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on the optimal hedge ratio

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**The asymmetric commodity inventory effect
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Abstract

Hedging strategies for commodity prices largely rely on dynamic models to compute optimal hedge ratios. This paper illustrates the importance of considering the commodity inventory effect (effect by which the commodity price volatility increases more after a positive shock than after a negative shock of the same magnitude) in modelling the variance-covariance dynamics. We show by in-sample and out-of-sample forecasts that a commodity price index portfolio optimized by an asymmetric BEKK-GARCH model outperforms the symmetric BEKK, static (OLS) or naïve models. Robustness checks on a set of commodities and by an alternative mean-variance optimization framework confirm the relevance of taking into account the inventory effect in commodity hedging strategies.

Keywords: BEKK, commodity, asymmetries, hedging, inventory effect.

JEL Classification: G13, C32, Q02.

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1 Introduction

Minimizing the investment risk through futures hedging is a technique widely used in many fields (currencies, stocks, commodities). Static regression techniques were first developed to estimate optimal hedge ratios (hereafter referred to as OHR). Dynamic versions, computed with GARCH models, were then proposed and shown to perform better than static ones (Baillie and Myers (1991), Kroner and Sultan (1993) and Lien, Tse, and Tsui (2002) among others).

One shortcoming of these papers is that they overlook the potential asymmetries characterizing the dynamics of some assets' return volatility. This is the case of stock prices, for which the increase in volatility is usually larger when the returns are negative than when they are positive. The traditional explanation of this asymmetry is related to the balance-sheet effect. Falling returns give rise to a deterioration of the debt-to-equity ratio, which raises the probability of default. This effect, originally put forward by Black (1976), is usually referred to as the "*leverage effect*".

Commodity prices are another example of assets characterized by an asymmetric dynamics in the volatility. On the contrary to the leverage effect, positive shocks to commodity returns give rise to an increase in volatility larger than the one subsequent to a negative shock. This phenomenon was already mentioned by Ng and Pirrong (1994) who, building on the theory of storage, find for metal prices that inventory conditions, proxied by the basis (i.e. the spot-futures spread), affect the volatility of the series asymmetrically. Intuitively, a shortage drives the prices up and increase the volatility/nervosity of the markets, while large inventories, tend to put downward pressure on prices, and lead to relatively low volatility. This inventory effect is documented by Carpentier 2010 and found to be relatively robust, for different subsamples, for diverse types of commodities and for different ways of specifying the asymmetry, though not so strong as the leverage effect regarding equity stocks.

Some papers take into account asymmetries in their dynamic OHR applications such as Lien and Yang (2006) for currency markets and Brooks, Henry, and Persaud (2002) for stock returns. With respect to commodities, Lien and Yang (2006) use a bivariate dynamic conditional correlation model (Engle (2002)) capturing the inventory asymmetries by adding a decomposition of the basis (into negative and positive terms) in the variance equation. Their approach relies on the theory of storage as laid out by Ng and Pirrong (1994) and takes the basis (as decomposed) as proxy of the state of the inventories.

We propose an alternative approach. The basis is not the sole proxy for inventories. As studied by Gorton, Hayashi, and Rouwenhorst (2007), spot return are also good indicators of the state of inventories. In addition, futures data is potentially more affected by speculation related to time-varying risk premium considerations. Since asymmetries are strictly related to inventory data, spot returns are potentially less noisy and good alternatives

to the basis. We thus use a model close to the asymmetric BEKK-GARCH specification developed by Kroner and Ng (1998), which is a multivariate extension of the univariate GJR-GARCH model developed by Glosten, Jagannathan and Runkle (1993). They share the feature of allowing asymmetric shocks in the models, but contrary to their specifications our model (hereinafter Asym-BEKK) captures specifically the positive shocks, instead of the negative ones, in view to highlight commodities inventory effects.

The goals of this paper are twofold. First, we investigate and illustrate the relevance of taking into account the commodities inventory effect in hedging techniques through an Asym-BEKK model. Second, we examine and compare the performance of our methodology through portfolio variance minimization, in-sample and out-of-sample forecasts. This paper is laid out as follows: Section 2 introduces to hedging and derives the OHR, Section 3 presents the econometric methodology, while Section 4 describes the data. Section 5 presents the results, Section 6 some extensions and Section 7 concludes.

2 Hedging

A hedge is achieved by taking opposite positions in the spot and futures market at the same time, so that losses supported in one market are, to some extent, offset by opposite price movements in the other market. The number of futures contract units needed to be issued depends on the covariance of the spot and futures contracts, as well as the variances of futures and spot contracts. The OHR of futures to spot contract, the one minimizing the variance of the hedged portfolio, is derived as follows.

Let S_t and F_t denote the logarithm of the spot and futures commodity prices, respectively. The return on a position in spot is $\Delta S_t = S_t - S_{t-1}$ and on a futures position is $\Delta F_t = F_t - F_{t-1}$. The expected return on a portfolio, R_t comprising one unit of the commodity and β units of the futures contract would then be:

$$E_{t-1}(R_t) = E_{t-1}(\Delta S_t) - \beta_{t-1}E_{t-1}(\Delta F_t) \quad (1)$$

where β_{t-1} is the hedge ratio for time t determined at $t - 1$. The variance of the portfolio is:

$$VAR(R_t) = VAR(\Delta S_t) + \beta_{t-1}^2 VAR(\Delta F_t) - 2\beta_{t-1} COV(\Delta S_t, \Delta F_t) \quad (2)$$

Combining the portfolio variance in a two-moment utility function we get:

$$U(E_{t-1}(R_t), VAR(R_t)) = E_{t-1}(R_t) - \theta VAR(R_t) \quad (3)$$

where θ is the risk aversion parameter. Maximizing the utility $U(E_{t-1}(R_t), VAR(R_t))$ is equivalent to solve:

$$\begin{aligned} \max \quad & E_{t-1}(\Delta S_t) - \beta_{t-1}E_{t-1}(\Delta F_t) \\ & -\theta(VAR(\Delta S_t) + \beta_{t-1}^2 VAR(\Delta F_t) - 2\beta_{t-1}COV(\Delta S_t, \Delta F_t)) \end{aligned} \quad (4)$$

Solving Equation 4 under the assumption that F_t follows a martingale process, i.e. $E_{t-1}(\Delta F_t) = 0$, yields the OHR β_{t-1}^* :

$$\beta_{t-1}^* = \frac{COV(\Delta S_t, \Delta F_t)}{VAR(\Delta F_t)} \quad (5)$$

$$= \rho_t \sqrt{VAR(\Delta S_t)/VAR(\Delta F_t)} \quad (6)$$

where ρ_t is the correlation between spot and futures returns.

3 Methodology

The simplest way to estimate the OHR laid out in Equation 6 is to assume that spot and futures returns are perfectly correlated and the spot and futures variances equal to each other. Under these assumptions, the OHR is $\beta_{t-1}^* = 1$. This is the *naïve* model, that we use later as comparison point for more sophisticated models.

Another simplist approach is to assume that the variances and covariance of spot and futures returns are constant over time. In this case, $\beta^* = \frac{COV(\Delta S, \Delta F)}{VAR(\Delta F)}$ is just the coefficient of a linear regression of the spot return on the futures return. This is the OLS model, that we also use later as a benchmark.

As reported by Baillie and Myers (1991), Kroner and Sultan (1993) and Lien, Tse, and Tsui (2002), early models assuming that the variance-covariance matrix of returns is constant over time proved too restrictive. Multivariate GARCH models relax such assumption and are largely used in this context (see Bauwens, Laurent, and Rombouts (2006) for a survey of MGARCH models).

Kroner and Sultan (1993) use the constant conditional correlation (CCC) model of Bollerslev (1990). The main weakness of this model is that conditional variances may vary over time but not covariances. As documented in Ng and Pirrong (1994), the covariance of spot and futures commodity returns tend to decrease when the inventories get tight, according to the theory of storage. Therefore, this multivariate GARCH model is not well-suited for our purposes.

Lien, Tse, and Tsui (2002) use a dynamic version of the CCC called dynamic conditional correlation model, as developed by Engle (2002), where the correlation is now allowed to vary over time. Again, this model is rather restrictive in the sense that the correlation is only affected by its own past and cannot be affected, for example, by past shocks of the spot variance. The theory of storage indeed suggests that past spot variance should affect negatively the conditional covariance.

The model used by Baillie and Myers (1991) is another multivariate GARCH model, labelled BEKK model, defined in Engle and Kroner (1995). The bivariate BEKK specification is the following:

$$y_t = \mu_t(\eta) + \epsilon_t \quad (7)$$

$$\epsilon_t = H_t^{1/2}(\eta)z_t \quad (8)$$

$$z_t \rightarrow N(0, I_2) \quad (9)$$

$$H_t(\eta) = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B \quad (10)$$

where y_t is the vector of spot and futures return (ΔS_t and ΔF_t), $\mu_t(\eta)$ their conditional mean depending on parameters θ such as an ARMA process, z_t is the vector of standardized residuals, H_t the 2×2 covariance matrix of the spot and futures return, A and B are 2×2 coefficient matrices and C is an upper triangular 2×2 matrix.

The advantages of this BEKK configuration is 1) the guarantee, by construction due to the quadratic form, that the variance-covariance matrix is positive-definite, 2) the time varying structure of all its components (variances and covariance of spot and futures returns) and 3) the potential cross-variables influences of the spot and futures components (each depend on a constant, on past variances of spot and futures, on past covariance, on past squared spot shock, on past squared futures shock and on cross product of past spot and futures shocks). We also use the BEKK model as a comparison point.

As mentioned in the introduction, the theory of storage predicts different channels of influence of the inventories on the volatility. Using the spot and futures shocks as proxies of the state of the inventory, following in this sense Gorton, Hayashi, and Rouwenhorst (2007), an appropriate multivariate GARCH model should capture the following features:

1. Spot variance should be more reactive to past shocks than futures variances
2. Spot and Futures covariance should decrease when the spot variance increases
3. Positive shocks should have a larger impact on the volatility than negative shocks (the inventory effect)

Since the models presented thus far, naïve, OLS and BEKK models, do not allow to measure and capture this last channel of influence, the inventory effect, and given its

relative importance as documented in Carpentier (2010), we use an extension of the BEKK model proposed by Kroner and Ng (1998) where the variance-covariance matrix not only depends on the magnitude of past squared return innovations and variance-covariance matrix but also on the sign of the past squared return innovations. The asymmetric model is the same as the BEKK model, except the covariance-variance matrix which is specified as follows:

$$H_t(\eta) = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B + D'\zeta_{t-1}\zeta'_{t-1}D \quad (11)$$

where D is a 2×2 matrix and ζ_t , the asymmetric term, is a two-variable vector defined as $\zeta_t = \max(\epsilon_t, 0)$. Hence, the elements of D represent the effect of past (positive signed) innovations corresponding to the inventory effect. As mentioned above, the Asym-BEKK model is a multivariate transposition of the univariate GJR-GARCH model, developed by Glosten Jagannathan and Runkle (1993) but where the asymmetric term captures the positive shocks instead of the negative ones. To have a full understanding of the channels of interactions, we detail in Equations 12, 13 and 14 the respective equations of the spot variance, futures variance and spot-futures covariance, with Greek letters representing combinations of the elements of coefficient matrices.

$$\begin{aligned} VAR(\Delta S_t) = h_{11,t} = \gamma_{11} & \\ & + \alpha'_{11}\epsilon_{1,t-1}^2 + \alpha''_{11}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha'''_{11}\epsilon_{2,t-1}^2 \\ & + \beta'_{11}h_{11,t-1} + \beta''_{11}h_{12,t-1} + \beta'''_{11}h_{22,t-1} \quad (12) \\ & + \delta'_{11}\zeta_{1,t-1}^2 + \delta''_{11}\zeta_{1,t-1}\zeta_{2,t-1} + \delta'''_{11}\zeta_{2,t-1}^2 \end{aligned}$$

$$\begin{aligned} COV(\Delta S_t, \Delta F_t) = h_{21,t} = h_{12,t} = \gamma_{12} & \\ & + \alpha'_{12}\epsilon_{1,t-1}^2 + \alpha''_{12}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha'''_{12}\epsilon_{2,t-1}^2 \\ & + \beta'_{12}h_{11,t-1} + \beta''_{12}h_{12,t-1} + \beta'''_{12}h_{22,t-1} \quad (13) \\ & + \delta'_{12}\zeta_{1,t-1}^2 + \delta''_{12}\zeta_{1,t-1}\zeta_{2,t-1} + \delta'''_{12}\zeta_{2,t-1}^2 \end{aligned}$$

$$\begin{aligned} VAR(\Delta F_t) = h_{22,t} = \gamma_{22} & \\ & + \alpha'_{22}\epsilon_{1,t-1}^2 + \alpha''_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + \alpha'''_{22}\epsilon_{2,t-1}^2 \\ & + \beta'_{22}h_{11,t-1} + \beta''_{22}h_{12,t-1} + \beta'''_{22}h_{22,t-1} \quad (14) \\ & + \delta'_{22}\zeta_{1,t-1}^2 + \delta''_{22}\zeta_{1,t-1}\zeta_{2,t-1} + \delta'''_{22}\zeta_{2,t-1}^2 \end{aligned}$$

In Table 1 we provide the correspondence between the Greek letters in Equations 12, 13 and 14 and the components of matrices C , B , A and D in Equation 11. It can be seen that instead of directly interpreting the estimated coefficients, we are referring to their

nonlinear combination as the effect of the past variables on the conditional variances and covariance. While the ML estimate of a non-linear function of parameters is calculated as the function of the ML estimate of the parameters, the standard error of the function is done by using “*Delta method*”. This method uses the jacobian matrix of the nonlinear function, evaluated at the estimated parameters.

Finally, the log-likelihood function when the Gaussian distribution is assumed becomes of the following form:

$$L(\eta) = -\frac{TN}{2}(\ln(2\pi)) - 1/2 \sum_{i=1}^T (\ln |H_i| + \epsilon'_i H_i^{-1} \epsilon'_i) \quad (15)$$

where T is the number of observations, N is the number of variables (series) in the system.

4 Data

To illustrate the potential importance of the asymmetries for commodity hedging, we use daily data of the S&P GSCI Index (formerly the Goldman Sachs Commodity Index). This index represents a benchmark measure of performance for the commodity market over time. The index is traded at Chicago Mercantile Exchange. The weight of energy is by far higher than other constituents and it amounts to slightly more than 68%. Other components, as of May 2010, are: Industrial Metals (8.49%), Precious Metals (3.74%), Agriculture (13.7%), and Livestock (5.24%), overall the index is composed of 24 commodities. Because of the broad diversification the index is also interpreted as a measure of general price movements and inflation in the world economy. In this paper we use GSCI Spot Month and GSCI Three Month Future indices, which span the data from January 17, 1995 to April 15, 2010 resulting in 3978 observations. The Data were collected from Datastream database under respective codes CGSYSPT and GS3MSPT.

The evolution of the two indices over the sample period is shown in Figure 1. When tested for unit root, using Augmented Dickey-Fuller (ADF) test, these series are found to be integrated of order one. However, after taking log-differences the data become stationary. The graph of these daily log-return series is given in Figure 2.

Summary statistics are presented in Table 2. As predicted by the theory of storage and documented by Ng and Pirrong (1994), the standard deviation of spot return is larger than for futures return. When inventories are large shocks affect both equally spot and futures volatility, but when stocks are low, spot volatility is more affected by a potential stock out than futures for which production has (more) time to adjust. Spot return also has more extreme values (min and max) than futures, following the same logic.

Box-Pierce test on squared series and Lagrange Multiplier test of Engle (1982), reported in Table 2, also indicate the presence of heteroskedasticity in the series, which together suggest that using GARCH models would be relevant.

5 Results

Our objective is to compare different models in terms of hedging performance and test if adding the asymmetric term to the model delivers better results for hedging a (potential) position in the GSCI index. To do so, we estimate time-varying hedge ratios using two different specifications for the conditional variance-covariance matrix, which are then compared with the constant hedge ratio, obtained from the OLS model and the naïve hedge, i.e. the hedge ratio being equal to 1. The naïve hedge implies opening an opposite position in futures contracts of the same amount as spot contracts are held at the same time (one-to-one relationship). Below we review the results obtained from estimation of the above models.

Most basic estimate of the OHR is obtained from the OLS model, where it represents simply an estimated slope coefficient, which is constant over time. Table 3 shows that the OHR estimated in this way equals to 1.129, meaning that an investor interested in reducing the variance of his/her portfolio would have to sell (buy) 1.129 futures contracts for every spot contract in which (s)he has a long (short) position.

However, relaxing the assumption of a constant OHR and using time-varying variance-covariance matrix as an input for calculating OHR gives more flexibility. The parameter estimates for the multivariate BEKK-GARCH model are presented in Table 4.

The first panel of Table 4 shows the results for the conditional mean Equations, which are specified as AR(1) processes for both spot and futures series. Following Baillie and Myers (1991), we estimate autoregressive models and do not work in a cointegration framework. We indeed found that error correction terms were not significant in the mean equations¹. *P*-values are presented in parenthesis and, as can be seen in both cases, the lag coefficient is statistically different from zero, while the constant is insignificant. Instead of this short-term dependence of the spot and futures returns, a random walk was expected according to the efficient market hypothesis. The significance of the lag return is however not an uncommon finding. Lien and Yang (2008), for example, also find significant lags in the mean equations of their commodities. Stock returns also often exhibit short-term dependence, for which potential sources are transaction costs or behavioral patterns (as discussed in the review of Malkiel (2003) or the chapter on predictability of asset returns in Campbell,

¹Results available on request from the authors.

Lo and MacKinlay (1996)).

Panel two of Table 4 displays the coefficient estimates for the conditional variance-covariance matrix. The elements of A and B matrices represent the ARCH and GARCH effects respectively and C is an upper-triangular matrix representing the constant term. P -values in parenthesis suggest high statistical significance of both ARCH and GARCH effects.

The results for the Asym-BEKK model, given in Table 5, suggest the presence of asymmetries in the conditional covariance structure. Particularly, all elements of matrix D are statistically significant even at 1% level.

However, as mentioned earlier, the BEKK specification does not allow direct interpretation of these elements as the impact of past variables on the contemporaneous volatility. Instead, we are interested in interpreting combinations of these parameters, which are reported in Table 6. The standard errors, reported in the columns next to the coefficients, were estimated using the delta method. The correspondence between Tables 5 and 6 is detailed in Table 1.

First, past shocks, ϵ_1^2 and ϵ_2^2 , have a larger impact on the spot series than on the futures one, which confirms the largest reactivity of spot prices. Second, when spot and futures shocks don't have opposite signs (which would signal potential inventory shortage), the spot variance decreases (coefficient of $\epsilon_1\epsilon_2$ negative). Third, all asymmetric coefficients are significantly. Past positive spot and futures shocks have a positive impact on the both variances, which illustrates the inventory effect.

Figure 3 graphs the time-varying OHRs as computed from the BEKK and Asym-BEKK models, as well as OLS and naïve hedge ratios. The average OHR for the asymmetric model is slightly lower than that obtained from the symmetric model (1.162967 and 1.16468 respectively) and both are higher than OLS hedge ratio. The two time-varying ratios are mostly above one, implying that more than one futures contract should be traded for each spot index contract².

To compare in-sample hedging performance we take the full sample. The four OHRs are used to choose optimal positions in futures contracts over the sample period. Then, the sample variance evaluated for the whole period is used as the measure of hedging performance. The naïve hedge is the worst model for the in-sample case. Using the OLS approach reduces the portfolio variance by about 20% and the time-varying OHR by about 27%. Using the Asym-BEKK provides the best performance (the lowest variance) as reported in Table 7.

The further step is comparing out-of-sample performance. One-day ahead forecasting is

²Augmented Dickey-Fuller (ADF) tests suggest that the time-varying ratios are stationary.

performed for 150 days (observations). For each $t + 1$ forecast, the model is reestimated by using the data available up to date t . Based on the variance-covariance matrix estimate, the expected OHR is used to build the portfolio (long in the spot and short in the futures up to the number of contracts determined by the OHR) and compute its return and variance. We start at the 3828th observation and repeat the procedure until the 3977th observation. The generated series are illustrated in Figure 4.

The results demonstrate that the asymmetric model yields superior hedging performance relative to all other models. As can be seen from Table 8 the portfolio variance is the lowest for the Asym-BEKK, 16% below the BEKK portfolio variance, and 6.55% below the naïve hedge which performs surprisingly well compared to the BEKK.

In their paper Brooks, Henry, and Persaud (2002) suggest linking the concept of the optimal hedge with the notion of the *News Impact Surface* developed by Kroner and Ng (1998). According to Kroner and Ng (1998) changes in prices can be viewed as additional information available to the market. Therefore, such innovations in prices, $S_t - S_{t-1} = \epsilon_{S,t}$ and $F_t - F_{t-1} = \epsilon_{F,t}$, represents a measure of news arriving to the market between periods t and $t - 1$. Consequently, the relationship between innovations in returns and the conditional variance-covariance structure is defined as the news impact surface. The OHR β_{t-1}^* , as defined in Equation 6, can be linked to news, by using news impact surfaces of futures variance and spot-futures covariance. The resulting news impact surface may be interpreted as the response of the OHR to arriving of the news.

Figure 5 plots the news impact surfaces for the conditional spot and futures variances, and the conditional spot-futures covariance, which are obtained from the Asym-BEKK model. Each surface is evaluated in the region $\epsilon_{j,t} \in [-1, 1]$, with $j = Spot, Futures$.

The asymmetric response of variances and covariance is clearly seen from these figures. Particularly, the conditional spot variance reaches slightly higher levels in magnitude when both shocks are positive compared to the case where both are negative.

Moreover, we see that negative correlation of the shocks largely increase the conditional variance of the spot (left and right side of upper graph in Figure 5 are higher). Indeed, periods where spot and futures dynamics coincide correspond to periods of large inventories and of relative quietness on the commodity markets.

In addition, large shocks in the spot returns contribute more to the conditional variance than large shocks in the futures return. The extreme left side of the Spot variance NIS in Figure 5 is higher than the extreme right side. This is also an implication of the theory of storage by which spot shocks better proxy state of the inventories. A similar pattern, though weaker, is found for the news impact surface of the futures variance, i.e. more dramatic response of the variance is found for the region where spot return innovations are positive.

6 Extensions

We propose two extensions. First, we check if taking into account the inventory effect through an Asym-BEKK model also improves the OHR performance for some other commodities. We then consider alternative optimality frameworks not restricted to the variance reduction. We more precisely relax the martingale assumption and maximize the (quadratic) expected utility by taking into account the expected return.

6.1 Other commodities

To check the relevance of using an Asym-BEKK model to estimate a dynamic OHR, we reiterate our in-sample and out-of-sample forecasts for 3 additional commodities: lead, nickel and aluminium. These metal price series are available as daily observations on Datastream under labels LEDCASH LED3MTH for lead, LNICASH LNI3MTH for nickel and LAHCASH LAH3MTH for aluminium. The suffix ‘cash’ and ‘3mth’ refer to spot and futures, respectively. The advantage of using these series, provided by the London Metal Exchange, is their availability in a continuous form (no need to compile successive futures contracts and correct for rollover discrepancies). Another advantage is their availability on the same time horizons as our benchmark S&P GSCI Index (spot and 3-month horizons).

We find in Table 9 that the Asym-BEKK models also dominate the BEKK model in terms of hedging performance for two other commodities (lead and aluminium). As regards lead, the variance of the hedged portfolio is reduced by 4.6% and 17.8% for in-sample and out-of-sample forecasts, respectively. The Asym-BEKK also improves hedging for aluminium in similar proportions. However, using an Asym-BEKK does not improve the hedging performance for nickel. On the contrary, it marginally increases the variance. These results confirm the relevance of checking for potential inventory effect in view to upgrade the commodity risk management.

6.2 Mean-variance framework

The OHR depends on the particular objective function to be optimized. Minimizing the variance is the most widely used approach, but there exist alternatives. Chen, Lee, and Shrestha (2003) and Lien and Tse (2002) present a review of different theoretical approaches to OHR and list, as alternatives to the variance minimization, the mean-variance, the expected utility, the mean extended-Gini coefficient as well as the semivariance approaches. These alternatives all consider a priori that minimizing the variance is too restrictive and

that the risk aversion or the expected return should be taken into account. Actually, most of these approaches are equivalent to variance minimization under (reasonable) assumptions.

A key assumption, on which we relied in Section 2 to derive the OHR from Equation 4, was that the futures return follows a martingale, i.e. $E_{t-1}(\Delta F_t) = 0$. Under this assumption, the mean-variance framework (whose objective function to maximize is $E(R_p) - \theta Var(R_p)$, where θ is the risk aversion and R_p the portfolio return) is equivalent to the variance minimization. Even optimizing the expected-utility, where the objective function to maximize is $E(U(Wealth))$, turns out to be equivalent to the variance minimization if the utility function is quadratic (as in Kroner and Sultan (1991)).

Assuming that futures return follows a martingale has some empirical grounds. Indeed, the constants in the mean equations reported in Tables 4 and 5 are not significant, this tends to give support to this assumption. Notwithstanding this empirical indication, we now relax it to check the robustness of our results. A first reason is that the return on the portfolio hedged according to the Asym-BEKK model does not dominate the return of the portfolio hedged following the BEKK modelling. In some cases, the portfolio return is lower. Improving the risk management performance in this case costs a lower return. The second reason why we check the results in a mean-variance framework is to implicitly check if the martingale assumption is reasonable. Radically different results, compared to Section 5 would seriously challenge the martingale assumption.

Alternative approaches are also proposed by Lien and Tse (2002) who consider that the quadratic form and joint normality assumptions are too restrictive and suggest to rely on stochastic-dominance framework. The extended mean-Gini approach is a method to construct portfolios efficient under the stochastic-dominance rules. Chen, Lee, and Shrestha (2003) note that the extended mean-Gini coefficient hedge ratio would be the same as the minimum variance hedge ratio if the spot and futures returns are jointly normally distributed. The mean-generalized semivariance hedge ratio approach, based on lower partial moments, takes into account the mean and the asymmetric risk aversion (to better capture the portfolio managers focus on lower tail return distributions). As noted in Chen, Lee and Shrestha (2003), this approach also provides same results as variance minimization when futures price follow a martingale process.

For convenience we rewrite here the quadratic utility function to maximize (cf Equation 4):

$$\begin{aligned} \max \quad & E_{t-1}(\Delta S_t) - \beta_{t-1} E_{t-1}(\Delta F_t) \\ & - \theta (VAR(\Delta S_t) + \beta_{t-1}^2 VAR(\Delta F_t) - 2\beta_{t-1} COV(\Delta S_t, \Delta F_t)) \end{aligned} \quad (16)$$

If we relax the martingale assumption for the futures return and solve this mean-variance

optimization problem, we now find that the OHR, β_{t-1}^{**} has an additional term:

$$\beta_{t-1}^{**} = \frac{COV(\Delta S_t, \Delta F_t)}{VAR(\Delta F_t)} - \frac{E_{t-1}(\Delta F_t)}{2\theta VAR(\Delta F_t)} \quad (17)$$

The OHR in this context thus includes a mean component (the conditional return of the futures) in addition to the variance. Following Lien and Yang (2008) and the empirical estimation of Chou (1988), we assume that θ , the degree of risk aversion, is equal to 4 and compare our strategies performances. We report our results in terms of utility in Table 10 and find that the Asym-BEKK model again dominates the BEKK modelling of the variance-covariance matrix even when a mean component is considered in the objective function. This is also the case now for nickel, while we found in the variance minimization approach that the Asym-BEKK did not improve the risk performance for this commodity.

7 Conclusions

Commodities are often found to exhibit the “inverse leverage effect”. In other words the response of the conditional variance to past negative shocks is weaker than the response to positive shocks of the same magnitude, i.e. opposite to the “leverage effect” documented for equity data in empirical finance literature. This paper aims to show that addressing optimal hedging problem in a multivariate setup that allows asymmetric response of the conditional variance-covariance matrix delivers a superior measure of OHR and reduces the risk that investors face when hedging their positions in spot contracts using futures contracts. To account for such asymmetries we estimate a flexible, asymmetric multivariate GARCH model in which the covariance structure is specified in a way that past positive and negative shocks have different effect on the conditional variances and covariance.

Our results confirm that such an asymmetric model yields improvements in both in-sample and out-of-sample forecasting and reduces the variance of realized returns on a portfolio consisting of spot and futures contracts on a broad commodity index (GSCI). Robustness checks operated on 3 other commodities and in a mean-variance optimization framework provide results confirming the relevance to consider the inventory effect. These findings imply that using asymmetric multivariate models would be more effective in hedging compared with those that assume symmetric response to past innovations. In addition to hedging optimization, the results have important implications for Value-at-Risk applications, as using the asymmetric model would improve assessment of this widespread risk measure.

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Tables

Table 1: “Greek letters” - nonlinear combinations of the estimated coefficients: Asym-BEKK specification

	$h_{11,t}$	$h_{12,t} = h_{21,t}$		$h_{22,t}$	
$\gamma_{11} =$	c_{11}^2	$\gamma_{12} =$	$c_{11}c_{12}$	$\gamma_{22} =$	$c_{12}^2 + c_{22}^2$
$\beta'_{11} =$	b_{11}^2	$\beta'_{12} =$	$b_{11}b_{12}$	$\beta'_{22} =$	b_{12}^2
$\beta''_{11} =$	$2b_{11}b_{21}$	$\beta''_{12} =$	$b_{11}b_{22} + b_{12}b_{21}$	$\beta''_{22} =$	$2b_{12}b_{22}$
$\beta'''_{11} =$	b_{21}^2	$\beta'''_{12} =$	$b_{21}b_{22}$	$\beta'''_{22} =$	b_{22}^2
$\alpha'_{11} =$	a_{11}^2	$\alpha'_{12} =$	$a_{11}a_{12}$	$\alpha'_{22} =$	a_{12}^2
$\alpha''_{11} =$	$2a_{11}a_{21}$	$\alpha''_{12} =$	$a_{11}a_{22} + a_{12}a_{21}$	$\alpha''_{22} =$	$2a_{12}a_{22}$
$\alpha'''_{11} =$	a_{21}^2	$\alpha'''_{12} =$	$a_{21}a_{22}$	$\alpha'''_{22} =$	a_{22}^2
$\delta'_{11} =$	d_{11}^2	$\delta'_{12} =$	$d_{11}d_{12}$	$\delta'_{22} =$	d_{12}^2
$\delta''_{11} =$	$2d_{11}d_{21}$	$\delta''_{12} =$	$d_{11}d_{22} + d_{12}d_{21}$	$\delta''_{22} =$	$2d_{12}d_{22}$
$\delta'''_{11} =$	d_{21}^2	$\delta'''_{12} =$	$d_{21}d_{22}$	$\delta'''_{22} =$	d_{22}^2

Notes. $h_{11,t}$, $h_{12,t}$ and $h_{22,t}$ refer to the spot, covariance and futures Equations 12, 13 and 14.

Table 2: Summary statistics

	LEVEL		RETURN	
	Spot	Futures	Spot	Futures
Mean	304.13	174.31	0.028438%	0.028970%
Standard Deviation	152.59	89.215	0.014473	0.012481
Minimum	128.89	76.250	-0.091451	-0.078958
Maximum	890.29	512.61	0.072147	0.070885
Normality test	2296.8**	2329.0**	725.58**	1089.8**
UR test	-2.189	-2.078	-30.27**	-29.82**
$Q^2(20)$ test	77232**	77335**	2575.6**	3698.0**
LM test	87552**	99196**	42.85**	57.38**
Observations	3977	3977	3977	3977

Note: The null of the normality test is normality of the series. The null of the unit root (UR) test is the presence of unit root in the series. The null of the Box-Pierce test ($Q^2(20)$) is the absence of autocorrelation in the squared series. The null of the Lagrange Multiplier (LM) test is the absence or ARCH effect in the series. ** stands for rejection at 1%. UR tests on level series based on logarithm of the series do not change the conclusions (not reported here).

Table 3: OLS regression for dependent variable ΔS_t

Exog. Var.	Coef.	Std. Err.	t value	Prob.> t
ΔF_t	1.129678	0.004151	272.18	0
<i>Constant</i>	-1.8E-05	2.25E-05	-0.82	0.414

Table 4: BEKK-GARCH Model Estimates

Conditional Mean Equations (P -values in parenthesis):

$$y_{1,t} = 0.000040 + 0.102380y_{1,t-1} + \epsilon_{1,t}$$

(0.6090) (0.0000)

$$y_{2,t} = 0.000082 + 0.108843y_{2,t-1} + \epsilon_{2,t}$$

(0.2307) (0.0000)

Conditional Variance-Covariance Matrix (P -values in parenthesis):

$$H_t(\eta) = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B$$

$$C = \begin{bmatrix} 0.000740 & 0.000439 \\ (0.0000) & (0.0000) \\ 0 & -0.000000 \\ & (0.0000) \end{bmatrix} \quad A = \begin{bmatrix} -0.851078 & -0.405689 \\ (0.0000) & (0.0000) \\ 1.031991 & 0.551631 \\ (0.0000) & (0.0000) \end{bmatrix}$$

$$B = \begin{bmatrix} 0.813052 & -0.070711 \\ (0.0000) & (0.0000) \\ 0.191031 & 1.067519 \\ (0.0000) & (0.0000) \end{bmatrix}$$

Table 5: Asym-BEKK Model Estimates

Conditional Mean Equations (P -values in parenthesis):

$$y_{1,t} = 0.000141 + 0.113007y_{1,t-1} + \epsilon_{1,t}$$

(0.4506) (0.0000)

$$y_{2,t} = 0.000262 + 0.116116y_{2,t-1} + \epsilon_{2,t}$$

(0.1051) (0.0000)

Conditional Variance-Covariance Matrix (P -values in parenthesis):

$$H_t(\eta) = C'C + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'H_{t-1}B + D'\zeta_{t-1}\zeta'_{t-1}D$$

$$C = \begin{bmatrix} 0.000734 & 0.000327 \\ (0.0000) & (0.0014) \\ 0 & -0.000003 \\ & (0.0000) \end{bmatrix} \quad A = \begin{bmatrix} -0.401618 & -0.045789 \\ (0.0000) & (0.1511) \\ 0.243888 & -0.127907 \\ (0.0004) & (0.0042) \end{bmatrix}$$

$$B = \begin{bmatrix} 0.921622 & -0.003593 \\ (0.0000) & (0.6909) \\ 0.066443 & 0.986246 \\ (0.0003) & (0.0000) \end{bmatrix} \quad D = \begin{bmatrix} -0.348236 & -0.206677 \\ (0.0002) & (0.0001) \\ 0.479650 & 0.328875 \\ (0.0000) & (0.0000) \end{bmatrix}$$

Table 6: Estimated nonlinear coefficients and standard errors for Asym-BEKK Model using Delta Method

	Equation $h_{11,t}$		Equation $h_{12,t}$		Equation $h_{22,t}$	
	Coefficient	St. Error	Coefficient	St. Error	Coefficient	St. Error
γ	1.0E-5***	1.3E-5	1.4E-7**	2.1E-7	2.9E-6	1.0E-6
α'	0.161***	0.045	0.018	0.015	0.002	0.003
α''	-0.196**	0.082	0.040*	0.022	0.012***	0.004
α'''	0.059*	0.034	-0.031***	0.005	0.016	0.011
β'	0.849***	0.030	-0.003	0.008	1.3E-6	6.5E-6
β''	0.122***	0.031	0.909***	0.011	-0.007	0.018
β'''	0.004*	0.002	0.066***	0.019	0.972***	0.022
δ'	0.121*	0.064	0.072**	0.033	0.043**	0.022
δ''	-0.334***	0.149	-0.214**	0.086	-0.136**	0.064
δ'''	0.230***	0.088	0.158***	0.059	0.108**	0.050

Note: Triple, double and single asterisk represents significance at 1, 5 and 10 per cent levels respectively.

Table 7: In-sample forecasts: variance of naïve, OLS, BEKK and Asym-BEKK portfolios

	naïve $\beta = 1$	OLS $\beta = 1.1296$	BEKK $\beta = h_{12,t}/h_{22,t}$	Asym-BEKK $\beta = h_{12,t}/h_{22,t}$
Variance	0.0133	0.0107	0.0097	0.0097
% of naïve Variance	100.00%	80.38%	73.38%	73.02%

Notes. For reporting purposes variance are multiplied by 1,000. The row “% of naïve Variance” reports the variances of portfolio returns for the different strategies as a percentage of the naïve variance.

Table 8: Out-of-sample forecasts: variance of naïve, OLS, BEKK and Asym-BEKK portfolios

	naïve $\beta = 1$	OLS $\beta = 1.1335$	BEKK $\beta = h_{12,t}/h_{22,t}$	Asym-BEKK $\beta = h_{12,t}/h_{22,t}$
Variance	0.0010	0.0017	0.0011	0.0009
% of naïve Variance	100.00%	176.86%	111.37%	93.46%

Notes. For reporting purposes variance are multiplied by 1,000. The row “% of naïve Variance” reports the variances of portfolio returns for the different strategies as a percentage of the naïve variance.

Table 9: In- and Out-of-sample forecasts: Portfolio variance reduction obtained by using an Asym-BEKK instead of a BEKK model

Commodity	in-sample	out-of-sample
Lead	-4.56%	-17.77%
Nickel	+0.15%	+2.45%
Aluminium	-4.52%	-28.25%

Note: The OHR used in this forecast exercise is β_{t-1}^* (see Equation 5). Variance reduction is computed by dividing the variance of the spot-futures portfolio resulting from an Asym-BEKK modelling strategy on the variance of the portfolio resulting from a BEKK modelling strategy.

Table 10: Mean-variance framework: In/Out-of-sample forecasts - Portfolio utility increase obtained by using an Asym-BEKK instead of a BEKK model

Commodity	In-sample	Out-of-sample
GSCI Index	0.0%	+0.1%
Lead	+6.1%	+12.4%
Nickel	+6.9%	+20.7%
Aluminium	+0.4%	+20.4%

Note: The OHR used in this forecast exercise is β_{t-1}^{**} (see Equation 17). Utility is defined as $E(R_p) - \theta Var(R_p)$. Utility increase is computed by comparing the utility of the spot-futures portfolio resulting from an Asym-BEKK modelling strategy on the utility of the portfolio resulting from a BEKK modelling strategy.

Figures

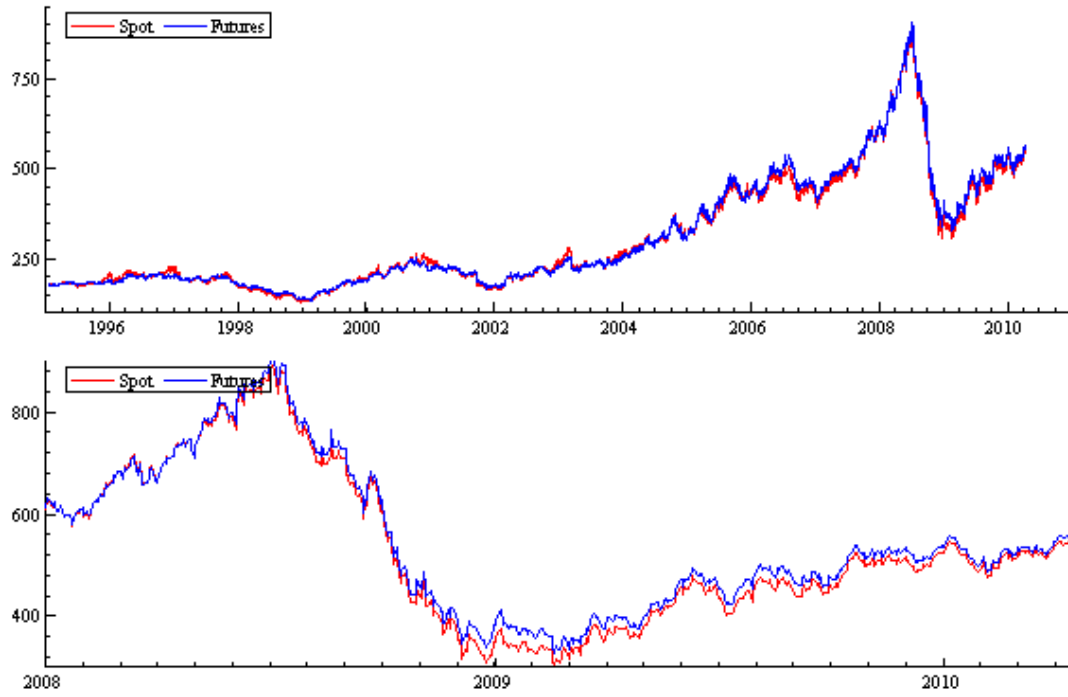


Figure 1: Goldman Sachs Commodity Price Index

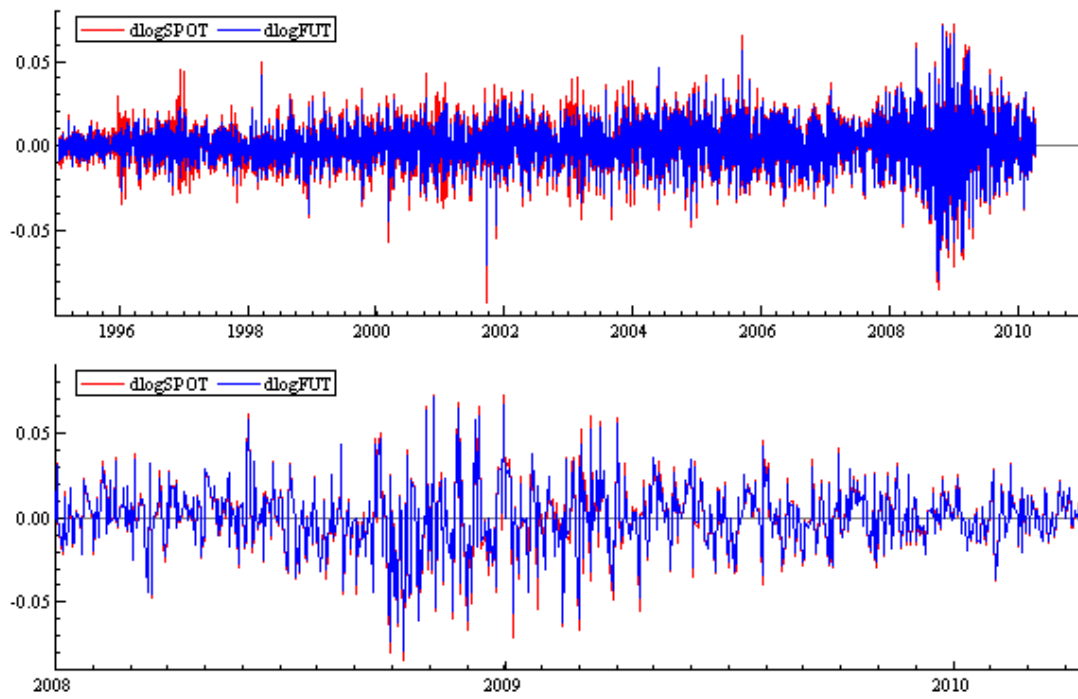


Figure 2: Spot and Futures Log-Return Series

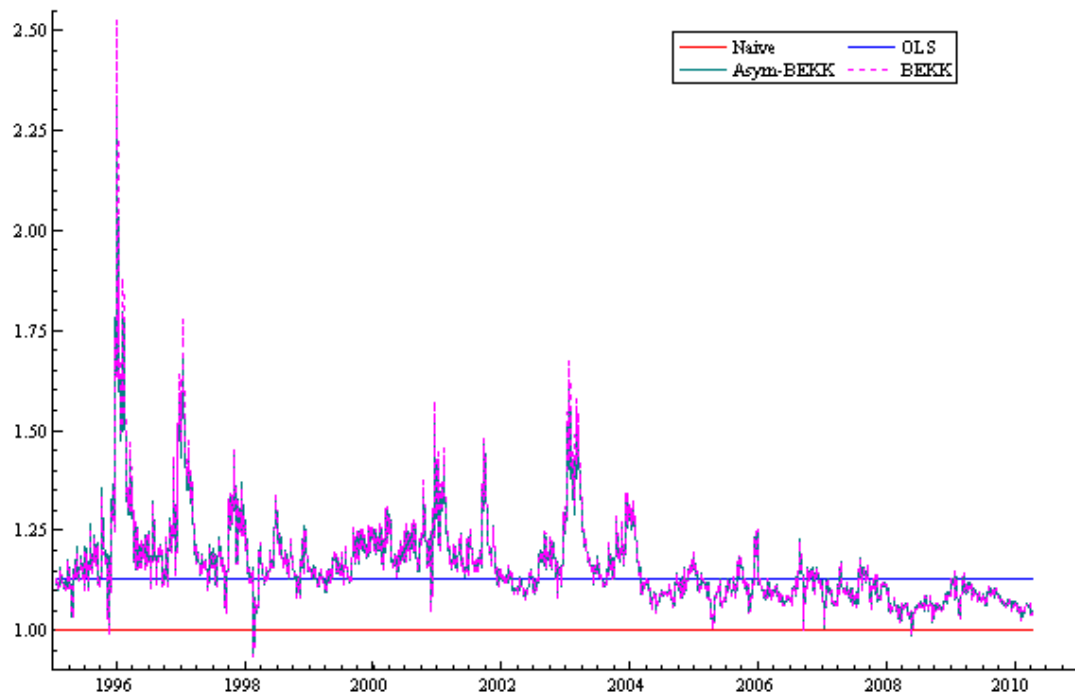


Figure 3: In-sample forecasts - Optimal Hedge Ratios

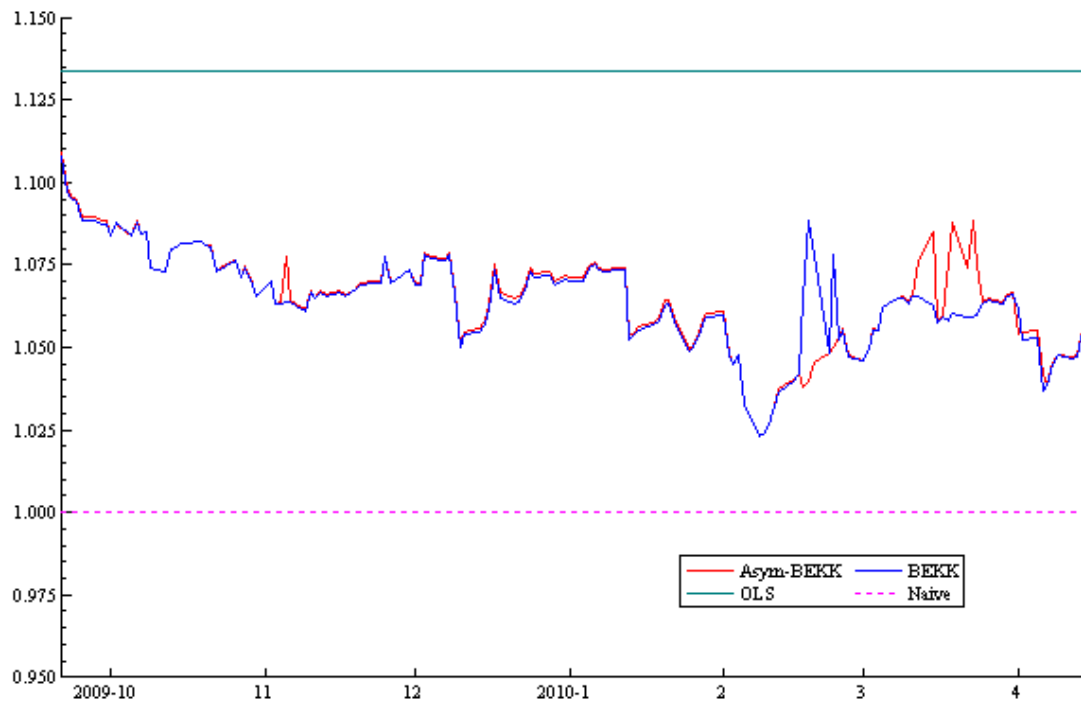
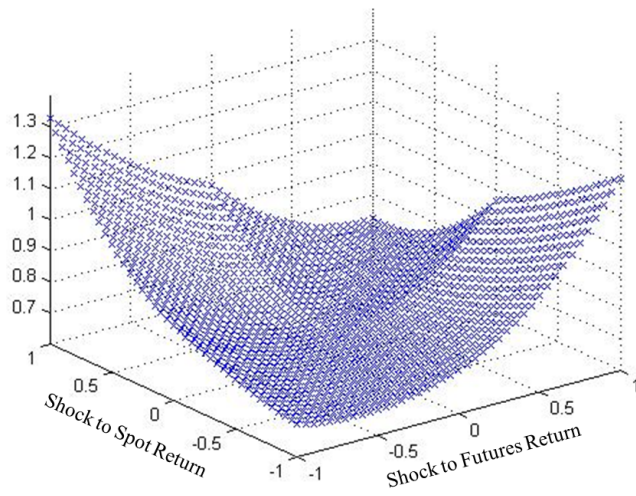
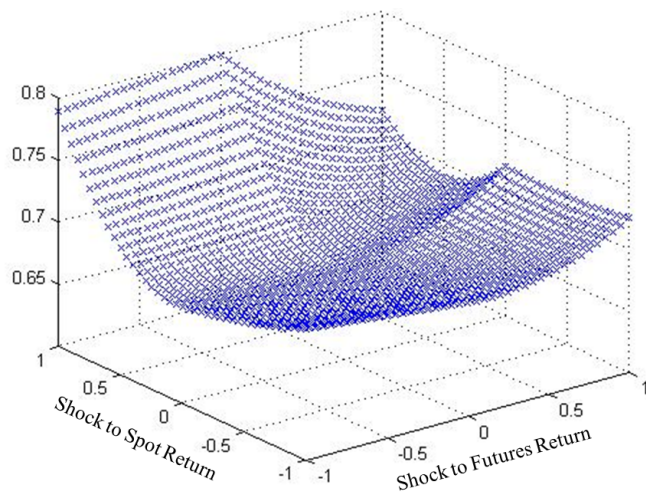


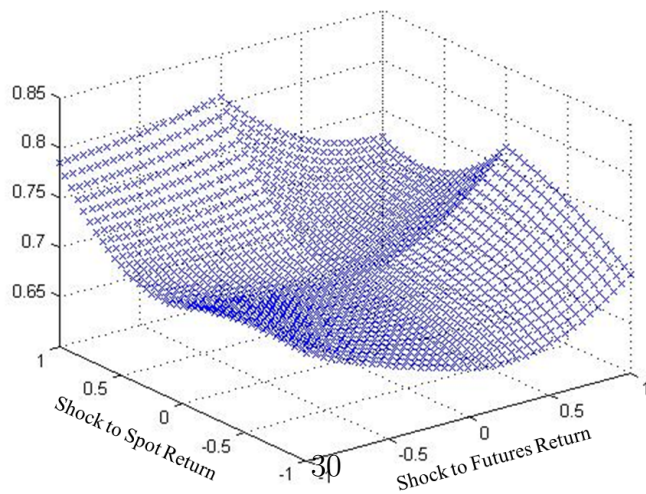
Figure 4: Out-of-sample forecasts - Optimal Hedge Ratios



(a) Spot Variance



(b) Futures Variance



(c) Spot-Futures Covariance

Figure 5: News Impact Surfaces

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