

2011/25



Sustainable growth and modernization
under environmental hazard and adaptation

Natali Hritonenko and Yuri Yatsenko



CORE

DISCUSSION PAPER

Center for Operations Research
and Econometrics

Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium

<http://www.uclouvain.be/core>

CORE DISCUSSION PAPER
2011/25

**Sustainable growth and modernization
under environmental hazard and adaptation**

Natali HRITONENKO¹ and
Yuri YATSENKO²

June 2011

Abstract

We develop an aggregated model to study rational environmental adaptation policies that compensate negative consequences of certain environmental hazards and changes. The model distinguishes three categories of adaptation measures that (a) compensate the decrease of the environmental amenity value, (b) compensate the decrease of total productivity, (c) develop and introduce new hazard-protected capital and technology. We analyze the optimal balance among investment, consumption, and different categories of adaptation investments. It appears that the environmental hazard and subsequent adaptation do not lead in the long run to a higher level of capital modernization compared to the benchmark case with no hazard. A synergism between productivity-related and amenity-related adaptation activities arises because the productivity-related adaptation positively impacts the economy and creates better possibilities for the amenity adaptation.

Keywords: environmental hazard, environmental adaptation, capital modernization, optimization.

JEL Classification: C00, O11, O13, Q01, Q54, Q57

¹ Prairie View A&M University, Texas, USA. E-mail: nahritonenko@pvamu.edu

² Houston Baptist University, Texas, USA. E-mail: yyatsenko@hbu.edu

The authors would like to thank Professor Raouf Boucekkine, Professor Thierry Br chet and other participants of the conference "Sustainable growth, technological progress and the environment" (Institut de Recherches Economiques et Sociales, Louvain-la-Neuve, Belgium, May 2011) for valuable comments. The authors are grateful to CORE for its continued support of their research. This paper was partially written during their visit to CORE in 2010. Natali Hritonenko acknowledges the support of NSF DMS 1009197.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1. Introduction

The environmental adaptation means adjustments in natural and human systems in response to actual or expected changes in the environment in order to alleviate negative impacts of the changes (IPCC 2001). If the predicted tendency of global climate change is correct, then the adaptation will become an essential trend in economic development in coming decades.

The human society can *mitigate* negative changes in the environment or *adapt* to the changes.

The Intergovernmental Panel on Climate Change emphasizes this choice in its Third Assessment Report (IPCC 2001) with separate documents for (1) Mitigation, and (2) Impacts, Vulnerability, and Adaptation. The first option, *mitigation* (actions to reduce emissions or increase carbon sinks), is expensive and provides no guarantee of success. New scientific results (e.g., Shakhova *et al* (2010) on Arctic methane venting) display the inadequateness of our current knowledge of the global climate change process. Also, there exist numerous political, cultural, and economic obstacles for mitigation actions as the 2009 UN Climate Change Conference in Copenhagen demonstrated. The second option, *adaptation*, is also expensive but practically feasible in many situations. The humans have always adapted. Adaptation to the environmental changes has been increasingly observed in preserving the economic well-being at different spatial and societal scales (Adger et al 2005). Much of this adaptation is triggered by past or current events but it is also anticipatory and based on some assessment of conditions in the future.

At the present time, the environmental adaptation possesses essential potential resources that should be used wisely. Proposals at the UNFCCC meeting in Bali in December 2007 suggested that a 2% levy on carbon trading could finance adaptation costs in developing regions (Hof *et al*

2009). Thus, modeling of environmental adaptation strategies is of increasing importance for economic theory and management practice. The aggregated modeling of rational adaptation measures can improve our understanding of adaptation and help in determining optimal government policies on adaptation to climate changes. Fankhauser *et al* (1999) point out several types of anticipatory adaptation policies that may be already studied, among them:

- Optimizing economic policies for earlier replacement of long-term weather-sensitive capital that will depreciate faster than initially anticipated.
- Revising long-term development plans such as coastal development or drought contingency plans to include climate change.

At present time, modeling of environmental adaptation is the subject of much fewer studies than mitigation modeling and is restricted solely to numeric simulation in the integrated assessment modeling framework (Hope *et al* 1993; de Bruin, Dellink, and Agrawala 2009; de Bruin, Dellink, and Tol 2009; Bosello 2008; Bosello *et al* 2010). In (Hope *et al* 1993), adaptation is an exogenous variable in the PAGE model, so, no optimization analysis is possible. De Bruin, Dellink, and Agrawala (2009) and de Bruin, Dellink, and Tol (2009) consider aggregate adaptation expenditures as an endogenous flow variable of the DICE model and demonstrate that adaptation and mitigation are complements, adaptation is stronger in the short run, mitigation is stronger in the long run, and the adaptation is better for low environmental damages. Bosello (2008) considers aggregate adaptation expenditures as an endogenous stock variable of the RICE model and argues that the optimal mitigation starts earlier, adaptation is postponed, and mitigation is better for low damages. The most comprehensive research is (Bosello *et al*, 2010) that distinguishes three adaptation categories (anticipatory stock, reactive flow, R&D stock) and solves a social planner problem in the AD-WITCH optimal growth model with the world split

into 12 regions. Their simulation results demonstrate that adaptation and mitigation are complements or imperfect substitutes, optimal mitigation should start earlier while adaptation is better in long run, and anticipatory adaptation is optimal for richer countries and some specific regions. One can see that the overall picture is conflicting and heavily depends in the choice of modeling tools. So, analytic modeling of environmental adaptation is an actual problem.

At the same time, relevant tools for analyzing global adaptation policies exist in the economic growth theory. Many authors (Gradus and Smulders 1993; Smulders and Gradus 1996; Stokey 1998; Elbasha and Roe 1996, Hallegatte 2009, and others) have studied how the optimal economic growth is affected by the environmental change and environmental policies. The negative impact of the environment in macroeconomic models is usually channeled through the utility function (a decreasing *amenity value* of the environment) or the production function (a decreasing *productive value* of the environment). Smulders and Gradus (1996) introduce a *critical environmental quality* at which the environment fails to act as a life-support system. Recent economic models with exhaustible resources (d'Autume and Shubert 2008) also assume that the exhaustible natural capital affects both the productivity and the amenity value. The choice of the environmental economic policy in macroeconomic models is usually represented by expenditures on mitigation measures (environmental abatement or clean-up). Models of optimal cleanup of hazardous wastes (see, Caputo and Wilen 1995) have been probably the first economic models with the negative impact of the environment on both production and utility functions. At the best of authors' knowledge, there is no specific macroeconomic growth model (except for Bréchet, Hritonenko and Yatsenko 2010) that provides a qualitative theoretical analysis of the optimal choice of expenses into adaptation measures. In this paper, we identify and analyze rational strategies of adaptation in the general context of economic-environmental

growth. In the terms of (Bosello *et al* 2010), we are focusing on a planned anticipatory adaptation.

Modeling of rational adaptation policies as a part of global economic-environmental modeling has its own scope and essential specifics (Reilly and Schimmelpfennig 2000, Smit et al 2000, Callaway 2004, Kahn 2003, and others). Namely, the scope of adaptation measures is restricted to the compensation of various negative consequences of climate change, in particular, of the average temperature increase. The spectrum of adaptation measures is wide and can be classified in accordance with negative changes into two categories:

- (a) The adaptation that compensates a decrease of the environmental *amenity value*.
- (b) The adaptation measures that compensate a decrease of the *productivity*.

Our specific motivation is to see whether the negative environmental changes can create certain adaptation-related economic advantages. Such a point of view exists in economic-environmental literature. In particular, Fankhauser et al (1999) emphasize the importance of capital modernization in adaptation and argue that:

- In the majority of cases, adaptation will probably involve the replacement of one type of capital by other.
- Climate changes will require increased flexibility and robustness of new capital, increase capital turn-over, and shorten the economic lifetime of capital.

In this paper we accentuate the role of modernization in adaptation policies and introduce a separate category of *modernization-related adaptation* inside the category (b) using a vintage capital approach. To keep analytic complexity reasonable, we restrict ourselves to comparative static analysis and use the vintage capital model with exponential technological change, in which modernization is implemented through installing new capital vintages and scrapping obsolete

vintages. We focus on long-run interior steady-state solutions of the formulated optimization problem.

The rest of the paper is as follows. In Section 2, we justify and formulate the central planner problem for a closed economy in which a negative environmental impact on consumer utility and economic productivity can be compensated by the environmental adaptation. Section 3 describes a balanced growth regime in the benchmark problem with no environmental change and adaptation. Section 4 analyzes the optimal balance among investment, consumption, and different categories of adaptation expenses and interprets the obtained results. Section 5 concludes. Some proofs are provided in Appendix.

2. Model

To construct a model of environmental adaptation, we start with a basic model of macroeconomic growth (Barro and Sala-i-Martin 2003) and develop it by including an environmental state variable and several adaptation expenses as decision variables. Namely, we assume that the social planner maximizes the utility of identical consumers as the following intertemporal optimization problem:

$$\max_{i, a_1, a_2, a_3} \int_0^{\infty} e^{-\rho t} u(c(t), h(t), a_1(t)) dt, \quad (1)$$

where ρ is the discount rate, h is an environmental hazard parameter, a_1 is the investment into the environmental *amenity* adaptation, and the utility function $u(c, h, a_1)$ decreases in h and increases in a_1 . The consumption c is

$$c(t) = y(t) - i(t) - a_1(t) - a_2(t) - a_3(t), \quad (2)$$

where y is the production output, i is the capital investment, a_2 is the investment into the productivity-related adaptation that uniformly impacts all capital in service, and a_3 is the investment into modernization-related adaptation that affects only the newly created capital. The abatement activities a_1 , a_2 , and a_3 are flows as in (Gradus and Smulders, 1993; Vellinga, 1999).

In order to illustrate the variety of adaptation activities, let us consider the following example³ of negative environmental impact: the water temperature in lakes and rivers will increase with the global temperature increase. It will create an amenity problem for consumers (at least, in warmer climate zones) and decrease the efficiency of nuclear power stations that use cold water for cooling down. Then the three categories of adaptation activities are: a_1 - cooling the water for consumers (using additional equipment inside households), a_2 - cooling the lake and river water on an industrial scale to offset the decreasing efficiency of existing power stations, and a_3 - design and construction of new more advanced power stations that will work efficiently using warmer water for cooling down.

Adaptation through modernization. Following the macroeconomic growth theory, a standard approach would be to describe the product output by a neoclassical production function with homogeneous capital and labor. In this paper, we would like to find out whether the environmental hazard creates economic incentives for intensified capital modernization (adaptation through modernization). To describe the adaptation through capital modernization, we depart from homogeneous capital and assume that the efficiency of different capital units depends on the time of their construction and the modernization effort is applied to the newest capital only (Boucekkine et al 2005). In doing so, we neglect the time-to-build effects and

³ The authors are grateful to Professor Thierry Bréchet for providing this insightful example.

assume that the instant of capital construction is the same as the time of its installation and only the new capital units can be installed to replace the oldest capital units. It leads us to the traditional Solow-type vintage capital models (Solow et al 1966, Boucekkine et al 1998, Hritonenko and Yatsenko 1996, 1999, 2008). Specifically, we describe the product output y by the following vintage model

$$y(t) = \int_{t-\tau(t)}^t f(h(t), a_2(t), a_3(\tau))i(\tau)d\tau \quad (4)$$

with embodied technological change and Leontief technology. In (4), the specific capital efficiency f of the capital vintage τ at time t , $\tau \leq t$, depends on the hazard parameter $h(t)$, the investment $a_2(t)$ into the efficiency of all capital vintages, and the vintage-specific investment $a_3(\tau)$. The endogenous decision variable $\tau(t)$ is the lifetime of the capital vintages scrapped at time t . One of our goals is to establish how the climate change and subsequent adaptation impact the capital lifetime τ .

Remark. Fankhauser and Tol (2005) describe the output y by a neoclassical production function and assume that the deterioration coefficient of a homogeneous capital K increases in the hazard parameter h . In this paper, we do not consider the physical depreciation of capital as a major reason for its replacement. We would like to find out whether the environmental hazard creates an economic need for intensified capital modernization (adaptation through modernization).

The Leontief technology means that the efficiency f does not depend on the amount of labor used by the capital unit. Following macroeconomic vintage models (Solow et al 1966, Boucekkine et al 1998, 2005), we assume that the labor used to produce the output y is

$$\int_{t-\tau(t)}^t e^{-\gamma\tau} i(\tau)d\tau = 1. \quad (5)$$

Equation (5) includes the exponential labor-saving technical progress with the rate $\gamma > 0$ that makes newer capital vintages less labor-consuming. In (5), the total labor is normalized to unity, which means that all variables y , c , i , a_1 , a_2 , and a_3 are per capita (Barro and Sala-i-Martin, 2003). In our model, the investments a_1 , a_2 , and a_3 must be per capita because they affect relative changes of the utility function and the productive function. Otherwise, the problem will have a strong scale effect in the sense that a more populated economy will be able to implement more adaptation measures. This effect is not supported by economic reality if we compare the adaptation efficiency in such countries as, for example, Denmark and India.

The environmental impact on amenity and productivity. Let us clarify the impact of the environment on the utility (1). Some authors (Gradus and Smulders 1993; Stokey 1998; Byrne 1997) choose the environment-dependent utility function to be additively separable with respect to the consumption and environmental hazard. We choose the following utility function

$$u(c, h, 0) = u_0(c)(1 - v_1(h)), \quad (6)$$

in (1) in the absence of adaptation, where the given function $v_1(h)$ describes the relative (rate) decrease of the amenity value of the economy due to the environmental hazard h caused by climate change. The justification of the relationship (6) follows from the fact that an increasing consumption cannot offset the environmental damage after its certain critical level (see Smulders and Gradus 1996). With respect to the consumption c , we choose the CES utility function

$$u_0(c) = c^{1-\sigma} / (1-\sigma), \quad 0 < \sigma < 1, \quad (7)$$

commonly used in related economic-environmental research (Smulders and Gradus 1996, Stokey 1998). We shall notice that the chosen utility specification (6)-(7) implies that the hazards

decrease the level of utility from consumption of all goods without affecting consumers' preferences. In doing so, we avoid the tough question of what kind of environmental damages impact the preference ordering. The dependence of (6) on the third parameter (described in (9) below) allows for a theoretic analysis of various adaptation strategies.

In economic-environmental modeling, the environmental hazard h is usually represented by the level of pollution (e.g., greenhouse gases) or the global average temperature. The amenity value decrease has been thoroughly estimated for specific hazards. In particular, such estimate for the global impact of carbon dioxide emissions is based on the "social cost of carbon", which is the marginal value of the damage done by an additional ton of CO₂ in atmosphere converted to emitting one ton of CO₂ at certain time (Pearce 2003). We assume that $h(t)$ monotonically increases with time from an initial value $h(0)=h_0$ to $h_1>h_0$. The initial value $h(0)$ corresponds to the current state of the economy, so we set $v_1(h_0)=0$ without loss of generality. As in (Adger et al 2005, Boyce 2002, and IPCC 2001) we assume that the environment change will not lead to an economic collapse, so it is reasonable to take $v_1(h_1)<1$. The collapse case corresponds to $v_1(h_1)=1$.

Similarly to (6), we describe the impact of the environmental hazard h on the vintage productivity $f(h,0,0)$ (in the absence of adaptation a_2 and a_3) as

$$f(h,0,0) = b(1 - v_2(h)), \quad (8)$$

where the increasing function $v_2(h)$, $0=v_2(h_0)<v_2(h)<1$, describes the relative productivity decrease because of the environmental hazard h , and b is a fixed productivity parameter. Conceptually, the productivity decrease is much easier to estimate than the amenity decrease in (6).

Adaptation versus induced technological change. A major novelty of the problem under study is that the negative environmental impact can be reduced by the investments a_1 , a_2 and a_3 into adaptation. In the output equation (4), an increase of the productivity (4) caused by investments a_2 and a_3 can also describe the induced output-augmenting technological change (disembodied with respect to a_2 and embodied in a_3). A similar concern can be made about the impact of the *amenity adaptation expense* a_1 on the utility (1). We do not want to mix the investments a_2 and a_3 with the induced technological change and introduce the following adaptation specifications:

$$u(c, h, a_1) = u(c)(1 - v_1(h)\varphi_1(a_1)) \quad \varphi_i(0)=1, \quad i=1,2,3, \quad (9)$$

$$f(h(t), a_2(t), a_3(\tau)) = b \left(1 - v_2(h(t))\varphi_2(a_2(t))\varphi_3(a_3(\tau)) \right) \quad (10)$$

Here, $\varphi_1(a_1)$, $\varphi_2(a_2)$, $\varphi_3(a_3)$ represent compensation abilities of adaption (*the adaptation efficiency functions*) that monotonically decrease from 1 to zero when the corresponding adaptation expense a_i increases, $i=1,2,3$. Formulas (9) and (10) restrict the scope of the decision variables a_1 , a_2 and a_3 to compensating (minimizing) the negative environmental impact of the hazard parameter h . Despite their relative simplicity, specifications (9)-(10) capture major qualitative features of adaptation investments.

Summarizing the above formulas (1)-(10), the optimization problem under study is:

$$\max \int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma}(t)}{1-\sigma} (1 - v_1(h(t))\varphi_1(a_1(t))) dt, \quad (11)$$

$$\alpha(t) = y(t) - i(t) - a_1(t) - a_2(t) - a_3(t), \quad (12)$$

$$y(t) = b \int_{t-T(t)}^t \left(1 - v_2(h(t))\varphi_2(a_2(t))\varphi_3(a_3(\tau)) \right) y(\tau) d\tau, \quad (13)$$

$$\int_{t-T(t)}^t e^{-\gamma\tau} i(\tau) d\tau = 1 \quad (14)$$

Problem (11)-(14) includes four decision variables i , a_1 , a_2 and a_3 and three dependent state variables y , c , T , that satisfy the constraints

$$\dot{i}(t) \geq 0, \quad T'(t) \leq 1, \quad y(t) \geq 0, \quad a_1(t) \geq 0, \quad a_2(t) \geq 0, \quad a_3(t) \geq 0, \quad (15)$$

and the initial conditions

$$T(0) = T_0 > 0, \quad i(\tau) = i_0(\tau), \quad \tau \in [-T_0, 0]. \quad (16)$$

The first two inequalities (15) are standard in vintage capital models. The first one implies that the withdrawal of new (just invested) capital vintages is inefficient. The differential constraint $T'(t) \leq 1$ means that the vintage scrapping time $t-T(t)$ does not decrease, i.e., removed obsolete vintages cannot be introduced again. The initial condition (16) describes the given profile $i_0(\tau)$ of installed capital vintages at time $t=0$ over the given prehistory interval $[-T_0, 0]$.

The properties of the given functions are:

$$dh/dt > 0, \quad h(0) = h_0 = 0, \quad h(t) \rightarrow h_1 \text{ at } t \rightarrow \infty$$

(the environmental hazard $h(t)$ increases from the initial value h_0 to $h_1 > h_0$);

$$\partial v_k / \partial h > 0 \text{ at } h_0 \leq h \leq h_1, \quad v_k(h_0) = 0, \quad v_k(h_1) < 1, \quad k=1,2$$

(both the utility and productivity decrease but remain positive when the environmental hazard increases up to h_1);

$$\partial \varphi_l / \partial a_l < 0 \text{ at } 0 \leq a_l < \infty, \quad \varphi_l(0) = 1, \quad \varphi_l(a_l) > 0, \quad l=1,2,3 \quad (17)$$

(the adaptation investments a_l partially compensate the negative impact of the environmental hazard in (9)-(10)).

3. Benchmark case: model without climate change and adaptation

Let us assume that the environmental hazard parameter h stays the same, $h(t)=h_0$, then $v_1(h(t))=v_2(h(t))=0$ and the optimization problem (11)-(16) is to find c , i , y , and T , such that

$$\max_i \int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma}(t)}{1-\sigma} dt, \quad (18)$$

$$\alpha(t) = y(t) - i(t), \quad (19)$$

$$y(t) = b \int_{t-T(t)}^t i(\tau) d\tau, \quad (20)$$

$$\int_{t-T(t)}^t e^{-\gamma\tau} i(\tau) d\tau = 1, \quad (21)$$

$$0 \leq i(t) \leq y(t), \quad T(t) \geq 0, \quad T'(t) \leq 1, \quad t \in [0, \infty), \quad (22)$$

$$T(0) = T_0, \quad i(\tau) = i_0(\tau), \quad \tau \in [-T_0, 0]. \quad (23)$$

The only decision variable of the problem (18)-(23) is $i \in L_{loc}^{\infty}[0, \infty)$ (the space of all measurable functions bounded on any finite interval from $[0, \infty)$), while y , c and T are the state variables. The *state* variable T is uniquely determined from the initial problem

$$e^{-\gamma(t-T(t))} i(t-T(t)) (1-T'(t)) = e^{-\gamma t} i(t), \quad T(0) = T_0, \quad (24)$$

obtained after differentiating (21).

Lemma 1 (necessary condition for an extremum). *Let (i, y, c, T) be a solution of the optimization problem (18)-(23), then*

$$l_i'(t) = 0 \text{ at } 0 < i(t) < y(t), \quad l_i'(t) \leq 0 \text{ at } i(t) = 0, \quad l_i'(t) \geq 0 \text{ at } i(t) = y(t),$$

where

$$l_i'(t) = \lambda(t) - b \int_t^{[t-T(t)]^{-1}} e^{-\rho(\tau-t)} [1 - e^{-\gamma(t+T(\tau)-\tau)}] \lambda(\tau) d\tau \quad (25)$$

$$\text{and} \quad \lambda(t) = -c^{-\sigma}(t) \quad (26)$$

is the dual variable for equality (19), and $(t-T(t))^{-1}$ is the inverse of $t-T(t)$.

See Appendix for the proof.

As usually in models with endogenous scrapping of obsolete capital, the upper limit of integrals in (25) represents the unknown future lifetime of capital and is related to the unknown scrapping time $t-T(t)$ as its inverse. Second-order conditions for an extremum are more complicated and obtained for similar problems in (Hritonenko and Yatsenko 2008).

We focus on long-run interior steady-state solutions of the optimization problem, which may include equilibrium states and balanced growth paths. Following (Boucekkine *et al* 1998; Hritonenko and Yatsenko 1996, 2008; and others), a *balanced growth path (BGP)* of the vintage model (18)-(23) is a solution to the equations (19)-(21),(25), such that $y(t)$, $i(t)$, and $c(t)$ grow exponentially (not necessarily with the same rate) and the capital lifetime $T(t)$ is constant.

The unknown interior trajectory (j, y, c, T) , if it exists, is determined by the equality $l_i(t)=0$ and (19)-(21). The decision variable i does not enter the optimality conditions (25) explicitly and is found from the state equation (21) under the known y and the initial conditions (23).

Theorem 1. *For $\gamma < r = \rho + \gamma\sigma < b$, there exists an optimal BGP:*

$$y(t) = \bar{y}e^{\gamma t}, \quad i(t) = \bar{i}e^{\gamma t}, \quad c(t) = \bar{c}e^{\gamma t}, \quad T(t) = \text{const}, \quad (28)$$

$$\bar{y} = \frac{b(1 - e^{-\gamma T})}{\gamma T}, \quad \bar{i} = \frac{1}{T}, \quad \bar{c} = \bar{y} - \bar{i}, \quad (29)$$

and the constant optimal lifetime $T > 0$ is a unique solution of the nonlinear equation

$$\frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} = \frac{1}{b}. \quad (30)$$

See Appendix for the proof.

Following (Barro and Sala-i-Martin, 2003), we refer to $r=\rho+\gamma\sigma$ as the *effective discount rate*.

Thus, the BGP exists and its growth rate is equal to the technological change rate γ . The optimal scrapping rule (30) has been thoroughly studied in vintage models of (Hritonenko and Yatsenko 1996, 2005, Boucekkine *et al* 1998). It has been shown that the value T is smaller for a larger γ at $\gamma < \rho \ll 1$; $T \rightarrow \infty$ as $\gamma \rightarrow 0$ and as $\rho \rightarrow b$. Moreover, if $\gamma < \rho \ll 1$, then $T \approx \sqrt{2/(b\gamma)}$. Substituting the last formula into (29), we obtain that the consumption c is positive at the conditions $\gamma \ll 1$ and $\gamma < 2b$, which cover all reasonable economic data.

4. Analysis of the model with adaptation

In this section, we analyze the model (11)-(16) with environmental impact and adaptation. To keep the complexity reasonable, we will continue comparative static analysis of the optimization problem (11)-(16). Our first question is to find out whether this problem has a BGP described by equalities (28) and

$$a_1(t) = \bar{a}_1 e^{\gamma t}, \quad a_2(t) = \bar{a}_2 e^{\gamma t}, \quad a_3(t) = \bar{a}_3 e^{\gamma t}. \quad (31)$$

A preliminary analysis shows that problem (11)-(16) may have a BGP (28),(31) *if only if* the adaptation efficiency functions depend on the levels \bar{a}_k of controls $a_k(t)$ rather than on the controls themselves. So, to continue the comparative static analysis and make meaningful predictions, we need two assumptions:

- The environmental hazard parameter $h(t)$ increases up to the value h_1 in a relatively short time and stays in a small neighborhood of h_1 after.
- The specifications (17) of the adaptation efficiency functions are:

$$\varphi_k(a_k(t)) = \varphi_k(\bar{a}_k), \quad (32)$$

$$\partial\varphi_k/\partial\bar{a}_k < 0 \text{ at } 0 \leq \bar{a}_k < \infty, \quad \varphi_k(0)=1, \quad \varphi_k(t)>0, \quad k=1,2,3. \quad (33)$$

Assumptions (31)-(33) simply mean that the efficiency of adaptation changes in time with the same rate as the technological change. We would like to stress that further qualitative analysis of problem (11)-(16) is possible only under these natural assumptions.

At (31)-(33), the final statement of the optimization problem (11)-(16),(31) is to find functions i , y , c , T and constants \bar{a}_1 , \bar{a}_2 , \bar{a}_3 , that deliver

$$\max \int_0^\infty e^{-\rho t} \frac{c^{1-\sigma}(t)}{1-\sigma} (1 - v_1(h_1)\varphi_1(\bar{a}_1)) dt \quad (34)$$

under

$$\alpha(t) = y(t) - i(t) - \bar{a}_1 e^{\gamma t} - \bar{a}_2 e^{\gamma t} - \bar{a}_3 e^{\gamma t}, \quad (35)$$

$$y(t) = b[1 - v_2(h_1)\varphi_2(\bar{a}_2)\varphi_3(\bar{a}_3)] \int_{t-T(t)}^t i(\tau) d\tau, \quad (36)$$

$$\int_{t-T(t)}^t e^{-\gamma\tau} i(\tau) d\tau = 1, \quad (37)$$

$$\alpha(t) \geq 0, \quad i(t) \geq 0, \quad y(t) \geq 0, \quad \bar{a}_1 \geq 0, \quad \bar{a}_2 \geq 0, \quad \bar{a}_3 \geq 0, \quad T(t) \geq 0, \quad T'(t) \leq 1, \quad (38)$$

$$T(0) = T_0, \quad i(\tau) = i_0(\tau), \quad \tau \in [-T_0, 0]. \quad (39)$$

The extremum condition for this optimization problem is given by the following lemma.

Lemma 2. *Let $(i, y, c, T, \bar{a}_1, \bar{a}_2, \bar{a}_3)$ be a solution of the problem (34)-(39), then the following inequalities hold:*

$$l_i(t) \leq 0 \text{ at } i(t)=0, \quad l_i(t) \geq 0 \text{ at } \alpha(t)=0, \quad l_i(t)=0 \text{ at } i(t)>0 \text{ and } \alpha(t)>0,$$

$$l_{\bar{a}_1}(t) \leq 0 \text{ at } \bar{a}_1=0, \quad l_{\bar{a}_1}(t) = 0 \text{ at } \bar{a}_1>0,$$

$$l_{\bar{a}_2}(t) \leq 0 \text{ at } \bar{a}_2=0, \quad l_{\bar{a}_2}(t) = 0 \text{ at } \bar{a}_2>0,$$

$$l'_{a_3}(t) \leq 0 \text{ at } \bar{a}_3=0, \quad l'_{a_3}(t) = 0 \text{ at } \bar{a}_3 > 0,$$

where

$$l'_i(t) = \lambda(t) - b[1 - v_2(h_1)\varphi_2(\bar{a}_2)\varphi_3(\bar{a}_3)] \int_t^{[t-T(t)]^{-1}} e^{-\rho(\tau-t)} [1 - e^{-\gamma(t+T(\tau)-\tau)}] \lambda(\tau) d\tau, \quad (40)$$

$$l'_{a_1}(t) = c^{-\sigma}(t) \left\{ \alpha(t) v_1(h_1) \partial \varphi_1 / \partial \bar{a}_1 + (1 - \sigma) [1 - v_1(h_1)\varphi_1(\bar{a}_1)] e^{\gamma t} \right\} \lambda(t), \quad (41)$$

$$l'_{a_2}(t) = \left\{ b v_2(h_1)\varphi_3(\bar{a}_3) \frac{\partial \varphi_2}{\partial \bar{a}_2} \int_{t-T(t)}^t i(\tau) d\tau + e^{\gamma t} \right\} \lambda(t), \quad (42)$$

$$l'_{a_3}(t) = \left\{ b v_2(h_1)\varphi_2(\bar{a}_2) \frac{\partial \varphi_3}{\partial \bar{a}_3} \int_{t-T(t)}^t i(\tau) d\tau + e^{\gamma t} \right\} \lambda(t), \quad (43)$$

$$\lambda(t) = -[1 - v_1(h_1)\varphi_1(\bar{a}_1)] c^{-\sigma}(t). \quad (44)$$

The proof extends the proof of Lemma 1.

To derive analytic expressions for the BGP, we choose more specific *adaptation efficiency functions*:

$$\varphi_l(\bar{a}_l) = e^{-k_l \bar{a}_l}, \quad k_l > 0, \quad l = 1, 2, 3, \quad (45)$$

that satisfy conditions (32)-(33). The constants k_1, k_2, k_3 in (45) can be interpreted as the *adaptation efficiency parameters*. According to (28) and (31), the BGP is

$$\begin{aligned} y(t) &= \bar{y} e^{\gamma t}, \quad i(t) = \bar{i} e^{\gamma t}, \quad c(t) = \bar{c} e^{\gamma t}, \quad T(t) = \text{const}, \\ a_1(t) &= \bar{a}_1 e^{\gamma t}, \quad a_2(t) = \bar{a}_2 e^{\gamma t}, \quad a_3(t) = \bar{a}_3 e^{\gamma t}, \end{aligned} \quad (46)$$

where the adaptation expenses are constant parts of the output.

By Lemma 2, an *interior* solution of the optimization problem (34)-(39),(46), if it exists, should satisfy the system of six nonlinear equations

$$\frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} = \frac{1}{b[1 - v_2(h_1)e^{-k_2\bar{a}_2 - k_3\bar{a}_3}]}, \quad (47)$$

$$\bar{y} = b[1 - v_2(h_1)e^{-k_2\bar{a}_2 - k_3\bar{a}_3}] \frac{1 - e^{-\gamma T}}{\gamma T}, \quad \bar{i} = \frac{1}{T}, \quad (48)$$

$$\bar{c} = \bar{y} - \bar{i} - \bar{a}_1 - \bar{a}_2 - \bar{a}_3, \quad (49)$$

$$e^{k_1\bar{a}_1} = v_1(h_1) \left(\frac{k_1\bar{c}}{1 - \sigma} + 1 \right), \quad (50)$$

$$k_2 e^{-k_2\bar{a}_2 - k_3\bar{a}_3} v_2(h_1) \frac{1 - e^{-\gamma T}}{\gamma T} = \frac{1}{b}, \quad (51)$$

$$k_3 e^{-k_2\bar{a}_2 - k_3\bar{a}_3} v_2(h_1) \frac{1 - e^{-\gamma T}}{\gamma T} = \frac{1}{b}, \quad (52)$$

with respect to the positive level variables \bar{y} , \bar{c} , \bar{i} , \bar{a}_1 , \bar{a}_2 , \bar{a}_3 and the constant lifetime T .

Let us first introduce the solution \hat{i} , \hat{y} , \hat{c} , \hat{T} of equations (47)-(49) at $\bar{a}_1 = \bar{a}_2 = \bar{a}_3 = 0$. It represents the optimal balanced growth in the case of climate change when the adaption is not possible. Analogously to Theorem 1, we can prove the following result.

Lemma 3. *If*

$$\gamma < r < b[1 - v_2(h_1)] \quad (53)$$

and $\bar{a}_1 = \bar{a}_2 = \bar{a}_3 = 0$, then the nonlinear system (47)-(49) has a positive solution $(\hat{i}, \hat{y}, \hat{c}, \hat{T})$

where \hat{T} is uniquely determined by (47) and

$$\hat{i} = \frac{1}{\hat{T}}, \quad \hat{y} = b\gamma[1 - v_2(h_1)] \frac{1 - e^{-\gamma\hat{T}}}{\gamma\hat{T}}, \quad \hat{c} = \hat{y} - \hat{i}. \quad (54)$$

In particular,

$$\hat{T} \approx \sqrt{\frac{2}{b\gamma[1 - v_2(h_1)]}} \quad \text{at small } \gamma < r \ll 1. \quad (55)$$

Now let us return to the system (47)-(52). It can be split into two systems: System A of (47), (48), (51), (52) with respect to \bar{y} , \bar{l} , \bar{a}_2 , \bar{a}_3 , T and System B of (49)-(50) in \bar{c} and \bar{a}_1 . System A should be solved first.

We start with the last two equations (51) and (52) of System A. One can see immediately that the adaptation can be positive only if the parameters k_2 , k_3 are larger than some threshold values. Namely, if the efficiency parameter k_2 is too small such that

$$k_2 < 1/bv_2(h_1), \quad (56)$$

then $l'_{a_2}(t) < 0$ by (42), that is, $l'_{a_2}(t)$ cannot be zero, the optimal $\bar{a}_2=0$ is corner by Lemma 2, and equation (51) is not relevant. Analogously, $\bar{a}_3=0$ if $k_3 < 1/bv_2(h_1)$. If both $\bar{a}_2=0$ and $\bar{a}_3=0$, then the positive optimal lifetime $T = \hat{T}$ is determined by Lemma 3.

Let us assume that at least one of \bar{a}_2 and \bar{a}_3 is non-zero. Then, at $k_2 \neq k_3$, two equations (51) and (52) are inconsistent and, by Lemma 2, one of two optimal controls \bar{a}_2 and \bar{a}_3 should be zero :

$$\bar{a}_2 = 0 \text{ if } k_2 < k_3 \quad \text{or} \quad \bar{a}_3 = 0 \text{ if } k_2 > k_3. \quad (57)$$

Hence, only one of (51) or (52) should be considered if $k_2 \neq k_3$, the second one leads to the coner solution. Let $k_2 > k_3$, $\bar{a}_2 \neq 0$, and $\bar{a}_3 = 0$ for definiteness. In this case, \bar{a}_3 is a corner solution and equation (52) is not relevant. Then $k_2 > 1/bv_2(h_1)$, the optimal

$$\bar{a}_2 = \frac{1}{k_2} \ln \left(k_2 b v_2(h_1) \frac{1 - e^{-\gamma T}}{\gamma T} \right) \quad (58)$$

is positive, the constant optimal lifetime T is uniquely determined from the equation

$$\frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} = \frac{1 - e^{-\gamma T}}{b(1 - e^{-\gamma T}) - \gamma T / k_2} \quad (59)$$

and

$$T \approx \sqrt{\frac{2}{b\gamma[1 - 1/(bk_2)]}} \quad \text{at } \gamma \ll r \ll 1. \quad (60)$$

At the given T , \bar{a}_2 , and \bar{a}_3 , we can easily find \bar{y} and \bar{l} from (48).

Finally, if $k_2=k_3$, then the equations (52) and (55) are the same, $\bar{a}_2=\bar{a}_3$, both adaptation possibilities are undistinguishable, and solving System A is similar to the case $k_2 \neq k_3$. Thus, System A has a solution under condition (53).

Let us examine System B next. The optimal amenity adaptation $\bar{a}_1 \geq 0$ can be determined from equation (50). If the efficiency parameter k_1 is small such that

$$k_1 \bar{c} < (1-\sigma)(1/v_1(h_1) - 1), \quad (61)$$

then (50) has no solution and the optimal $\bar{a}_1=0$ by Lemma 2. By (49) and (58), the optimal consumption at $\bar{a}_1=0$ is

$$\bar{c} = \begin{cases} \hat{c} & \text{at } k_2 b < 1/v_2(h_1), \\ \hat{c}_{k_2} & \text{at } k_2 b > 1/v_2(h_1), \end{cases} \quad (62)$$

where \hat{c} is determined by Lemma 3 and

$$\hat{c}_{k_2} = b \frac{1 - e^{-\gamma T}}{\gamma T} - \frac{1}{T} - \frac{1}{k_2} - \frac{1}{k_2} \ln \left(k_2 b v_2(h_1) \frac{1 - e^{-\gamma T}}{\gamma T} \right) \quad (63)$$

The differentiation of (63) in k_2 demonstrates that \hat{c}_{k_2} increases in k_2 , hence, $\hat{c}_{k_2} > \hat{c} > 0$ and the optimal consumption (62) is always positive at $\bar{a}_1=0$.

Let us exclude the endogenous \bar{c} from the condition (61). Substituting (62) into (61), the condition (61) for $\bar{a}_1=0$ becomes

$$\begin{aligned} k_1 \hat{c} &< (1-\sigma)(1/v_1(h_1) - 1) && \text{at } k_2 b < 1/v_2(h_1), \\ k_1 \hat{c}_{k_2} &< (1-\sigma)(1/v_1(h_1) - 1) && \text{at } k_2 b > 1/v_2(h_1). \end{aligned} \quad (64)$$

Inequalities (64) always hold for small k_1 .

Let us now assume that k_1 is not small but such that (64) fails. Then, by (50), the optimal amenity adaptation level is

$$\bar{a}_1 = \frac{1}{k_1} \ln \left[v_1(h_1) \left(\frac{k_1 \bar{c}}{1-\sigma} + 1 \right) \right] > 0. \quad (65)$$

Substituting (65) into equation (49) at known \bar{y} and T , we obtain the equation for \bar{c} :

$$\bar{c} + \frac{1}{k_1} \ln \left(\frac{k_1 \bar{c}}{1-\sigma} + 1 \right) + \frac{1}{k_1} \ln v_1(h_1) = \hat{c}_{k_2}, \quad (66)$$

where $\hat{c}_{k_2} > 0$ is given by (63). The left-hand side of this equation is negative at $\bar{c} = 0$ and increases in \bar{c} up to ∞ . Hence, equation (66) has a unique solution $\bar{c} > 0$. Knowing \bar{c} , we can find \bar{a}_1 by (65). Thus, System B is solved.

We can summarize the above results in the following statement.

Theorem 2. If $r < b[1 - v_2(h_1)]$, then the problem (34)-(39) possesses an optimal BGP (46) with positive \bar{y} , \bar{c} , \bar{l} and finite capital lifetime T . The optimal adaptation investments \bar{a}_1 , \bar{a}_2 , \bar{a}_3 are zero if the corresponding adaptation efficiency parameters k_1 , k_2 , k_3 are small and satisfy (56) and (64). Specifically, $\bar{a}_2 = \bar{a}_3 = 0$ at $k_2 < 1/bv_2(h_1)$ and $k_3 < 1/bv_2(h_1)$. If $k_2 > 1/bv_2(h_1)$ and/or $k_3 > 1/bv_2(h_1)$, then one of \bar{a}_2 and \bar{a}_3 is zero:

$$\bar{a}_2 = 0 \text{ if } k_2 < k_3 \quad \text{or} \quad \bar{a}_3 = 0 \text{ if } k_2 > k_3,$$

and the other component is

$$\bar{a}_i = \frac{1}{k_i} \ln \left(k_i b v_2(h_1) \frac{1 - e^{-\gamma T}}{\gamma T} \right) > 0.$$

At $\gamma < \rho \ll 1$, the optimal capital lifetime T is

$$T \approx \begin{cases} \sqrt{\frac{2}{b\gamma[1-v_2(h_1)]}} & \text{at } \bar{a}_i = 0 \\ \sqrt{\frac{2}{b\gamma[1-1/(bk_i)]}} & \text{at } \bar{a}_i > 0 \end{cases}$$

The optimal adaptation investment \bar{a}_1 is positive and given by (65) if the adaptation efficiency parameters satisfy (64).

Theorem 2 leads to several relevant conclusions about adaptation strategies.

1. *The choice of optimal adaption expenditures.* The optimal strategy is to invest simultaneously into, at most, two adaptation venues: the amenity adaptation and the more efficient out of two productivity adaptations. In the general case, we do not invest simultaneously into both average-productivity and modernization-related adaptations but into the one with larger efficiency parameter k_2 or k_3 .

2. *Similarity of productivity-related adaptation measures in the long run.* The effect of the modernization-related (embodied) adaptation and total productivity (disembodied) adaptation appear to be identical in the long run. Therefore, the relevance of modernization in the whole spectrum of adaptation measures should not be overestimated. The key in making adaptation decisions lies in the adaptation efficiencies k_2 and k_3 . The optimal strategy is to invest into adaptation measures with the higher efficiency, regardless whether they belong to modernization adaptation or average-productivity adaptation. For example, building new dams and seawalls around coastal cities may be less efficient (per unit of investments) than implementing new warning scheme or evacuation plan for the coastal zone.

The second phenomenon is unexpected because the impact of embodied and disembodied TC on modernization is rather different in economic theory (Boucekkine et al 2005). We should notice that this outcome is not trivial and is not a general feature of vintage model (11)-(16). It occurs *only along the BGP* because then the improvements in new capital vintages are being averaged over the future capital life (which is constant along the BGP). A preliminary analysis shows that this outcome will be quite different if we analyze transition dynamics of the model or lift the additional model specifications (32)-(33). We leave that issue for a future study. The next two outcomes have clear and important policy implications.

3. *Synergism between productivity and amenity adaptation activities.* Comparing two threshold conditions (64) demonstrates a certain level of synergism between productivity and amenity adaptation. Namely, if the productivity efficiency parameter k_2 is small, then the amenity efficiency parameter k_1 should satisfy the first threshold condition (64) to guarantee a positive amenity investment \bar{a}_1 . However, if the productivity efficiency parameter k_2 becomes larger, then the second threshold condition (64) for the amenity adaptation k_1 becomes less restrictive (because $\hat{c}_{k_2} > \hat{c}$). It means that the presence of the productivity adaptation increases the final productivity and stimulates the amenity adaptation. For example, accelerated economic recovery of a coastal zone affected by hurricane makes rebuilding houses for the affected population more efficient.

4. *The impact of adaptation on production, consumption, and capital modernization.* Under made assumptions, the climate change and adaptation affect the optimal levels of the economy but do not impact its growth rate (which is determined in our model by the technological change

rate γ). It is an expected result because we have assumed that the climate change affects the amenity and productivity at some percentage but does not decrease them to zero (Adger et al 2005, Boyce 2002). Otherwise, it will be a model of catastrophic change not just adaption.

Let us compare the optimal levels of economy with and without climate change and adaptation. To do that, we denote the BGP components (28) and (29) without climate change as $\tilde{\gamma}$, \tilde{y} , \tilde{c} , \tilde{T} and recall the BGP $\hat{\gamma}$, \hat{y} , \hat{c} , \hat{T} with climate change and no adaption given by (54) and (55). Theorems 1 and 2 and Lemma 3 produce the following

Corollary. If the adaptation efficiency parameters k_1, k_2, k_3 are not small (the optimal adaptation investments are positive), then $\hat{T} > T > \tilde{T}$, $\hat{y} < \bar{y} < \tilde{y}$, $\hat{c} < \bar{c} < \tilde{c}$.

So, as expected, an environmental damage suppresses economic activity and leads to smaller amenity. The optimal adaptation increases the production output, consumption, and investments compared to the case of no adaptation, however, their levels remain smaller than in the case of no damage.

The non-trivial result is that the economically optimal capital lifetime T appears to be *always* larger under environmental damage (with and without adaptation) and the corresponding economic depreciation of capital is always lower (it equals $1/T$). The economic intuition behind this is the same: the environmental damage decreases economic productivity and the rational adaptation can compensate this decrease only partially.

5. Conclusion

1. The paper applies traditional macroeconomic modeling tools (optimal growth, vintage capital models, and comparative static analysis) to study rational environmental adaptation policies to compensate negative consequences of certain environmental hazards, particularly, the average temperature increase. Naturally, an environmental damage decreases both productivity and utility and the rational adaptation measures can compensate them only partially. We focus on the efficiency of specific adaptation measures and, in particular, analyze the role of technological modernization in adaptation activities. The constructed model completely separates the environmental adaptation investments from its more famous counterpart (environmental mitigation) and investments into technical progress. The variety of possible adaptation measures is classified into three categories that: (a) compensate the decrease of the environmental amenity value, (b) compensate the decrease of average productivity, (c) develop and introduce new hazard-protected capital and technology (modernization-related adaptation). The adaptation efficiency parameters are essential in making optimal adaptation decisions and the optimal strategy appears to invest into the adaptation measures with higher efficiency.
2. An important feature of the environmental adaptation is the *synergism* between the productivity-related and amenity-related adaptation activities. It occurs because the productivity-related adaptation positively impacts the economy, which in turn creates better possibilities for the amenity adaptation. The constructed model takes this effect into account when calculating the optimal investments into different adaptation venues.

3. Our results demonstrate that, despite expectations of some authors (Fankhauser *et al*/ 1999), the environmental hazard and subsequent adaptation do not lead to higher level of capital modernization compared to the benchmark case with no hazard, at least, in the long run. The economically optimal lifetime of capital is larger under the environmental damage regardless whether adaptation takes place or not. Thus, the environmental hazard does not create an economic need for permanently intensified capital modernization. Therefore, the relevance of modernization-related adaptation measures should not be overestimated. The alternative point of view is expressed by Fankhauser and Tol (2005) who analyze macroeconomic growth under climate change in four economic frameworks (Solow-Swan, Cass-Koopmans, Mankiw-Romer-Weil, and Romer). They do not use vintages and adaptation expenses and describe the impact of global temperature increase on the longevity of capital postulating that the deterioration rate of a homogeneous capital is higher for larger temperatures. We expect that a similar effect may occur in our model during transition dynamics (compare to (d'Autume and Shubert 2008)) but leave this issue for future study.

6. Appendix

Proof of Lemma 1. Let us substitute (20) to (19) and write the Lagrangian for the optimization problem (18)-(23)

$$L(t) = \int_0^\infty (\theta^{-\rho t} \frac{c^{1-\sigma}(t)}{1-\sigma} + \lambda(t)(\alpha(t) - b \int_{t-T(t)}^t i(\tau) d\tau + i(t)) + \psi(t)(\int_{t-T(t)}^t \theta^{-\gamma\tau} i(\tau) d\tau - 1)) dt$$

where the Lagrange multipliers λ for (19) and ψ is for (21) are dual variables.

The increment of δL with respect to admissible increments δc , δT , δi is

$$\begin{aligned} \delta L &= \int_0^\infty (\theta^{-\rho t} \frac{(\alpha(t) + \delta\alpha(t))^{1-\sigma}}{1-\sigma} + \lambda(t)((\alpha(t) + \delta\alpha(t)) - b \int_{t-T(t)-\delta T(t)}^t (i(\tau) + \delta i(\tau)) d\tau + (i(t) + \delta i(t)) \\ &\quad + \psi(t)(\int_{t-T(t)-\delta T(t)}^t \theta^{-\gamma\tau} (i(\tau) + \delta i(\tau)) d\tau - 1)) dt \\ &\quad - \int_0^\infty (\theta^{-\rho t} \frac{c^{1-\sigma}(t)}{1-\sigma} + \lambda(t)(\alpha(t) - b \int_{t-T(t)}^t i(\tau) d\tau + i(t)) + \psi(t)(\int_{t-T(t)}^t \theta^{-\gamma\tau} i(\tau) d\tau - 1)) dt \\ &= \int_0^\infty (\theta^{-\rho t} \frac{(\alpha(t) + \delta\alpha(t))^{1-\sigma} - c^{1-\sigma}(t)}{1-\sigma} + \lambda(t)(\delta\alpha(t)) - b \int_{t-T(t)}^t \delta i(\tau) d\tau - b \int_{t-T(t)-\delta T(t)}^{t-T(t)} i(\tau) d\tau + \delta i(t)) \\ &\quad + \psi(t)(\int_{t-T(t)}^t \theta^{-\gamma\tau} \delta i(\tau) d\tau + \int_{t-T(t)-\delta T(t)}^{t-T(t)} \theta^{-\gamma\tau} i(\tau) d\tau)) dt \end{aligned}$$

Interchanging limits of integration $\int_0^\infty f(t) \int_{t-T(t)}^t g(\tau) \delta i(\tau) d\tau dt = \int_0^\infty g(t) \int_t^{t-T(t)^{-1}} f(\tau) d\tau \delta i(t) dt$,

applying Taylor series $\frac{(\alpha(t) + \delta\alpha(t))^{1-\sigma} - c^{1-\sigma}(t)}{1-\sigma} = c^{-\sigma}(t) \delta\alpha(t) + \alpha(\delta\alpha(t))$ and

$$\int_{t-T(t)-\delta T(t)}^{t-T(t)} f(\tau) d\tau = f(t-T(t))\delta T(t) + \alpha(\delta T(t)), \text{ excluding } \psi(t) = \lambda(t)b\theta^{\gamma(t-T(t))}, \text{ and collecting}$$

like terms we obtain (25)-(26) .

The conditions $l_i'(t)=0$ at $0 < i(t) < y(t)$, $l_i'(t) \leq 0$ at $i(t)=0$, and $l_i'(t) \geq 0$ at $i(t)=y(t)$ follow from the general necessary extremum condition of the form: $\delta L = L(i+\delta i) - L(i) \leq 0$ for any admissible variation δi . The lemma is proven.

Proof of Theorem 1. According to the definition of the BGP, the solution is to be expressed as $y(t) = \bar{y}e^{\gamma_1 t}$, $i(t) = \bar{i}e^{\gamma_2 t}$, $c(t) = \bar{c}e^{\gamma_3 t}$, $T(t) = T = \text{const}$. Substituting $i(t) = \bar{i}e^{\gamma_2 t}$ and $T(t) = T$ to (21), we obtain that (21) can be held only if $\gamma_2 = \gamma$ and then $\bar{i} = 1/T$. Now, rewriting (20) we obtain

$$y(t) = \bar{y}e^{\gamma_1 t} = b \int_{t-T}^t \frac{e^{\gamma \tau}}{T} d\tau = \frac{b}{\gamma T} (1 - e^{-\gamma T}) e^{\gamma t},$$

from which follows $\gamma_1 = \gamma$ and the first formula of (29). Substitution of $c(t) = \bar{c}e^{\gamma_3 t}$ to (19) leads to $\gamma_3 = \gamma$ and $\bar{c} = \bar{y} - \bar{i}$, that is, (28)-(29) are justified.

Then, from (25)-(26) and (28)-(29) we obtain that the interior solution is obtained from $\left(e^{\gamma t} \right)^{\rightarrow \sigma} e^{-\rho t} = b \int_t^{t+T} \left(e^{\gamma \tau} \right)^{\rightarrow \sigma} e^{-\rho \tau} (1 - e^{-\gamma T} e^{\gamma(\tau-T)}) d\tau$. Evaluating the integral and introducing the notation $r = \rho + \gamma \sigma$, we obtain (30).

To prove that equation (30) has a unique solution, let us denote its left-hand side as

$$F(T) = \frac{1 - e^{-rT}}{r} - \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma}. \text{ Because } F(0) = 0 \text{ and } \lim_{T \rightarrow \infty} F(T) = 1/r, \text{ equation (30) has a solution}$$

if $r < b$. The solution is unique because the function $F(T)$ is strictly increasing in T :

$$F'(T) = \gamma \frac{e^{-\gamma T} - e^{-rT}}{r - \gamma} > 0 \text{ as } r > \gamma. \text{ The theorem is proven.}$$

References

1. Adger, W.N., Arnell, W.A., Tomkins, E.L. (2005). Successful adaptation to climate change across scales, *Global Environmental Change* 15, 77–86
2. Barro, R.J. and X.Sala-i-Martin (2003) *Economic Growth*, The MIT Press, Massachusetts
3. Boucekine, R., Germain, M., Licandro, O., Magnus, A. (1998). Creative destruction, investment volatility and the average age of capital, *Journal of Economic Growth*, 3, 361-384.
4. Boucekine, R., del Rio, F., Licandro, O. (2005). Obsolescence and modernization in the growth process, *Journal of Development Economics*, 77, 153-171.
5. Bosello, F (2008). Adaptation, mitigation and green R&D to combat global climate change: Insights from an empirical integrated assessment exercise, CCMC Research Paper 20.
6. Bosello, F., Carraro, C., de Cain, E. (2010). Climate policy and the optimal balance between mitigation, adaptation and unavoided damage, *Climate Change Economics*, 1, 71-92.
7. Boyce, J.K., (2002). *The Political Economy of the Environment*. Edward Elgar, Cheltenham.
8. Bréchet, T., Hritonenko, N., Yatsenko, Y. (2010). Adaptation and mitigation in long-term climate policies. CORE Discussion Paper 2010/65.
9. Byrne, M. (1997). Is growth a dirty word? Pollution, abatement and economic growth, *Journal of Development Economics* 54, 261–284.
10. Callaway, J.M., (2004). Adaptation benefits and costs: are they important in the global policy picture and how can we estimate them? *Global Environmental Change* 14, 273–282.
11. Caputo, M., Wilen, J. (1995). Optimal cleanup of hazardous wastes, *International Economic Review*, 36, 217-243.
12. d’Autume, A., Shubert, K. (2008). Hartwick’s rule and maximin paths when the exhaustible resource has an amenity value, *Journal of Environmental Economics and Management*, 56, 260-274.
13. De Bruin, K., Dellink, R., Agrawala, S. (2009). Economic aspects of adaptation to climate change: Integrated assessment modelling of adaptation costs and benefits, OECD Environment Working Papers 6.
14. de Bruin, K., Dellink, R., Tol, R. (2009). AD-DICE: An implementation of adaptation in DICE model, *Climatic Change*, 95, 63-81.
15. Elbasha, E.H., Roe, T.L. (1996). On endogenous growth: the implications of environmental externalities, *Journal of Environmental Economics and Management*, 31, 240-268.
16. Fankhauser, S., Smith, J.B., Tol, R.S.J. (1999). Weathering climate change: some simple rules to guide adaptation decisions, *Ecological Economics* 30, 67–78.
17. Fankhauser, S., Tol, R.S.J., (2005). On climate change and economic growth, *Resource and Energy Economics* 27, 1–17.
18. Gradus, R., Smulders, S. (1993). The trade-off between environmental care and long-term growth–pollution in three prototype models, *Journal of Economics*, 58, 25-51.
19. Hallegatte, S. (2009) Strategies to adapt to an uncertain climate change, *Global Environmental Change*, 19, 240-247.

20. Hof, A.F., de Bruin, K.C., Dellink, R.B., den Elzen, M.G.J., van Vuuren, D.P. (2009) The effect of different mitigation strategies on international financing of adaptation, *Environmental Science and Policy*, 12, 832-843.
21. Hope, C., Anderson, J., Wenman, P. (1993). Policy analysis of the greenhouse effect: An application of the PAGE model, *Energy Policy*, 21, 327-338.
22. Hritonenko, N., Yatsenko, Yu. (1996), Integral-functional equations for optimal renovation problems, *Optimization*, 36, 249-261.
23. Hritonenko, N., Yatsenko, Yu. (1999). *Mathematical Modeling in Economics, Ecology and the Environment*, Kluwer Academic Publishers, Dordrecht /Boston /London.
24. Hritonenko, N., Yatsenko, Yu. (2005). Turnpike and optimal trajectories in integral dynamic models with endogenous delay, *Journal of Optimization Theory and Applications*, 127, 109-127.
25. Hritonenko, N., Yatsenko, Yu. (2008). Optimal control of Solow vintage capital model with nonlinear utility, *Optimization*, 57, 581-590.
26. Hritonenko, N., Yatsenko, Yu. (2010). Technological innovations, economic renovation, and anticipation effects, *Journal of Mathematical Economics*, 46, 1064-1078.
27. Intergovernmental Panel on Climate Change (IPCC), (2001). *Climate Change 2001: Impacts, Adaptation and Vulnerability. Summary for Policy Makers*. World Meteorological Organisation, Geneva.
28. Kahn, M.E. (2003) Two measures of progress in adapting to climate change. *Global Environmental Change* 13, 307–312.
29. Pearce, D. (2003) The social cost of carbon and its policy implications, *Oxford Review of Economic Policy*, 19, 362-384
30. Reilly, J., Schimmelpfennig, D., (2000). Irreversibility, uncertainty, and learning: portraits of adaptation to long-term climate change, *Climatic Change* 45, 253–278.
31. Shakhova, N., Semiletov, I., Salyuk, A., Yusupov, V., Kosmach, D., Gustafsson, Ö. (2010). Extensive methane venting to the atmosphere from sediments of the East Siberian Arctic Shelf, *Science* 327, 1246-1250.
32. Smit, B., Burton, I., Klein, R.J.T., Wandel, J., (2000). An anatomy of adaptation to climate change and variability. *Climatic Change* 45, 223–251.
33. Smulders, S., Gradus, R. (1996). Pollution abatement and long-term growth, *European Journal of Political Economy* 12, 505–532.
34. Solow, R., Tobin, J., Von Weizsacker, C., Yaari, M. (1966) Neoclassical growth with fixed factor proportions, *Review of Economic Studies*, 33, 79-115.
35. Stokey, N. (1998). Are there limits to growth? *International Economic Review*, 39, 1-31
36. Vellinga, N. (1999). Multiplicative utility and the influence of environmental care on the short-term economic growth rate, *Economic Modelling* 16, 307-330.

Recent titles

CORE Discussion Papers

- 2010/70. Fred SCHROYEN and Adekola OYENUGA. Optimal pricing and capacity choice for a public service under risk of interruption.
- 2010/71. Carlotta BALESTRA, Thierry BRECHET and Stéphane LAMBRECHT. Property rights with biological spillovers: when Hardin meets Meade.
- 2010/72. Olivier GERGAUD and Victor GINSBURGH. Success: talent, intelligence or beauty?
- 2010/73. Jean GABSZEWICZ, Victor GINSBURGH, Didier LAUSSEL and Shlomo WEBER. Foreign languages' acquisition: self learning and linguistic schools.
- 2010/74. Cédric CEULEMANS, Victor GINSBURGH and Patrick LEGROS. Rock and roll bands, (in)complete contracts and creativity.
- 2010/75. Nicolas GILLIS and François GLINEUR. Low-rank matrix approximation with weights or missing data is NP-hard.
- 2010/76. Ana MAULEON, Vincent VANNETELBOSCH and Cecilia VERGARI. Unions' relative concerns and strikes in wage bargaining.
- 2010/77. Ana MAULEON, Vincent VANNETELBOSCH and Cecilia VERGARI. Bargaining and delay in patent licensing.
- 2010/78. Jean J. GABSZEWICZ and Ornella TAROLA. Product innovation and market acquisition of firms.
- 2010/79. Michel LE BRETON, Juan D. MORENO-TERNERO, Alexei SAVVATEEV and Shlomo WEBER. Stability and fairness in models with a multiple membership.
- 2010/80. Juan D. MORENO-TERNERO. Voting over piece-wise linear tax methods.
- 2010/81. Jean HINDRIKS, Marijn VERSCHELDE, Glenn RAYP and Koen SCHOORS. School tracking, social segregation and educational opportunity: evidence from Belgium.
- 2010/82. Jean HINDRIKS, Marijn VERSCHELDE, Glenn RAYP and Koen SCHOORS. School autonomy and educational performance: within-country evidence.
- 2010/83. Dunia LOPEZ-PINTADO. Influence networks.
- 2010/84. Per AGRELL and Axel GAUTIER. A theory of soft capture.
- 2010/85. Per AGRELL and Roman KASPERZEC. Dynamic joint investments in supply chains under information asymmetry.
- 2010/86. Thierry BRECHET and Pierre M. PICARD. The economics of airport noise: how to manage markets for noise licenses.
- 2010/87. Eve RAMAEKERS. Fair allocation of indivisible goods among two agents.
- 2011/1. Yu. NESTEROV. Random gradient-free minimization of convex functions.
- 2011/2. Olivier DEVOLDER, François GLINEUR and Yu. NESTEROV. First-order methods of smooth convex optimization with inexact oracle.
- 2011/3. Luc BAUWENS, Gary KOOP, Dimitris KOROBILIS and Jeroen V.K. ROMBOUTS. A comparison of forecasting procedures for macroeconomic series: the contribution of structural break models.
- 2011/4. Taoufik BOUEZMARNI and Sébastien VAN BELLEGEM. Nonparametric Beta kernel estimator for long memory time series.
- 2011/5. Filippo L. CALCIANO. The complementarity foundations of industrial organization.
- 2011/6. Vincent BODART, Bertrand CANDELON and Jean-François CARPANTIER. Real exchanges rates in commodity producing countries: a reappraisal.
- 2011/7. Georg KIRCHSTEIGER, Marco MANTOVANI, Ana MAULEON and Vincent VANNETELBOSCH. Myopic or farsighted? An experiment on network formation.
- 2011/8. Florian MAYNERIS and Sandra PONCET. Export performance of Chinese domestic firms: the role of foreign export spillovers.
- 2011/9. Hiroshi UNO. Nested potentials and robust equilibria.
- 2011/10. Evgeny ZHELOBODKO, Sergey KOKOVIN, Mathieu PARENTI and Jacques-François THISSE. Monopolistic competition in general equilibrium: beyond the CES.
- 2011/11. Luc BAUWENS, Christian HAFNER and Diane PIERRET. Multivariate volatility modeling of electricity futures.

Recent titles

CORE Discussion Papers - continued

- 2011/12. Jacques-François THISSE. Geographical economics: a historical perspective.
- 2011/13. Luc BAUWENS, Arnaud DUFAYS and Jeroen V.K. ROMBOUTS. Marginal likelihood for Markov-switching and change-point GARCH models.
- 2011/14. Gilles GRANDJEAN. Risk-sharing networks and farsighted stability.
- 2011/15. Pedro CANTOS-SANCHEZ, Rafael MONER-COLONQUES, José J. SEMPERE-MONERRIS and Oscar ALVAREZ-SANJAIME. Vertical integration and exclusivities in maritime freight transport.
- 2011/16. Géraldine STRACK, Bernard FORTZ, Fouad RIANE and Mathieu VAN VYVE. Comparison of heuristic procedures for an integrated model for production and distribution planning in an environment of shared resources.
- 2011/17. Juan A. MAÑEZ, Rafael MONER-COLONQUES, José J. SEMPERE-MONERRIS and Amparo URBANO. Price differentials among brands in retail distribution: product quality and service quality.
- 2011/18. Pierre M. PICARD and Bruno VAN POTTELSBERGHE DE LA POTTERIE. Patent office governance and patent system quality.
- 2011/19. Emmanuelle AURIOL and Pierre M. PICARD. A theory of BOT concession contracts.
- 2011/20. Fred SCHROYEN. Attitudes towards income risk in the presence of quantity constraints.
- 2011/21. Dimitris KOROBILIS. Hierarchical shrinkage priors for dynamic regressions with many predictors.
- 2011/22. Dimitris KOROBILIS. VAR forecasting using Bayesian variable selection.
- 2011/23. Marc FLEURBAEY and Stéphane ZUBER. Inequality aversion and separability in social risk evaluation.
- 2011/24. Helmuth CREMER and Pierre PESTIEAU. Social long term care insurance and redistribution.
- 2011/25. Natali HRITONENKO and Yuri YATSENKO. Sustainable growth and modernization under environmental hazard and adaptation.

Books

- P. VAN HENTENRYCKE and L. WOLSEY (eds.) (2007), *Integration of AI and OR techniques in constraint programming for combinatorial optimization problems*. Berlin, Springer.
- P-P. COMBÈS, Th. MAYER and J-F. THISSE (eds.) (2008), *Economic geography: the integration of regions and nations*. Princeton, Princeton University Press.
- J. HINDRIKS (ed.) (2008), *Au-delà de Copernic: de la confusion au consensus ?* Brussels, Academic and Scientific Publishers.
- J-M. HURIOT and J-F. THISSE (eds) (2009), *Economics of cities*. Cambridge, Cambridge University Press.
- P. BELLEFLAMME and M. PEITZ (eds) (2010), *Industrial organization: markets and strategies*. Cambridge University Press.
- M. JUNGER, Th. LIEBLING, D. NADDEF, G. NEMHAUSER, W. PULLEYBLANK, G. REINELT, G. RINALDI and L. WOLSEY (eds) (2010), *50 years of integer programming, 1958-2008: from the early years to the state-of-the-art*. Berlin Springer.
- G. DURANTON, Ph. MARTIN, Th. MAYER and F. MAYNERIS (eds) (2010), *The economics of clusters – Lessons from the French experience*. Oxford University Press.
- J. HINDRIKS and I. VAN DE CLOOT (eds) (2011), *Notre pension en héritage*. Itinera Institute.

CORE Lecture Series

- D. BIENSTOCK (2001), Potential function methods for approximately solving linear programming problems: theory and practice.
- R. AMIR (2002), Supermodularity and complementarity in economics.
- R. WEISMANTEL (2006), Lectures on mixed nonlinear programming.