

$$\min \sum_{c \in C, f \in F} \frac{\lambda_c^f O_c^f}{CAP}$$

In which the transportation costs  $\sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma_f} \right)}$  have been replaced by  $O_c^f$ .

The problem is thus more difficult because we cannot easily opt for a specific cost function. Indeed, this cost will differ if the lot size reaches the capacity limit or not and this depends on the  $L$  value.

The solution we suggest consists in finding first, on the basis of a given  $L$  which lot sizes will remain below the threshold capacity  $CAP$  and which ones will not. This solution procedure relies on the following very intuitive property.

*Property. The optimal solution of a variable  $L$  capacitated problem for which the holding cost is replaced by a fictitious unit value  $W$  of the inventory and that implies a logistic stock  $L^*(W)$ , is the optimal solution of the problem with  $L$  fixed to  $L^*(W)$  for any value of  $H$ .*

Since the problem with variable  $L$  is easy, we can develop a solution procedure for the fixed  $L$  problem. We start with any value  $W$  for the fictitious holding cost and solve the variable  $L$  capacitated problem. This leads to some optimal logistic cost  $L^*(W)$ , most likely different than the given  $L$ . We then increase or decrease  $W$  until we reach our target  $L$ .

The technical proof of this property can be sketched as follows. With  $L$  fixed, we aim at minimizing the transportation cost with a constraint on the total logistic inventory  $L$ . We can introduce this constraint into the objective function by means of a Lagrangian multiplier  $W$ . The objective function is now the sum of a transportation cost and a fictitious holding cost like in the variable  $L$  case. The optimal solution is found by tuning the Lagrangian multiplier until the target  $L$  value is reached.

Different approaches can be used to find the exact value of  $W$ . One could browse systematically all possible values or use an approach based on iteratively adapted intervals.

Next, we describe a very simple alternative to the computation of the solution:

1. Set  $W_{LOW} = 0$  and  $W^{HIGH} = \text{very large value}$
2. Choose an adequate value for the fictitious holding cost  $W$  comprised between  $W_{LOW}$  and  $W^{HIGH}$
3. Solve the corresponding variable  $L$  capacitated problem
  - Compute the lot sizes  $Q_c^{f*}(ppc)[W]$  using (7.2)
  - Limit those that exceed the capacity limit using (7.3)
  - Solve the capacitated variable  $L$  customer allocation problem. This gives the optimal  $\lambda_c^{f*}$
4. Knowing the non-zero  $\lambda_c^{f*}$ , derive the corresponding total logistic inventory  $L^*(W)$
5. If  $L^*(W) = L$  or if  $L^*(W) < L$  and all lot sizes are equal to CAP, then STOP
6. Otherwise, update the interval, that is
  - if  $L^*(W) > L$ , Set  $W_{LOW} = W$

- if  $L^*(W) < L$ , Set  $W_{HIGH} = W$
- go back to step 2.

## 7.4 Managerial implications for the multi-factory case

Let us consider on a small example the allocation of two customers to two factories. If we allow the accumulation of the empty containers by means of a global logistic inventory, we showed that we obtain the following objective to minimize

$$\min \sum_{c \in C, f \in F} \lambda_c^f \sqrt{O_c^f \left( \frac{1}{\Lambda_c} + \frac{1}{\Gamma_f} \right)}$$

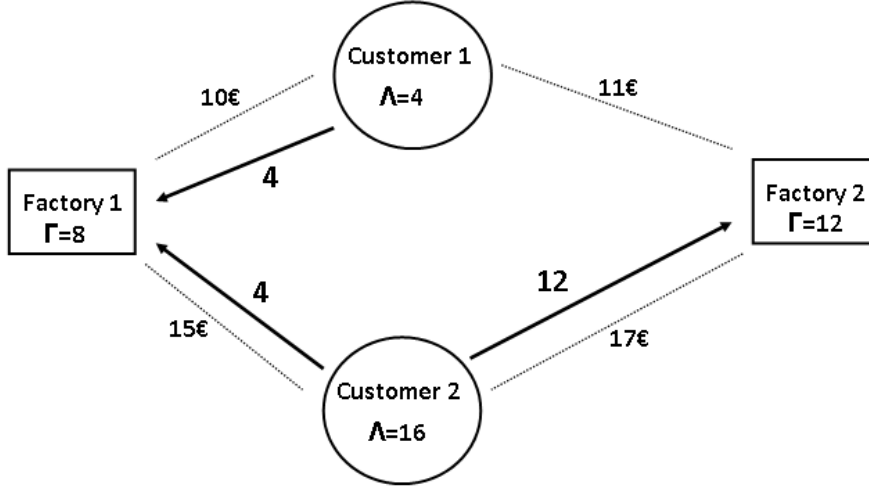


Figure 7: A small allocation example based on logistic costs.

The solution is illustrated on Figure 7. Customer 1 is allocated to factory 1 as well as 4 units of customer 2. This solution is explained by the fact that, for customer 2, reallocating 1 unit from factory 2 to factory 1 brings an increase of the total cost which is

$$\sqrt{15 \left( \frac{1}{16} + \frac{1}{8} \right)} - \sqrt{17 \left( \frac{1}{16} + \frac{1}{12} \right)} = 0.10$$

while for customer 1, the corresponding cost increase is only

$$\sqrt{10 \left( \frac{1}{4} + \frac{1}{8} \right)} - \sqrt{11 \left( \frac{1}{4} + \frac{1}{12} \right)} = 0.02$$

The underlying reasons for the use of this objective function are (i) the existence of an inventory which allows larger lot sizes for distant factories and (ii) the rotation of that inventory.

These reasons explain the difference of our objective function with the classical objective function of the transportation problem based on the transportation costs

$$\min \sum_{c \in C, f \in F} \lambda_c^f O_c^f$$

The solution provided by this more classical objective function is depicted on Figure 8. Customer 1 is allocated to factory 2. Allocating customer 1 to factory 1 would bring a profit of 1€ per unit but would also result in a loss of 2€ per unit for transferring some flow of customer 2 from factory 1 to factory 2.

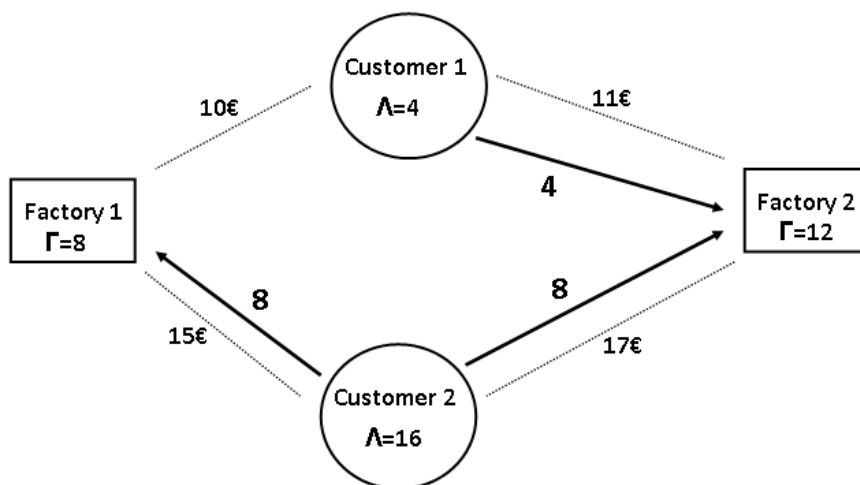


Figure 8: A small allocation example based on transportation costs.

We showed, however, that the effects allowing us to use the first objective function in the uncapacitated case may be tempered by a capacity limit. In the capacitated case, we have seen that, if the logistic inventory is small, no lot size would reach the capacity limit and the solution found for the uncapacitated case, represented on Figure 7, remains valid. However, when the logistic inventory progressively increases, some capacity limits will be reached, and the proportion between the quantities will begin to change. Eventually, if the logistic inventory is very large, all quantities are limited by the capacity, and we retrieve the classical objective based on transportation

costs. In our example, with a transport capacity limit of 20, if the logistic inventory exceeds 36 units, the solution depicted on Figure 8, based on a classical allocation on the transportation costs, becomes the optimal solution.

In the multi-factory multi-customer case, the gain of pooling appears at two levels. First, resource pooling allows the customers to send their containers back to any factory. This reallocation provides a benefit in terms of total travelled distance. This benefit cannot be measured analytically. It depends on the actual data and can only be computed numerically. Secondly, inside each factory cluster, the pooling provides an additional benefit. This benefit can amount up to  $1/\sqrt{2}$ . When capacity constraints are active, they can limit this benefit. In the worst case, capacity constraints are all binding and the pooling provides no benefits on the transportation costs. However, a reduction of the logistic stock due to an increase rotation rate of the units at the factories and at the clients remains.

## 8 Conclusion and future work

The objective of the paper was to answer 3 strategic questions for the management of the return flows of empty reusable logistic containers

- Q1: to which factory should each customer return the containers?
- Q2: at which frequency should they be returned ?
- Q3: how many containers are needed in the network?

and to get some insight on the benefit of pooling such containers among the players. To tackle all these questions, we considered cases of increasing complexity from the “1-factory 1-customer” case, to the “F-factory C-customer” case. We also considered the impact of capacity constraints and of a fixed/variable logistic cost. Solutions were provided in all cases.

At first, we showed that the single-factory single-customer case with variable  $L$  was easily solved by an EOQ-like formula. The optimal quantity, however, was smaller than a classical EOQ by a factor  $\sqrt{2}$  since we consider the inventory both at the customer and at the factory sites.

The case with a single-factory and several customers reduces to  $c$  independent single-factory single-customer problems if the logistic units are not pooled. Each customer will select its own optimal lot size. Dependence between the customers appears if we suppose that the logistic units can be

pooled at the factory. However, the benefit of the pooling depends on the coordination of the return flows. If they all occur at the same moment then we have no coordination and no benefit. In the other extreme, namely perfect coordination, each return arrives at the factory when the inventory had just fallen to zero. This allows the quickest rotation of the inventory at the factory and thus the best use of the inventory of containers. The pooled perfect coordination case leads to lot sizes that are  $\sqrt{2}$  larger and a total benefit of up to  $1/\sqrt{2}$ , that is, about 30 percent. A set of experiments developed by the same authors in a forthcoming paper shows that the perfect coordination assumption is very realistic.

The multi-factory multi-customer case introduces the problem of allocating the customers to the factories. The first important result is that the allocation problem, in the uncapacitated case, is independent from the actual level of the logistic stock. It defines an allocation of the customers and relative frequencies for all customer-factory pairs. As before, the logistic stock will be used to scale those relative frequencies such as to meet the constraint implied by the number of units in the network. The allocation problem appears as a transportation problem with costs including features of the EOQ-like model. It was shown that the cost comparison between the customer-factory relationships is based on a square root of the distances weighted by a factor depending on the consumption rates at the customer and at the factory.

If a transport capacity limit is added to the model, then the allocation problem is not independent of  $L$  any more. For a low level of logistic inventory, when the capacity has no impact, the EOQ-like costs remain valid, as well as the uncapacitated allocation. However, for large values of  $L$ , the holding cost effect tends to disappear and a classical allocation based on the distances becomes optimal. We have suggested an iterative procedure in order to determine, for the given value of the logistic stock, which lot size will be limited by the capacity and which ones will not. On the basis of this result, the optimal customer allocation can be found.

Our analysis allowed to identify the economic logic which is relevant for the model, the trade-offs that appear in the management of the return flows of empty logistic units and how the costs would react to variations of the parameters. It also allows us to identify the different problems and the logic for solving them. This analysis provides a good basis for tackling strategic comparisons when deciding for some network structures. The analysis relies on the perfect coordination assumption. But our preliminary results, to

be presented in a forthcoming paper, are already quite reassuring in that respect. One of our current research directions is thus the development of robust heuristics which guarantee almost perfect coordination in the daily management of these return flows.

Besides, other extensions are also under study. In the model we presented, the transportation cost is assumed to be fixed from a customer to a factory. Milkruns or consolidation strategies are being studied. Finally, networks with different types of containers that can share the same transportation means are also possible extensions.

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## References

- [1] Anily S. Multi-item Replenishment and Storage Problems (MIRSP): Heuristics and Bounds. *Operations Research* 1991; 39; 233-239.
- [2] Axsäter S. Economic lot sizes and vehicle scheduling. *European Journal of Operational Research* 1980; 4; 395-398.
- [3] Churchman C W, Ackoff R L, Arnoff E L. *Introduction to Operations Research*. John Wiley and Sons, New York; 1957.
- [4] Dixon P S, Poh C L. Heuristic procedures for multiitem inventory planning with limited storage. *IIE Transactions* 1990;6; 112-123.
- [5] Gallego G, Shaw D, Simchi-Levi D. The Complexity of the Staggering Problem and Other Classical Inventory Problems. *Operation Research Letters* 1992; 12; 47-52.
- [6] Gallego G, Queranne M, Simchi-Levi D. Single resource multi-item inventory systems. *Operations Research* 1996; 44; 580-595.
- [7] Goyal S K.() Optimum ordering policy for a multi-item single supplier system. *Operations Research Quarterly* 1974; 25; 293.
- [8] Günther H-O. Ein Verfahren zur Bestellmengenplanung bei beschränkter Lagerkapazität. In Bühler W, Feichtinger G, Hartl R F, Rademacher



- F J, Stähly P. Operations Research Proceedings. Springer, Berlin Heidelberg;491-498; 1990.
- [9] Hadley G, Whitin T M. Analysis of Inventory Systems. Prentice Hall, Englewood Cliffs, New Jersey; 1963.
- [10] Haksever C, Moussourakis J. A model for optimizing multi-product inventory systems with multiple constraints. International Journal of Production Economics 2005; 97; 18-30.
- [11] Hall N G. A Multi-Item EOQ Model with Inventory Cycle Balancing. Naval Research Logistics 1988; 35; 319-325.
- [12] Hariga M, Jackson P L. The warehouse scheduling problem: Formulations and algorithms. IIE Transactions 1996; 28; 115-127.
- [13] Harris F W. What quantity to make at once. The library of factory management 1915; 5; 47-52.
- [14] Holt C C. Decision Rules for Allocating Inventory to Lots and Cost Foundations for Making Aggregate Inventory Decisions. Journal of Industrial Engineering 1958; 9; 14-22.
- [15] Hommer E D. Space-limited aggregate inventories with phased deliveries. Journal of Industrial Engineering 1966; 17; 327.
- [16] Huang W, Romeijn H E, Geunes J. The Continuous-Time Single-Sourcing Problem with Capacity Expansion Opportunities. Naval Research Logistics 2005; 52; 193-211.
- [17] Krone L H. A note on economic lot sizes for multi-purpose equipment. Management Science 1964; 10; 461-464.
- [18] Maxwell W L. The scheduling of economic lot sizes. Naval Research Logistics Quarterly 1964; 11; 89-124.
- [19] Page E, Paul R J. Multi-product Inventory Situation with one Restriction. Journal of the Operational Research Society 1976; 27; 815-834.
- [20] Parsons J A. A note on Krone's economic lot-size formulas. Management Science 1965; 12; 314-315.
- [21] Parsons J A. Multiproduct lot size determination when certain restrictions are active. Journal of Industrial Engineering 1966; 17; 360-365.

- [22] Rosenblatt M J. Multi-item inventory system with budgetary constraint: A comparison between the Lagrangian and the fixed cycle approach. *International Journal of Production Research* 1981; 19; 331-339.
- [23] Silver E A. A simple method of determining order quantities in joint replenishments under deterministic demand. *Management Science* 1976; 22; 1351-1361.
- [24] Zoller K. Deterministic multi-item inventory system with limited capacity. *Management Science* 1977; 24; 451-455.

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