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Asymmetric information and overeducation

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Abstract

We consider an economy where production may use labor of two different skill levels. Workers are heterogeneous and, by investing in education, self-select into one of the two skills. Ex-ante, when firms choose their investments in physical capital, they do not know the level of human capital prevailing in the labor market they will be active in. We prove existence and constrained inefficiency of competitive equilibria, which are always characterized by overeducation. An increase in total expected surplus can be obtained by shrinking, at the margin, the set of workers investing in high skill. This can be implemented by imposing taxes on the cost of investing in high skill or by imposing a progressive labor earning tax.

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1. INTRODUCTION

Existence and effects of the externalities related to the investments in human capital (HC) have been important topics of research for many years. The source of the externalities is usually identified in the existence of knowledge spillovers creating a wedge between private and social returns to HC. An alternative mechanism is related to the existence of distortions creating pecuniary externalities.¹ This paper analyzes this sort of externalities in the framework of a competitive economy where workers are heterogeneous and self-select themselves into different labor markets by investing in different types of HC. Our main focus is on the efficiency properties of the equilibria. We establish that they are always characterized by overeducation at the extensive margin, since the set of workers investing in high skills is always larger than the (constrained) efficient one.

Our result should be contrasted with the conclusion obtained by Acemoglu (1996), a classical reference on pecuniary externalities in economies with HC. He considers a frictional economy where workers and firms invest in human and physical capital, respectively, before they are randomly matched. Thus, their individual investments depend upon the distribution of the investments of their potential future partners. Due to the income allocation mechanism,² agents do not fully appropriate the marginal returns of their irreversible investments. This hold-up problem generates undereducation at the equilibrium, so that an increase in the investments of workers (and/or of firms) is always welfare improving. This result rests crucially on the adoption of a pure efficiency unit set-up. In a companion paper (2012), we study a similar framework with frictional labor markets, but with two separate sectors using different kinds of HC, i.e., we adopt a Roy's model with elastic investments in both human and physical capital. We establish that the equilibrium allocation may be inefficient due to overeducation, because, at the equilibrium, too many workers are investing in high skills. Moreover, each one of them is investing too little effort. This difference is explained by the fact that, in Roy's models, workers sort themselves into the two types of skills without considering the effects of their choices on the distribution of HC in the two labor markets. In our set-up, the composition of the labor force in terms of HC matters because it affects firms' investments. The role of this "composition effect" has been previously analyzed in several papers considering two-sectors, random-matching economies with investments in HC (among the recent ones, see Charlotte and Decreuse (2005) and Mendolicchio *et al.* (2010)).

In this paper, we study the welfare impact of the composition effect in its simplest set-up, with self-selection and asymmetric information, but without additional distortions.³ In our economy, workers are heterogeneous in terms of ability and choose to invest effort and real resources to acquire one of two types of HC, low or high skill. The two skills are used, together with capital goods, in two distinct production processes. Contrary to the papers mentioned above, we consider a situation where spot labor markets are perfectly competitive. The only distortion in

¹The empirical and theoretical literature on externalities and HC is quite large. For fairly recent surveys, see Moretti (2004) and Halfdanarson *et al.* (2010).

²Income distribution takes place via a Nash bargaining process after a worker is randomly matched with a firm.

³In the matching model of Mendolicchio *et al.* (2012), the hold-up problem related to the bargaining income allocation mechanism tends to induce undereducation. The composition effect induced by the existence of two different skills tends to induce overeducation. The net effect depends on the quantitative relevance of the two opposite distortions.

the economy is that firms choose their investments knowing the skill characteristics of the workers active in the labor market they face, but without knowing, at the time these investments are made, the actual level of their innate ability (which affects the actual level of HC they acquire). This allows us to clarify the potential impact of the composition effect on the efficiency properties of equilibria, without the complications due to several distortions simultaneously at play. Our model has two driving features. First, the asymmetric information between workers and firms on their innate ability, which is observable only ex-post. Secondly, the lack of contractibility between firms and workers. Since spot labor markets are perfectly competitive, the choices at the intensive margin (i.e., the level of the investments) are always at their efficient level, given the informational and contractibility constraints and contingent on the partition of the agents with respect to their skill. However, the choices at the extensive margin, i.e., the self-selection of workers into the different skills, are always inefficient, and the set of workers acquiring high skill is always larger than the constrained efficient one. Hence, overeducation always holds at the equilibrium.

An additional difference with respect to the previous literature is that, since we consider standard production functions where the inputs are quantities of (efficiency units of) labor of the two skills, what matters is the *total* supply of human capital in the different labor markets, not the average per-capita level of HC.⁴ It is easy to see that one could reformulate our analysis in terms of average per capita HC by simply imposing an appropriate reinterpretation of the production technology. The results would not substantively change. The main point is that, in economies with people investing in different, non fungible, skills, the composition of the labor force has a key role. Since agents do not internalize the effect of their choices on the distribution of HC, the sorting is typically not at its socially efficient level. In our framework, this translates into overeducation at the equilibrium.

Different policy instruments can be adopted to mitigate the welfare effects of the distortion. First, taxes on the direct costs of education in high skills can reduce the overinvestment problem. Since we consider economies with quasi-linear preferences, these taxes do not affect the workers' choices at the intensive margin. They have a positive effect on welfare (measured by total surplus) because of the composition effect. Secondly, earning taxes may also have a positive welfare effect. In particular, some small degree of progressivity of the tax system may be welfare improving. This is because a higher marginal tax rate on high-skill, high-wage earnings shrinks the set of people investing in these skills. Of course, positive marginal tax rates have a negative incentive effect on the effort invested in acquiring HC. Still, when the initial tax rate (or the elasticity of the labor supply) is sufficiently small, the negative impact on welfare of the - distortionary - incentive effect of the earning tax can be dominated, from the quantitative view point, by its positive impact via the composition effect.

We consider a parametric and stylized set-up. In particular, as already mentioned, workers have quasi-linear preferences and the production processes using the two skills are completely independent. The first restriction is convenient for analytic simplicity. Moreover, it allows us to focus exclusively on efficiency issues, which are our main concern. On the downside, quasi-linearity of preferences rules out any meaningful consideration of distributional issues which are obviously an

⁴In the formal model, average human capital and aggregate human capital do, in fact, coincide. However, in terms of interpretation what matters is the total human capital in each labor market and its average across different, "local", labor markets for the same skill.

important consideration in the definition of public policies related to investments in HC. It also implies that our results concerning the role of taxes are best seen as a way to better understand the effects of the distortion we are considering and, definitely, not as a contribution to the blossoming literature on the optimal taxation in economies with HC.⁵ Still, our results on the welfare impact of the composition effect may be of some interest for this literature, too. In particular, they suggest that some of the conclusions reached in pure efficiency unit models may fail to be robust when one considers Roy's economies. To focus the analysis on independent production processes (one for each skill level) simplifies the analysis. However, the main qualitative results hold in richer structures of the production sector.

Finally, we focus on economies with purely pecuniary externalities. It is quite intuitive that a similar phenomenon may take place in economies where investments in HC generate technological externalities, provided that the spillover effects are not homogeneous across the different types of HC.

The structure of the paper is the following. Section 2 describes the class of economies, the notion of equilibrium and presents existence and main properties of the equilibrium. Section 3 is the core of the paper and contains the welfare analysis. Some final considerations follow.

2. THE MODEL

The economy has two key features. First, workers are heterogeneous with respect to some parameter which affects their choice of the type of skills they acquire investing in education and the actual level of HC that they obtain. Secondly, when choosing their optimal investments in physical capital, firms do not know precisely the level of HC of the workers they will actually be able to hire. The simplest way to formalize these properties is the following. The economy is composed by a continuum of islands, denoted by $\delta \in [\underline{\delta}, \bar{\delta}]$, endowed with the Lebesgue measure, and interpreted as local labor markets. On each island there is an interval $[0, 1]$ of workers also endowed with the Lebesgue measure. Workers are denoted by a pair $(i, \delta) \in [0, 1] \times [\underline{\delta}, \bar{\delta}]$. They are identical within each island (i.e., with respect to the index i), but heterogeneous across islands, because of the parameter δ , whose realization in a given island is, ex-ante, private information of the workers. This parameter can be interpreted as describing their "innate ability" and it affects the workers' choices related to the acquisition of HC. Workers self-select into one of the two labor markets characterized by different skills. We will denote all the variables related to "high skills" with a superscript $s = e$, to "low skills" with $s = ne$. Given their choice of a skill type, workers then choose their optimal level of HC. Their choices at the extensive margin induce a partition $\Delta^P \equiv \{\Delta^{ne}, \Delta^e\}$, with workers of type δ acquiring high skills if and only if $\delta \in \Delta^e$. In the sequel, given a partition Δ^P , we will use $\delta^s \in \Delta^s$ to refer to the realization of δ , with $\delta \in \Delta^s$.

There is a large number of potential firms, denoted by j , endowed with the same technology. They may use one or both of two different production processes. The first uses capital and high skill labor. The second, capital and low skill labor. Production takes place under constant return to scale, and each firm has production function $Y_j(\cdot) = \sum_s A^s K_j^{s(1-a)} L_j^{sa}$. Hence, there are two perfectly substitutable

⁵The classical references on the effects of income taxes on the accumulation of HC are Ben-Porath (1970), Heckman (1976), and Eaton and Rosen (1980). More recent contributions include Andenberg (2009), Andenberg and Andersson, (2003), Bovenberg and Jacobs (2005), Jacobs (2005), Jacobs and Bovenberg (2011), Jacobs *et al.* (2012).

production processes, one using high skill labor and one type of physical capital, the other using low skill inputs. To simplify, but without any loss of generality, we assume that the unit price of capital goods (which is anyway exogenous) is invariant across s , and equal to 1. The price of output is also fixed and equal to one. Furthermore, we assume that production using high skill labor has higher total factor productivity, i.e., that $A^e > A^{ne}$. What really matters is that, for a given level of inputs, the production process using high skill labor is more valuable, i.e., that, for each $(\bar{K}_j^e, \bar{L}_j^e) = (\bar{K}_j^{ne}, \bar{L}_j^{ne})$, $p^e A^e \bar{K}_j^{e(1-a)} \bar{L}_j^{ea} > p^{ne} A^{ne} \bar{K}_j^{ne(1-a)} \bar{L}_j^{nea}$.⁶ This can happen because $A^e > A^{ne}$ or because the two processes produce different commodities and the one obtained using high skill labor has an higher price. For notational convenience, we fix $p^{ne} = p^e = 1$ and assume $A^e > A^{ne}$. Before choosing their investments in physical capital, firms are matched with one local labor market for one, or both, the skills levels.⁷ Firms' decisions take place in two stages. In the first period, an interval $[0, 1]$ of firms, endowed with the Lebesgue measure, is matched with an island. These firms learn the types of workers they will be able to hire (i.e., if $s = e$ or $s = ne$), and the equilibrium partition $\bar{\Delta}^P$. However, they do not know the actual realization $\delta \in \Delta^s$. In the same period, firms choose their optimal investments \bar{K}_j^s , $s = ne, e$. In the second stage, the actual levels of HC of the workers become observable, labor markets open and clear at the competitive wages and production takes place.

In the sequel, we will show that, at each equilibrium, $\bar{\Delta}^P = ([\underline{d}, \bar{\delta}), [\bar{\delta}, \bar{d}])^s$ for some threshold $\bar{\delta}$, so that the partition $\bar{\Delta}^P$ can be written $\Delta(\bar{\delta})$ and it can be directly identified with $\bar{\delta}$. Therefore, ex-ante, given the equilibrium wage map $w(\cdot) \equiv \{w^{ne}(\cdot), w^e(\cdot)\}$ and $\bar{\delta}$, each firm chooses its optimal plan solving the optimization problem

$$\max_{(K_j, L_j)} E((Y_j - \sum_s w^s(\cdot) L_j^s) | \bar{\delta}) - \sum_s K_j^s, \quad (\text{II})$$

where $E(\cdot | \bar{\delta})$ is the conditional expectation of its argument with respect to the partition induced by the threshold $\bar{\delta}$. Let $K_j(w(\cdot); \bar{\delta}) = \{K^{ne}(w^{ne}(\cdot); \bar{\delta}), K^e(w^e(\cdot); \bar{\delta})\}$ and $L_j(w(\cdot); \bar{\delta}) = \{L_j^{ne}(w^{ne}(\cdot); \bar{\delta}), L_j^e(w^e(\cdot); \bar{\delta})\}$ denote firm j 's demand functions. Evidently, they are j -invariant. Given the time-structure of the economy, the demand for capital goods does not depend upon the realization (δ^{ne}, δ^e) , while, in general, the demand for labor depends upon this realization and, indirectly, upon the partition $\Delta(\bar{\delta})$, because this affects the level of physical capital.

Consider now the workers. They self-select into one of the two labor markets characterized by different kinds of skills and, then, choose their optimal level of HC. Since we are just interested in efficiency issues, it is convenient to impose that individual preferences are described by a quasi-linear utility function, and, without any essential loss of generality, we set $U(C^s, h^s) = C^s - \frac{1}{\delta} \frac{h^{s(1+\Gamma)}}{1+\Gamma}$. Our preferred interpretation is that each worker inelastically supplies one unit of time. h^s is his effort in the acquisition of HC, which converts 1-to-1 into efficiency units of

⁶We impose sector invariance of the exponent α to simplify notation. Nothing essential depends upon this assumption.

⁷Given the production function, it is immaterial if they can actually hire both low-skill and high-skill workers. For notational simplicity, we will usually assume that each firm hires just one type of workers, i.e. that it is matched with a labor market for just one skill level.

⁸Workers with $\delta = \bar{\delta}$ are actually indifferent between the two skills. Without loss of generality, we assume that they choose $s = e$.

labor of skill s , for $s = ne, e$. Acquisition of high skills requires effort, h^e , and a direct cost T .⁹ Hence, $C^e = w^e h^e - T$, while $C^{ne} = w^{ne} h^{ne}$. The parameter δ affects the marginal disutility of effort and, consequently, all the choices related to the investment in HC. To fix ideas, we may think of it as a parameter defining the innate ability of the individuals. This utility function delivers two convenient properties: first, it allows for a simple parameterization of the marginal disutility of effort, hence of the optimal level of HC across islands. Secondly, it allows us to explicitly compute the supply function of HC.¹⁰

For future reference, we study workers' optimal behavior given a vector of taxes $\xi \equiv (\tau^{ne}, \tau^e, \Delta T)$, where τ^s is the marginal tax rate on labor income in sector s . ΔT are taxes (or subsidies) on the direct costs of education in the high skills. Since, at the equilibrium, labor income is always higher for workers in sector e , (τ^{ne}, τ^e) , with $\tau^e > \tau^{ne}$, describes the usual progressive, step-linear, labor income tax system.

Notice that, given the characteristics of the production function, for each s and each pair (δ^e, δ^{ne}) , w^s only depends upon the investments in capital of type s and the realization δ^s . Hence, workers face no actual uncertainty, and we can write the wage equilibrium maps as $w(\delta^{ne}, \delta^{ne}; \bar{\delta}, \xi) \equiv \{w^{ne}(\delta^{ne}; \bar{\delta}, \xi), w^e(\delta^e; \bar{\delta}, \xi)\}$, giving the equilibrium wages associated with the partition $\Delta(\bar{\delta})$ and the realization $(\delta^{ne}, \delta^e) \in \Delta^{ne} \times \Delta^e$. Within each island, workers are identical, thus we will mostly omit the index i . Their choices can be described considering first their optimal solution to the two optimization problems

$$\max_{h^s} U^s(C^s, h^s; \delta), \quad (U^s)$$

for $s = ne, e$. Let $H^s(\delta; \bar{\delta}, \xi)$ be the supply function, conditional on s . Let $V^s(\delta; \bar{\delta}, \xi)$ be the associated value functions. Evidently, a worker acquires HC of type e if and only if $V^e(\delta; \bar{\delta}, \xi) \geq V^{ne}(\delta; \bar{\delta}, \xi)$.

Equilibrium is defined by market clearing and individual optimization, under the assumption that expectations are rational.

DEFINITION 1. An equilibrium is a measurable partition $\Delta(\bar{\delta})$ and a wage map $w(\delta^{ne}, \delta^{ne}; \bar{\delta}, \xi)$ such that

- i. $\{K_j, L_j(\cdot)\}$ solves (Π_j) , for each $(\delta^{ne}, \delta^e) \in \Delta^{ne}(\bar{\delta}) \times \Delta^e(\bar{\delta})$,
- ii. $\{C^s(\cdot), H^s(\cdot)\}$ solves (U^s) , for each $\delta^s \in \Delta^s(\bar{\delta})$, for $s = ne, e$,
- iii. $V^e(\delta; \bar{\delta}, \xi) \geq V^{ne}(\delta; \bar{\delta}, \xi)$ for each $\delta^e \in \Delta^e(\bar{\delta})$,
- iv. $V^{ne}(\delta; \bar{\delta}, \xi) \geq V^e(\delta; \bar{\delta}, \xi)$ for each $\delta^{ne} \in \Delta^{ne}(\bar{\delta})$,
- v. for each pair (δ^e, δ^{ne}) and each s , $H^s(\delta; \bar{\delta}, \xi) = L^s(w^s(\cdot); \bar{\delta})$.

Evidently, $(i - ii)$ impose individual optimization. (iii) imposes that the utility of a worker with $\delta \in \Delta^e(\bar{\delta})$ is at least as large if he acquires high skills than if he invests in low skills. (iv) imposes the analogous condition for unskilled workers. (v) are the market clearing conditions.¹¹

For this economy, an equilibrium exists. We start with an arbitrary threshold $\hat{\delta}$ and an arbitrary pair of firm-invariant investments, \hat{K} . We then compute the wage

⁹Similar results would hold if there were time costs, too.

¹⁰Given the structure of production, similar results hold for a more general class of quasi-linear utility functions, with a general, strictly convex function $\frac{1}{\delta} v^s(h_i^s)$.

¹¹Since there is a measure 1 of identical workers and firms on each labor market, it would be pedantic to distinguish between individual and aggregate demand and supply.

equilibrium maps conditional on $(\widehat{\delta}, \widehat{K})$, $\widehat{w}(\widehat{\delta}, \widehat{K})$. Next, given $\widehat{\delta}$, we can use the map $\widehat{w}(\widehat{\delta}, \widehat{K})$ to determine the optimal investments in physical capital, contingent on the threshold. They are described by a function $\widehat{K}(\delta)$. Finally, the map $\widehat{w}(\delta, \widehat{K}(\delta))$ can be used to find the actual equilibrium threshold.

The equilibrium threshold is defined by the condition

$$V^e(\delta = \bar{\delta}; \bar{\delta}, \xi) - V^{ne}(\delta = \bar{\delta}; \bar{\delta}, \xi) = 0.$$

Using the wage maps conditional on an arbitrary threshold (see (A3) in Appendix), we can rewrite this equality as

$$\begin{aligned} 0 = F(\bar{\delta}) &\equiv \bar{\delta}^{\gamma-1} (1 - \tau^e)^{\frac{1+\Gamma}{\Gamma}} \left(E(\delta^{e(\gamma-1)} | \bar{\delta})^{1-\alpha} A^e \right)^{\frac{1+\Gamma}{\alpha\Gamma}} \\ &\quad - \bar{\delta}^{\gamma-1} (1 - \tau^{ne})^{\frac{1+\Gamma}{\Gamma}} \left(E(\delta^{ne(\gamma-1)} | \bar{\delta})^{1-\alpha} A^{ne} \right)^{\frac{1+\Gamma}{\alpha\Gamma}} - b(T + \Delta T), \end{aligned} \quad (1)$$

where $b \equiv \frac{1+\Gamma}{\Gamma} \left((1 - \alpha)^{\frac{1-\alpha}{\alpha}} \alpha \right)^{-\frac{1+\Gamma}{\Gamma}}$ and $\gamma \equiv \frac{1+\Gamma}{1+\Gamma-\alpha}$. A solution to eq. (1) defines an equilibrium threshold $\bar{\delta}$. Hence, to establish the existence of an equilibrium we need two properties. First, we need to show that the equilibrium partition has indeed structure $\Delta^e(\bar{\delta}) = [\bar{\delta}, \bar{d}]$, as postulated above. Secondly, a solution to $F(\bar{\delta}) = 0$ must exist. The proof is straightforward and the details are in Appendix. There, we also establish uniqueness and comparative statics of the equilibrium. These additional properties do not hold in general. Indeed, numerical examples show that, for some values of the parameters, $F(\cdot)$ is not strictly monotonic, so that there are several equilibria, at least for some values of T . A sufficient condition for uniqueness of the solution to $F(\bar{\delta}) = 0$ is that the total factor productivity in sector e is sufficiently larger than the one in sector ne .¹²

Proposition 1 presents our existence and uniqueness results, taking as benchmark the economy with a flat labor income tax and no taxes on the direct costs of education.

PROPOSITION 1. *Fix (Γ, α) and $\xi = (\tau, \tau, 0)$. Given (A^{ne}, A^e, ξ) ,*

(a) *there is a set (\underline{T}, \bar{T}) such that, for each T with $\frac{bT}{1-\tau} \in (\underline{T}, \bar{T})$, there is an equilibrium with threshold $\bar{\delta}(\xi) \in (\underline{d}, \bar{d})$;*

(b) *given A^{ne} , there is \underline{A}^e such that, for each $A^e > \underline{A}^e$, the equilibrium is unique and described by a C^1 function satisfying $\frac{\partial \bar{\delta}(\xi)}{\partial \tau^e} < 0$, $\frac{\partial \bar{\delta}(\xi)}{\partial \tau^{ne}} > 0$, and $\frac{\partial \bar{\delta}(\xi)}{\partial \Delta T} > 0$.*

From (A3) in Appendix, at each equilibrium, the wages (and earnings) of the high skill workers are always strictly larger than the ones of the low skilled agents. Hence, any skill-conditional earning tax profile (τ^{ne}, τ^e) is equivalent to the standard tax system with variable marginal tax rates, as we have claimed above.

3. CONSTRAINED INEFFICIENCY OF EQUILIBRIA

Our main purpose is to analyze the efficiency properties of the equilibria. Their Pareto inefficiency is self-evident since firms' investments are invariant with respect to the realizations (δ^{ne}, δ^e) . This is due to the lack of contractibility of investments and it reflects the basic structure of the economy. More interesting is to consider

¹²There are other alternative sufficient conditions delivering the same property.

the efficiency properties of the economy taking as a reference point a notion of constrained optimal (CO) allocation. We adopt the usual fiction of a benevolent planner maximizing total expected surplus under the appropriate constraints, reflecting the fundamental structure of the economy. In our set-up, this is the constraint that investments in physical capital must be selected ex-ante, before firms are actually matched with labor markets, so that they must be (δ^{ne}, δ^e) -invariant. The objective function of the social planner is $E(W(.))$, the total expected surplus:¹³

$$E(W(.)) = \sum_s \mu^s(\tilde{\Delta}) \left[A^s K^{s(1-a)} E(L^{sa} | \tilde{\Delta}) - E\left(\frac{1}{\delta} \frac{L^{s(1+\Gamma)}}{1+\Gamma} | \tilde{\Delta}\right) \right] - \sum_s K^s - \mu^e(\tilde{\Delta})T,$$

where $\mu^s(\tilde{\Delta})$ is the measure of agents of type s . Evidently, for each s ,

$$\left[A^s K^{s(1-a)} E(L^{sa} | \tilde{\Delta}) - E\left(\frac{1}{\delta} \frac{L^{s(1+\Gamma)}}{1+\Gamma} | \tilde{\Delta}\right) \right] - K^s$$

is the expected surplus (including education costs) for the interval of firms matched with an island with $\delta \in \tilde{\Delta}^s$. Bear in mind that $E(W(.))$ coincides with the actual total surplus. The policy instruments of the planner are two maps $\{\tilde{L}^{ne}(\delta^{ne}), \tilde{L}^e(\delta^e)\}$ fixing the labor inputs for each δ^{ne} and δ^e , the investments in physical capital, $\{\tilde{K}^{ne}, \tilde{K}^e\}$, and a measurable partition $\tilde{\Delta}$. It is fairly intuitive that the CO partition always has structure $\tilde{\Delta} = \{[\underline{d}, \tilde{\delta}), [\tilde{\delta}, \bar{d}]\}$ so that it can be identified with a threshold $\tilde{\delta}$.

We establish three results: first agents' choices at the *intensive* margin are always at their efficient values, conditional on the equilibrium partition $\Delta(\bar{\delta})$. However, the equilibrium threshold is always different from the efficient one, so that inefficiency is entirely due to the agents' choices at the *extensive* margin. Second, and more relevant in terms of policy implications, the equilibrium threshold is always too low, so that equilibria are *always* characterized by overeducation. These two results are summarized in Prop. 2. The third result is established in Prop. 3 and 4, where we analyze the effects on welfare of several combinations of earnings taxes and taxes on the direct costs of education.

PROPOSITION 2. *Under the maintained assumptions, there is a CO allocation with threshold $\tilde{\delta}$. Any equilibrium allocation with $\bar{\delta} \in (\underline{d}, \bar{d})$ is CO if and only if $\bar{\delta} = \tilde{\delta}$. For a generic set of economies, $\bar{\delta} < \tilde{\delta}$, so that overeducation holds.*

The details of the proof are in the Appendix. The core of the argument is the following. Since, at an equilibrium, expected profits are zero, total expected surplus, given any arbitrary threshold $\hat{\delta}$, can be also written, at $\xi = 0$, as

$$E(W(\hat{\delta})) \equiv \int_{\hat{\delta}}^{\bar{d}} V^e(\delta; \hat{\delta}) d\delta + \int_{\underline{d}}^{\hat{\delta}} V^{ne}(\delta; \hat{\delta}) d\delta.$$

In Appendix we show that, conditional on the value of the threshold, the surplus is at its maximum. Hence, to establish the constrained inefficiency of equilibria, we

¹³Since all firms and all workers in each labor market are identical, no confusion should arise by dropping the indexes (i, j) .

just need to evaluate

$$\begin{aligned} \frac{\partial E(W(\widehat{\delta}))}{\partial \widehat{\delta}} \Big|_{\widehat{\delta}=\bar{\delta}} &= - [V^e(\delta = \bar{\delta}; \bar{\delta}) - V^{ne}(\delta = \bar{\delta}; \bar{\delta})] \\ &+ \int_{\bar{\delta}}^{\bar{d}} \frac{\partial V^e(\delta; \widehat{\delta})}{\partial \widehat{\delta}} \Big|_{\widehat{\delta}=\bar{\delta}} d\delta^e + \int_{\underline{d}}^{\bar{\delta}} \frac{\partial V^{ne}(\delta; \widehat{\delta})}{\partial \widehat{\delta}} \Big|_{\widehat{\delta}=\bar{\delta}} d\delta^{ne}. \end{aligned} \quad (2)$$

The first term is zero, by definition of $\bar{\delta}$. This is the private gain from investing in high skill for the workers at the margin. The second component measures the effects of the change in the composition of the labor force on the wages, and, hence, on the utilities of the inframarginal workers. This is the external effect. For each s , $V^s(\delta; \widehat{\delta})$ is increasing in $E(\delta^{s(\gamma-1)}|\widehat{\delta})$. Since $E(\delta^{s(\gamma-1)}|\widehat{\delta})$ is the expectation of a strictly increasing function of $\widehat{\delta}$, $\frac{\partial V^s(\delta; \widehat{\delta})}{\partial \widehat{\delta}} \Big|_{\widehat{\delta}=\bar{\delta}} > 0$, for each s , so that, at each equilibrium, $E(W(\widehat{\delta}))$ is increasing in the value of the threshold. Hence, equilibria are *always* characterized by overeducation. The basic intuition is simple. When, at the margin, a set of workers switches from sector e to sector ne , this increases the expected HC in both sectors, therefore decreasing the expected equilibrium wages. This stimulates investments in physical capital and, in turn, has a positive effect on the equilibrium wages for all the inframarginal workers. Given that workers optimally self-select themselves into the two labor markets, the decrease in the wage rate for the workers switching sector is exactly compensated by the schooling costs. Hence, at the equilibrium, an increase in the threshold has always a positive effect on welfare.

It is worthwhile to notice two additional properties of this class of economies. First, investments in high skills are productive, as long as $A^e > A^{ne}$. However, positive investments in these skills prevail even at the limit, when $A^e = A^{ne}$. Indeed, it is easy to see that, for each $A^e \geq A^{ne}$, $\lim_{\widehat{\delta} \rightarrow \bar{d}} f(\widehat{\delta}) > 0$. Hence, for T sufficiently small, the equilibrium threshold satisfies $\bar{\delta} < \bar{d}$, for each $A^e \geq A^{ne}$. At the limit, to invest in high skills is socially unproductive, but it is still individually profitable, because it allows to enter a sector where investments in physical capital are higher and, therefore, labor earnings are also higher.

Secondly, the chain of causation from a change in the threshold to a welfare improvement is via the investments in physical capital. Given any pair $\{K^{ne}, K^e\}$, an increase in the value of $\bar{\delta}$ implies that the expected wages (per efficiency unit of labor) are decreasing for both skills. This stimulates investments and, in turn, induces an increase in the equilibrium wages in the inframarginal markets.

From Prop. 2, it is clear that any policy just affecting the value of the threshold can be used to obtain a welfare improvement. Since preferences are quasi-linear, taxes on the direct cost of education, $\Delta T > 0$, have no effect on the choices at the intensive margin, while they can affect in the appropriate way the ones at the extensive margin and, therefore, the composition of the labor force in terms of HC. When $F(\cdot)$ is increasing in the threshold, an appropriate, positive value of ΔT is welfare improving since it moves $\bar{\delta}$ up. Therefore, the constrained optimal allocation can be implemented choosing an appropriate (and positive) value ΔT , so that the equilibrium threshold coincides with the CO one. Fig. 1 shows the values of the equilibrium thresholds (the solid curve on top) and the CO thresholds (the dashed

curve lying below) for a specific economy,¹⁴ corresponding to different levels of T .

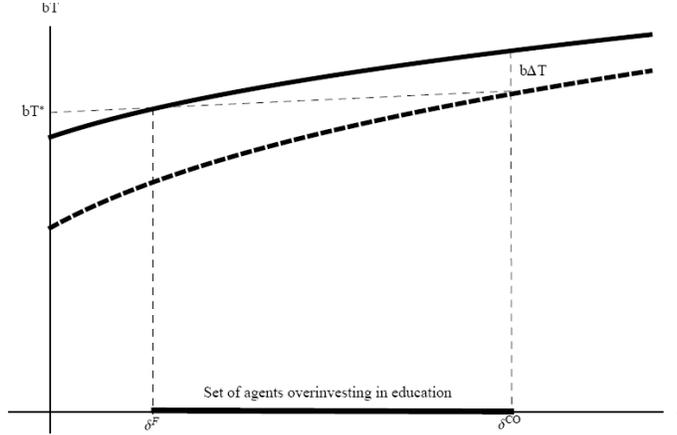


Figure 1

For each value of the cost of education T , the equilibrium value δ^F is below the constrained efficient level δ^{CO} . Hence, for each T , overeducation holds. For each T , the vertical distance between the two curves (measured at the value of δ^{CO}) determines the value of ΔT required to implement the constrained efficient allocation as an equilibrium.

Less obvious is that a small increase in the marginal tax rate on high earnings (the ones of the high skilled people) can also be welfare improving. Conditional on the threshold, private investments are at their optimal level. Hence, an increase in the marginal tax rate always has a direct (i.e., given the threshold) negative impact on surplus. However, it may have an indirect positive effect due to the induced change of the equilibrium threshold. To put it differently: an increase in the marginal tax rate distorts the supply of HC at the intensive margin, but it can improve welfare because of its effects on the overall composition of the labor force, i.e., at the extensive margin. An increase in the marginal rate on high income individuals has a negative direct incentive effect, but a positive composition effect, because it moves up the equilibrium threshold, so that the sign of the change in welfare is, in general, indeterminate. For a perfectly inelastic supply function, an increase in the marginal tax rate has no negative incentive effect. It has a strictly positive composition effect because the choice at the extensive margin depends upon the difference between the net wages in the two sectors. Hence, the effect on welfare is positive. By continuity, an increase in the marginal tax rate for high labor income is welfare improving whenever the elasticity of the supply function is sufficiently small. The same result holds if the initial tax rate is sufficiently small. On the other hand, an increase in the marginal tax rate on the low income workers has a negative incentive and composition effect. Hence, it has, unambiguously, a negative impact on welfare.

In the next two propositions, we take as a benchmark an economy with no taxes and subsidies on the direct costs of education and with a sufficiently small

¹⁴The parameters are $\alpha = \frac{2}{3}$, $\Gamma = 4$, $[\underline{d}, \bar{d}] = [1, 4]$, $A^e/A^{ne} = 1.2$. The function $E(W(\hat{\delta}))$ is strictly concave in the threshold, so that $\frac{\partial E(W(\hat{\delta}))}{\partial \hat{\delta}} = 0$ is a necessary and sufficient condition for a CO allocation.

flat earning tax. Assume that workers and firms choose their optimal behavior given a policy vector ξ . $E(W(\xi))$ is the total expected surplus at the equilibrium associated with ξ . Once again, we can ignore expected profits as a component of total surplus, since they are always zero, at the equilibrium. Using (A4) in Appendix, total expected surplus (including the tax revenues $R(\xi)$) is given by

$$E(W(\xi)) \equiv \frac{1}{b} \sum_s \frac{(\Gamma + \tau^s)(1 - \tau^s)^{\frac{1}{\Gamma}}}{\Gamma} \mu^s(\xi) A^{s \frac{1+\Gamma}{\alpha\Gamma}} E(\delta^{s(\gamma-1)} \bar{\delta}(\xi))^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} - \mu^e(\bar{\delta}(\xi))T,$$

where, for each s , $\mu^s(\bar{\delta}(\xi))$ is the measure of the set of agents investing in skill s , i.e., $\mu^e(\bar{\delta}(\xi)) = (\bar{d} - \bar{\delta}(\xi))$, and $\mu^{ne}(\bar{\delta}(\xi)) = (\bar{\delta}(\xi) - \underline{d})$.

PROPOSITION 3. Consider an equilibrium associated with $\bar{\xi} = (\bar{\tau}, \bar{\tau}, \Delta\bar{T})$, $\Delta\bar{T} = 0$, $\bar{\tau} > 0$ and sufficiently small. Also, assume that $\left. \frac{\partial f(\cdot)}{\partial \delta} \right|_{\bar{\delta}} > 0$. Then,

- i. $d\Delta T > 0$ increases total surplus,
- ii. $d\tau^{ne} > 0$ decreases total surplus,
- iii. $d\tau^e > 0$ increases total surplus.

The proof is in Appendix. The condition $\left. \frac{\partial f(\cdot)}{\partial \delta} \right|_{\bar{\delta}} > 0$ determines the comparative statics of the equilibrium threshold, and it guarantees that $\frac{\partial \bar{\delta}(\cdot)}{\partial \tau^e} > 0$, $\frac{\partial \bar{\delta}(\cdot)}{\partial \tau^{ne}} < 0$ and $\frac{\partial \bar{\delta}(\cdot)}{\partial \Delta T} > 0$. As established in Prop. 1 above, it always holds if the ratio A^e/A^{ne} is sufficiently large. A value $\bar{\tau}$ not "too large" is sufficient to guarantee that $\left. \frac{\partial E(W(\xi))}{\partial \delta} \right|_{\bar{\delta}} > 0$ at the equilibrium threshold $\bar{\delta}(\xi)$. Remember that $E(W(\xi))$ includes tax revenues. Since the wage rate is always larger in sector e , they are decreasing in the threshold. For τ sufficiently large, the behavior of the tax revenues is the dominant component of $E(W(\xi))$ and it may be $\frac{\partial E(W(\xi))}{\partial \delta} < 0$. Sufficiently small values of $\bar{\tau}$ rule out this possibility and guarantee that $\left. \frac{\partial E(W(\xi))}{\partial \delta} \right|_{\bar{\delta}} > 0$. These conditions are sufficient to establish (i – ii). Property (iii) requires an additional restriction. τ (or, alternatively, $\frac{1}{\Gamma}$) must be sufficiently small, because, for general values of the parameters, the sign of $\frac{\partial E(W(\xi))}{\partial \tau^e}$ is indeterminate.

Consider now policies where reductions in the income taxes are financed through taxes on the direct costs of education, or by revenue neutral changes ($d\tau^e, d\tau^{ne}$).

PROPOSITION 4. Let $\xi = (\bar{\tau}, \bar{\tau}, \Delta\bar{T})$, with $\Delta\bar{T} = 0$, $\bar{\tau} > 0$ and sufficiently small. Assume that $\frac{\Gamma}{1+\Gamma} > \tau$ and $\left. \frac{\partial f(\cdot)}{\partial \delta} \right|_{\bar{\delta}} > 0$. Consider balanced budget policies ($d\tau^e, d\tau^{ne}$) and ($d\tau^s, d\Delta T$). Then,

- i. $(d\tau^e, -d\tau^{ne}) \gg 0$ increases total surplus,
- ii. $(-d\tau^{ne}, d\Delta T) \gg 0$ increases total surplus,
- iii. $(-d\tau^e, d\Delta T) \gg 0$ increases total surplus.

The proof is, once again, in Appendix. The restriction $\frac{\Gamma}{1+\Gamma} > \tau$ implies that, given the value of the threshold, in each sector we are on the increasing part of the Laffer's curve. The Prop. shows that it may be welfare improving to move from a flat income tax to a progressive one. A welfare improvement also follows by a decrease in the marginal tax rate on "low" labor incomes, financed by increasing taxes on the direct costs of education. This is quite intuitive, since the two tax changes have positive effects both at the extensive and at the intensive margins. Consider now a decrease in the marginal tax rate on "high" labor incomes, similarly

financed. Now, the two tax changes have opposite effects on the choices at the extensive margin, since $d\Delta T > 0$ moves up the equilibrium threshold, while $d\tau^e < 0$ does the opposite. In Appendix, we show that the net effect on welfare is positive, for sufficiently small tax rates.

4. CONCLUSION

We have established how asymmetric information and lack of contractibility may generate *negative* pecuniary externalities related to HC accumulation. One of the key features of the economy is that skills are not perfectly fungible. We consider the polar case where the two skills are used in distinct production processes. However, the results are robust to more general specifications of the production process. The essential point is that workers, by choosing their education, sort themselves into distinct labor markets. This is consistent with the Roy's approach. When this happens, overeducation may hold at equilibrium. To the contrary, in a pure efficiency unit models, without sorting and with perfectly competitive spot labor markets, equilibrium allocations would always be constrained efficient.

Our results illustrate the trade-off which exists between extensive and intensive accumulation of HC. When agents are heterogeneous and different skills are used in production, to enlarge the set of agents with high skills may imply a reduction of the (across labor markets) average level of high skills HC. Provided that the distribution of HC for a given skill matters, this may have adverse welfare effects. Simple policy interventions can internalize the externality for the workers, implementing the constrained efficient allocation.

Two additional features of our results should be stressed. First, workers' investment choices affect social welfare through their impact on the investments in physical capital. Workers' choices at the extensive margin affect the distribution of HC and, consequently, firms' investments. It follows that an elastic demand for physical capital is an essential ingredient of the analysis. Secondly, there is asymmetric information ex-ante, because workers know their ability (i.e., the value of their δ) while firms do not know its value for the workers they will actually hire. However, this asymmetry disappears ex-post. Once labor markets open, firms perfectly observe the actual HC of their employees. Therefore, the informational problem at play is completely different from the one considered in the canonical signalling model.

5. APPENDIX

Given the structure of the production function, the economy essentially decomposes into two distinct sub-economies, one for each s . Therefore, in the sequel, we analyze separately, in particular, profits and surpluses obtained by the same firm using the two processes.

Let $\gamma \equiv \frac{1+\Gamma}{1+\Gamma-\alpha}$.

We start solving for the competitive wages given an arbitrary pair (δ^{ne}, δ^e) and any arbitrary pair $\bar{K}_j^s = \bar{K}^s$, for each s and j . A straightforward computation shows that, for each s , market clearing requires

$$w^s(\delta^s, \bar{K}^s) = \frac{(\alpha A^s \bar{K}^{s(1-\alpha)})^{\frac{\Gamma}{1+\Gamma-\alpha}}}{[(1-\tau^s)\delta^s]^{\frac{1-\alpha}{1+\Gamma-\alpha}}}. \quad (A1)$$

Given any arbitrary threshold $\widehat{\delta}$, ex-ante expected profits of firm j in sector s are

$$E\left(\Pi_j^s(\cdot)|\widehat{\delta}\right) = (1-\alpha)\alpha^{\frac{1-\alpha}{1-\alpha}}A^{s\frac{1}{1-\alpha}}E\left(w^s(\delta^s, \overline{K}^s)^{\frac{-\alpha}{1-\alpha}}|\widehat{\delta}\right)\overline{K}_j^s - \overline{K}_j^s. \quad (A2)$$

Since expected profit must be zero, it must be

$$E\left(w^s(\delta^s, \overline{K}^s)^{\frac{-\alpha}{1-\alpha}}|\widehat{\delta}\right) = \frac{1}{(1-\alpha)\alpha^{\frac{1-\alpha}{1-\alpha}}A^{s\frac{1}{1-\alpha}}},$$

so that, using (A1),

$$K^s(\widehat{\delta}) = (1-\alpha)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}}\alpha^{\frac{1}{\Gamma}}A^{s\frac{1+\Gamma}{\alpha\Gamma}}(1-\tau^s)^{\frac{1}{\Gamma}}E(\delta^{s(\gamma-1)}|\widehat{\delta})^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}}.$$

Substituting it into (A1), we obtain the $\widehat{\delta}$ -conditional equilibrium wage map

$$w^s(\delta^s; \widehat{\delta}) = \frac{(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha^{\frac{1}{\Gamma}}A^{s\frac{1}{\alpha}}E(\delta^{s(\gamma-1)}|\widehat{\delta})^{\frac{1-\alpha}{\alpha}}}{\delta^{s\frac{1-\alpha}{1+\Gamma-\alpha}}}. \quad (A3)$$

Hence, given $\widehat{\delta}$, each worker's optimal supply of HC is

$$\widehat{H}^s(\delta^s; \widehat{\delta}, \xi) = \left[(1-\alpha)^{\frac{1-\alpha}{\alpha\Gamma}}\alpha^{\frac{1}{\Gamma}}\right](1-\tau^s)^{\frac{1}{\Gamma}}A^{s\frac{1}{\alpha\Gamma}}\delta^{s\frac{1}{1+\Gamma-\alpha}}E(\delta^{s(\gamma-1)}|\widehat{\delta})^{\frac{1-\alpha}{\alpha\Gamma}},$$

and the value of the indirect utility function is

$$V^e(\delta; \widehat{\delta}, \xi) = \frac{\Gamma\left[(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha(1-\tau^e)A^{e\frac{1}{\alpha}}E(\delta^{e(\gamma-1)}|\widehat{\delta})^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1+\Gamma}{\Gamma}}\delta^{e(\gamma-1)}}{1+\Gamma} - (T + \Delta T), \quad (A4)$$

for $s = e$. For $s = ne$, we obtain a similar expression. The equilibrium condition $F(\widehat{\delta}) = 0$, eq. (1) in the text, follows immediately.

Proof of Prop. 1. (a) Consider an arbitrary ξ , with $\tau^e = \tau^{ne} = \tau$ and $\Delta T = 0$. $F(\widehat{\delta})$ can be rewritten as

$$\begin{aligned} F(\widehat{\delta}) &= \widehat{\delta}^{\gamma-1} \left(\left[A^e E(\delta^{e(\gamma-1)}|\widehat{\delta})^{1-\alpha} \right]^{\frac{1+\Gamma}{\alpha\Gamma}} - \left[A^{ne} E(\delta^{ne(\gamma-1)}|\widehat{\delta})^{1-\alpha} \right]^{\frac{1+\Gamma}{\alpha\Gamma}} \right) - \frac{bT}{(1-\tau)^{\frac{1+\Gamma}{\Gamma}}} \\ &\equiv f(\widehat{\delta}) - \widehat{bT} = 0 \end{aligned} \quad (A5)$$

Pick the partition induced by any interior threshold $\widehat{\delta}$. Evidently,

$$\delta'^{\gamma-1} \left(\left[A^e E(\delta^{e(\gamma-1)}|\widehat{\delta})^{1-\alpha} \right]^{\frac{1+\Gamma}{\alpha\Gamma}} - \left[A^{ne} E(\delta^{ne(\gamma-1)}|\widehat{\delta})^{1-\alpha} \right]^{\frac{1+\Gamma}{\alpha\Gamma}} \right) - \widehat{bT} \geq 0$$

if and only if $\delta' \geq \widehat{\delta}$. Hence, any non-empty equilibrium partition satisfies $\overline{\Delta}^e = [\widehat{\delta}, \overline{d}]$.

For each threshold $\widehat{\delta}$, and each s , $E(\delta^{s(\gamma-1)}|\widehat{\delta})$ is the conditional expectation of a strictly increasing function, hence it is strictly increasing in $\widehat{\delta}$ and well-defined on $[\underline{d}, \overline{d}]$. Thus, $f(\widehat{\delta})$ is continuous and strictly positive on $[\underline{d}, \overline{d}]$, including the boundary points. Let $\min_{[\underline{d}, \overline{d}]} f(\widehat{\delta}) \equiv \widehat{bT} \geq 0$ and $\widehat{bT} \equiv \max_{[\underline{d}, \overline{d}]} f(\widehat{\delta}) > \widehat{bT}$, because $f(\widehat{\delta})$ is clearly

not constant over $[\underline{d}, \bar{d}]$. Then, by the intermediate value theorem, for each T such that $T \in [\underline{T}, \bar{T}]$, there is $\bar{\delta}(\xi)$ such that $F(\bar{\delta}) = 0$. Hence, an equilibrium exists.

(b) Clearly, $\frac{\partial F(\cdot)}{\partial T} < 0$, $\frac{\partial F(\cdot)}{\partial \tau^e} < 0$ and $\frac{\partial F(\cdot)}{\partial \tau^{ne}} > 0$. Hence, the signs of the comparative statics properties, and uniqueness of equilibrium, follow immediately from the sign of $\frac{\partial F(\cdot)}{\partial \hat{\delta}}$. Let $\eta^s(\hat{\delta})$ be the elasticity of $E(\delta^{s(\gamma-1)}|\hat{\delta})$ with respect to $\hat{\delta}$. From (A5), simple computations show that

$$\text{sign} \frac{\partial f(\cdot)}{\partial \hat{\delta}} = \text{sign} \left(\begin{array}{c} (\gamma - 1) \left(1 - \left(\frac{A^{ne}}{A^e} \right)^{\frac{1+\Gamma}{\alpha}} \left(\frac{E(\delta^{ne(\gamma-1)}|\hat{\delta})}{E(\delta^{e(\gamma-1)}|\hat{\delta})} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \right) \\ + \frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma} \left(\eta^e(\hat{\delta}) - \left(\frac{A^{ne}}{A^e} \right)^{\frac{1+\Gamma}{\alpha}} \left(\frac{E(\delta^{ne(\gamma-1)}|\hat{\delta})}{E(\delta^{e(\gamma-1)}|\hat{\delta})} \right)^{\frac{(1-\alpha)(1+\Gamma)}{\alpha\Gamma}} \eta^{ne}(\hat{\delta}) \right) \end{array} \right).$$

Fix (α, Γ) . The first term is strictly positive. It is easy to check that $\eta^e(\delta) \geq 0$, for each $\delta \in [\underline{d}, \bar{d}]$, while $\eta^{ne}(\delta)$ is bounded. It follows that $\frac{\partial f(\cdot)}{\partial \hat{\delta}} > 0$, for $\frac{A^{ne}}{A^e}$ sufficiently small, because $\frac{E(\delta^{ne(\gamma-1)}|\hat{\delta})}{E(\delta^{e(\gamma-1)}|\hat{\delta})} < 1$. ■

Proof of Prop. 2. We start restricting the analysis to partitions $\tilde{\Delta} = \{[\underline{d}, \tilde{\delta}], [\tilde{\delta}, \bar{d}]\}$. Fix an arbitrary pair $(\hat{K}^e, \hat{K}^{ne})$. For each δ^s , the optimal investment in HC is obtained solving

$$\max_{h_i^s} A^s \hat{K}^{s(1-\alpha)} h^{s\alpha} - \frac{1}{\delta^s} \frac{h^{s1+\Gamma}}{1+\Gamma}$$

with optimal solution $H^s(\delta^s, \hat{K}^s) = (\alpha A^s \hat{K}^{s(1-\alpha)} \delta^s)^{\frac{1}{1+\Gamma-\alpha}}$, for each s . The associated surplus is

$$W(\delta^s, \hat{K}^s) = \frac{1}{\gamma} \alpha^{(\gamma-1)} A^{s\gamma} \hat{K}^{s(1-\alpha)\gamma} \delta^{s(\gamma-1)} - \hat{K}^s.$$

Fix an arbitrary threshold $\hat{\delta}$. Let $\mu^s(\hat{\delta})$ be the measure of the set of workers investing in skill s , given the partition induced by $\hat{\delta}$. It coincides with the measure of firms investing in physical capital of type s .¹⁵ The optimal level of the investments in physical capital is obtained solving the optimization problem

$$\max_{\hat{K}^s} \frac{1}{\gamma} \alpha^{\gamma-1} A^{s\gamma} \hat{K}^{s(1-\alpha)\gamma} \int_{\Delta^s(\hat{\delta})} \delta^{s(\gamma-1)} d\delta - \mu^s(\hat{\delta}) \hat{K}^s.$$

Since $(1-\alpha)\gamma < 1$, the problem is strictly concave. Its optimal solution is $\tilde{K}^s(\hat{\delta}) = (1-\alpha)^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} \alpha^{\frac{1}{\Gamma}} A^{s\frac{1+\Gamma}{\alpha\Gamma}} E(\delta^{s(\gamma-1)}|\hat{\delta})^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}}$, with associated

$$\tilde{H}^s(\delta^s; \hat{\delta}) = \delta^{\frac{1}{1+\Gamma-\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha\Gamma}} \alpha^{\frac{1}{\Gamma}} A^{s\frac{1}{\alpha\Gamma}} E(\delta^{s(\gamma-1)}|\hat{\delta})^{\frac{1-\alpha}{\alpha\Gamma}}.$$

Conditional on the threshold, these functions coincide with the equilibrium functions computed above. Replacing $\tilde{K}^s(\hat{\delta})$ and $\tilde{H}^s(\delta^s; \hat{\delta})$, for each s , in $E(W(\cdot))$ we obtain that, given any threshold $\hat{\delta}$, the maximum total expected surplus is

$$E(W(\hat{\delta})) = \frac{1}{b} \sum_s \mu^s(\hat{\delta}) A^{s\frac{1+\Gamma}{\alpha\Gamma}} E(\delta^{s(\gamma-1)}|\hat{\delta})^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}} - \mu^e(\hat{\delta}) T, \quad (A6)$$

¹⁵We are implicitly assume that each firm is matched with a labor market for just one skill. This simplifies the notation and it does not entail any loss of generality, given the production function.

a continuous function. Consider now the optimization problem

$$\max_{\tilde{\delta}} E(W(\tilde{\delta})).$$

Continuity implies that an optimal solution $\tilde{\delta}$ exists, either interior or on the boundary. Evidently, together with $\{\tilde{K}^{ne}, \tilde{K}^e\}$ and $\{\tilde{H}^e(\delta^e; \tilde{\delta}), \tilde{H}^{ne}(\delta^{ne}; \tilde{\delta})\}$, $\tilde{\delta}$ defines the CO allocation.

To conclude this part of the argument, we still need to show that the partition $\{[\underline{d}, \tilde{\delta}], [\tilde{\delta}, \bar{d}]\}$ is not dominated by any other measurable partition $\hat{\Delta}$. Consider (A6), replacing $E(\delta^{s(\gamma-1)}|\tilde{\delta})$ with $E(\delta^{s(\gamma-1)}|\hat{\Delta}^s)$. Pick any measurable $\hat{\Delta}$, such that there are sets B^{ne} and B^e of positive (and, without loss of generality, identical) measure such, for each pair $(\delta^{ne}, \delta^e) \in B^{ne} \times B^e$, $\delta^{ne} > \delta^e$. Since $A^e > A^{ne}$, it is quite obvious that we can increase $E(W(\cdot))$ simply switching the two sets, i.e., setting $B^{ne} \subset \Delta^e$, and $B^e \subset \Delta^{ne}$. Hence, the optimal partition $\hat{\Delta}$ must have structure $\{[\underline{d}, \tilde{\delta}], [\tilde{\delta}, \bar{d}]\}$, modulo some zero measure set.

We now show that an equilibrium allocation is CO if and only if $\tilde{\delta} = \bar{\delta}$. The ‘‘if’’ part is obvious, since the CO levels of the investments coincide with their equilibrium values, conditional on the threshold.

Assume that an CO allocation is interior. Then, it must be true that $\frac{\partial E(W(\hat{\delta}))}{\partial \hat{\delta}}|_{\hat{\delta}=\bar{\delta}} = 0$ at the equilibrium threshold $\bar{\delta}$. Consider eq. (2) in the text. We have already pointed out that, at any equilibrium $\bar{\delta}$, the term in square brackets is zero. Moreover, the last two terms are positive, since $V^s(\delta; \bar{\delta})$ is increasing in $E(\delta^{s(\gamma-1)}|\hat{\delta})$ which is the expected value of a strictly increasing function of $\hat{\delta}$. Thus, $\frac{\partial E(W(\hat{\delta}))}{\partial \hat{\delta}}|_{\hat{\delta}=\bar{\delta}} > 0$. ■

Observe that, for $\Delta T = 0$ and $\tau^e = \tau^{ne} = \tau$ sufficiently small, $\frac{\partial E(W(\cdot))}{\partial \hat{\delta}}|_{\hat{\delta}=\bar{\delta}(\xi)} > 0$, since, at $\xi = 0$, $\frac{\partial E(W(\hat{\delta}))}{\partial \hat{\delta}}|_{\hat{\delta}=\bar{\delta}(0)} > 0$.

For each s , define

$$S^s(\xi) \equiv \mu^s(\bar{\delta}(\xi)) \frac{1}{b} (1 - \tau^s)^{\frac{1+\Gamma}{\Gamma}} A^s \frac{1+\Gamma}{\alpha\Gamma} E(\delta^{s(\gamma-1)}|\bar{\delta}(\xi))^{\frac{1+\Gamma-\alpha}{\alpha\Gamma}},$$

the total utility (gross of education costs) obtained by workers in sector s .

Proof of Prop. 3. Let $\Delta T = 0$ and pick $\tau = \tau^e = \tau^{ne}$ such that $\frac{\partial E(W(\xi))}{\partial \hat{\delta}}|_{\hat{\delta}=\bar{\delta}(\xi)} > 0$. Clearly,

$$\frac{\partial E(W(\xi))}{\partial \tau^s} = \frac{\partial E(W(\xi))}{\partial \tau^s} \Big|_{\bar{\delta}(\xi)} + \frac{\partial E(W(\xi))}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} \frac{\partial \bar{\delta}(\cdot)}{\partial \tau^s}, \text{ for each } s,$$

$$\frac{\partial E(W(\xi))}{\partial \Delta T} = \frac{\partial E(W(\xi))}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} \frac{\partial \bar{\delta}(\cdot)}{\partial \Delta T}.$$

By assumption, $\frac{\partial \bar{\delta}(\cdot)}{\partial \tau^e} > 0$, $\frac{\partial \bar{\delta}(\cdot)}{\partial \tau^{ne}} < 0$ and $\frac{\partial \bar{\delta}(\cdot)}{\partial \Delta T} > 0$. Hence, $\frac{\partial E(W(\xi))}{\partial \Delta T} > 0$. By direct computation, for each s ,

$$\frac{\partial E(W(\xi))}{\partial \tau^s} = -\frac{(1+\Gamma)\tau^s}{\Gamma^2(1-\tau^s)^2} S^s(\xi) < 0.$$

Therefore, $\frac{\partial E(W(\xi))}{\partial \tau^{ne}} < 0$. The sign of $\frac{\partial E(W(\xi))}{\partial \tau^e}$ is not well-defined, because the direct effect, $\frac{\partial E(W(\xi))}{\partial \tau^e} \Big|_{\bar{\delta}(\xi)}$, is negative, while the indirect one is positive. Evidently, $\lim_{\tau^e \rightarrow 0} \frac{\partial E(W(\xi))}{\partial \tau^e} = 0$, while $\frac{\partial E(W(\xi))}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)}$ and $\frac{\partial \bar{\delta}(\cdot)}{\partial \tau^e}$ are positive and bounded away from 0, for each τ^e . Hence, for τ^e sufficiently small, $\frac{\partial E(W(\xi))}{\partial \tau^e} > 0$. ■

Proof of Prop. 4. Let $\Delta T = 0$ and $\tau = \tau^e = \tau^{ne}$. Then,

$$\begin{aligned} R(\xi) &\equiv \sum_s \frac{(1+\Gamma)\tau^s}{\Gamma(1-\tau^s)} S^s(\xi) + \mu^e(\bar{\delta}(\xi))\Delta T \\ &\text{and} \\ W(\xi) &\equiv \sum_s S^s(\xi) - \mu^e(\bar{\delta}(\xi))(T + \Delta T) + R(\xi) \\ &= \sum_s \frac{\Gamma + \tau^s}{\Gamma(1-\tau^s)} S^s(\xi) - \mu^e(\bar{\delta}(\xi))T, \end{aligned} \tag{A7}$$

By direct computation,

$$\frac{\partial R(\xi)}{\partial \tau^s} \Big|_{\bar{\delta}(\xi)} = \frac{1+\Gamma}{\Gamma^2} \frac{\Gamma - (1+\Gamma)\tau^s}{(1-\tau^s)^2} S^s(\xi) > 0,$$

since, by assumption $\frac{\Gamma}{1+\Gamma} > \tau$. Also, with a straightforward manipulation, we can write

$$\frac{\partial R(\xi)}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} = \sum_s \frac{(1+\Gamma)\tau^s}{\Gamma(1-\tau^s)} \frac{\partial S^s(\xi)}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} - \Delta T.$$

Since $\frac{\partial S^s(\xi)}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)}$ is bounded, $\lim_{\tau \rightarrow 0} \frac{\partial R(\xi)}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} = 0$ at $\Delta T = 0$.

The map given by the equilibrium and the invariance of the tax revenues conditions is

$$G(\bar{\delta}, \xi) \equiv \begin{bmatrix} F(\bar{\delta}; \xi) \\ R(\bar{\delta}; \xi) - \bar{R} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Consider first the balanced budget policy $(d\tau^e, d\tau^{ne})$. By the implicit function Thm.,

$$\begin{bmatrix} \frac{\partial \bar{\delta}}{\partial \tau^{ne}} \\ \frac{\partial \tau^e}{\partial \tau^{ne}} \end{bmatrix} = \begin{bmatrix} \frac{F_{\tau^{ne}} R_{\bar{\delta}} - F_{\bar{\delta}} R_{\tau^{ne}}}{F_{\tau^e} R_{\bar{\delta}} - F_{\bar{\delta}} R_{\tau^e}} \\ \frac{F_{\bar{\delta}} R_{\tau^{ne}} - F_{\tau^{ne}} R_{\bar{\delta}}}{F_{\tau^e} R_{\bar{\delta}} - F_{\bar{\delta}} R_{\tau^e}} \end{bmatrix}, \tag{A8}$$

where we use, for instance, F_{τ^e} to denote $\frac{\partial F(\cdot)}{\partial \tau^e} \Big|_{\bar{\delta}}$. Since $\lim_{\tau \rightarrow 0} \frac{\partial R(\xi)}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} = 0$, while the other terms are bounded away from zero, $[-F_{\bar{\delta}} R_{\tau^e} + F_{\tau^e} R_{\bar{\delta}}] < 0$ for τ sufficiently small. Since, by construction, $R(\xi)$ is invariant,

$$\begin{aligned} \frac{\partial E(W(\xi))}{\partial \tau^{ne}} &= \frac{\partial E(W(\xi))}{\partial \tau^{ne}} \Big|_{\bar{\delta}} + \frac{\partial E(W(\xi))}{\partial \tau^e} \Big|_{\bar{\delta}} \frac{\partial \tau^e}{\partial \tau^{ne}} + \frac{\partial E(W(\xi))}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} \frac{\partial \bar{\delta}(\xi)}{\partial \tau^{ne}} \\ &= \frac{\partial S^{ne}(\xi)}{\partial \tau^{ne}} \Big|_{\bar{\delta}} + \frac{\partial S^e(\xi)}{\partial \tau^e} \Big|_{\bar{\delta}} \frac{\partial \tau^e}{\partial \tau^{ne}} + \left[\sum_s \frac{\partial S^s(\xi)}{\partial \bar{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} + T \right] \frac{\partial \bar{\delta}(\xi)}{\partial \tau^{ne}}. \end{aligned}$$

Hence, using (A8),

$$\begin{aligned}
& \frac{\partial E(W(\xi))}{\partial \tau^{ne}} [F_{\tau^e} R_{\bar{\delta}} - F_{\bar{\delta}} R_{\tau^e}] \\
&= \frac{1 + \Gamma}{\Gamma(1 - \tau)} [(S^{ne}(\xi) R_{\tau^e} - S^e(\xi) R_{\tau^{ne}}) F_{\bar{\delta}} + (S^e(\xi) F_{\tau^{ne}} - S^{ne}(\xi) F_{\tau^e}) R_{\bar{\delta}}] \\
&+ \left[\sum_s \frac{\partial S^s(\xi)}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} + T \right] [F_{\tau^{ne}} R_{\tau^e} - F_{\tau^e} R_{\tau^{ne}}]. \tag{A9}
\end{aligned}$$

At $\tau^{ne} = \tau^e$, $(S^{ne}(\xi) R_{\tau^e} - S^e(\xi) R_{\tau^{ne}}) = 0$. $(S^e(\xi) F_{\tau^{ne}} - S^{ne}(\xi) F_{\tau^e})$ is positive since $F_{\tau^{ne}} > 0$ and $F_{\tau^e} < 0$. For $\chi(ne) = -1$ and $\chi(e) = 1$, we can write

$$\left[\sum_s \frac{\partial S^s(\xi)}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} + T \right] = \frac{(1 + \Gamma)(1 - \alpha)}{\alpha \Gamma} \sum_s S^s(\xi) \chi(s) \left(E(\delta^{s(\gamma-1)} | \bar{\delta}(\xi)) - \bar{\delta}^{\gamma-1} \right) > 0,$$

where the inequality holds because $E(\delta^{e(\gamma-1)} | \bar{\delta}(\xi)) - \bar{\delta}^{\gamma-1} > 0$ and $E(\delta^{ne(\gamma-1)} | \bar{\delta}(\xi)) - \bar{\delta}^{\gamma-1} < 0$. Hence, given that $[F_{\tau^{ne}} R_{\tau^e} - F_{\tau^e} R_{\tau^{ne}}] > 0$, the second addendum in (A9) is also positive. Therefore, for $R_{\bar{\delta}}$ (i.e., τ) sufficiently small in absolute value, $\frac{\partial E(W(\xi))}{\partial \tau^{ne}} [F_{\tau^e} R_{\bar{\delta}} - F_{\bar{\delta}} R_{\tau^e}] > 0$, and, therefore, $\frac{\partial E(W(\xi))}{\partial \tau^{ne}} < 0$, as claimed, since $[F_{\tau^e} R_{\bar{\delta}} - F_{\bar{\delta}} R_{\tau^e}] < 0$. This establishes (i).

Let's now consider a balanced budget policy $(d\tau^s, d\Delta T)$. By direct computation,

$$\begin{bmatrix} \frac{\partial \bar{\delta}}{\partial \Delta T} \\ \frac{\partial \tau^s}{\partial \Delta T} \end{bmatrix} = \begin{bmatrix} \frac{R_{\tau^s} + \mu^e(\cdot) F_{\tau^s}}{F_{\bar{\delta}} R_{\tau^s} - R_{\bar{\delta}} F_{\tau^s}} \\ -\frac{R_{\bar{\delta}} + \mu^e(\cdot) F_{\bar{\delta}}}{F_{\bar{\delta}} R_{\tau^s} - R_{\bar{\delta}} F_{\tau^s}} \end{bmatrix},$$

so that

$$\frac{\partial E(W(\xi))}{\partial \Delta T} = \frac{\partial E(W(\xi))}{\partial \tau^s} \frac{\partial \tau^s}{\partial \Delta T} + \frac{\partial E(W(\xi))}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} \frac{\partial \bar{\delta}(\xi)}{\partial \Delta T}$$

Consider the policy $(-d\tau^{ne}, d\Delta T) \gg 0$. For τ sufficiently small, $[F_{\bar{\delta}} R_{\tau^{ne}} - R_{\bar{\delta}} F_{\tau^{ne}}] > 0$, so that $\frac{\partial \bar{\delta}(\xi)}{\partial \Delta T} > 0$ and $\frac{\partial \tau^{ne}}{\partial \Delta T} < 0$. Since $\frac{\partial E(W(\xi))}{\partial \tau^{ne}} < 0$ (see the proof of Prop. 3) and $\frac{\partial E(W(\xi))}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} > 0$, this policy is welfare improving.

For the policy $(-d\tau^e, d\Delta T) \gg 0$, $\frac{\partial \bar{\delta}(\xi)}{\partial \Delta T}$ can be negative, because $F_{\tau^e} < 0$. However,

$$\lim_{\tau \rightarrow 0} \frac{\partial E(W(\xi))}{\partial \Delta T} = \frac{\partial E(W(\xi))}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)} \frac{1}{F_{\bar{\delta}}} \left[1 - \frac{\bar{\delta}(\xi)^{\gamma-1}}{E(\delta^{e(\gamma-1)} | \bar{\delta}(\xi))} \right].$$

Given that $\frac{\bar{\delta}(\xi)^{\gamma-1}}{E(\delta^{e(\gamma-1)} | \bar{\delta}(\xi))} < 1$, and $(\frac{\partial E(W(\xi))}{\partial \hat{\delta}} \Big|_{\hat{\delta}=\bar{\delta}(\xi)}, F_{\bar{\delta}}) \gg 0$, $\frac{\partial E(W(\xi))}{\partial \Delta T} > 0$ and

the policy is welfare improving. ■

REFERENCES

- [1] Acemoglu, D.T., 1996, A microfoundation for social increasing returns in human capital accumulation, *Quarterly Journal of Economics* **111**, 779-804.

- [2] Andenberg, D., 2009, Optimal policy and the risk-properties of human capital reconsidered, *Journal of Public Economics* **93**, 1017-1026.
- [3] Andenberg, D. and F. Andersson, 2003, Investments in human capital, wage uncertainty, and public policy, *Journal of Public Economics* **87**, 1521-1537.
- [4] Ben-Porath, Y., 1970, The production of human capital over time, in *Education, Income and Human Capital* by W. Hansen, Ed., Cambridge MA: NBER, 129-154.
- [5] Bovenberg, A.L. and B. Jacobs 2005, Redistribution and education subsidies are Siamese twins, *Journal of Public Economics* **89**, 2005-2035.
- [6] Charlotte, O., and B. Decreuse, 2005, Self-selection in education with matching frictions, *Labour Economics* **12**, 251-267.
- [7] Eaton, J. and H.S. Rosen, 1980, Taxation, human capital, and uncertainty, *American Economic Review* **70**, 705-715.
- [8] Halfdanarson, B., D.F. Heuermann, and J. Südekum, 2010, Human capital externalities and the urban wage premium: Two literatures and their interrelations, *Urban Studies* **47**, 749-767.
- [9] Heckman, J.J., 1976, A life-cycle model of earnings, learning, and consumption, *Journal of Political Economy* **84**, S9-S44.
- [10] Jacobs, B. 2005, Optimal income taxation with endogenous human capital, *Journal of Public Economic Theory* **7**, 295-315.
- [11] Jacobs, B. and A.L. Bovenberg, 2011, Optimal taxation of human capital and the earnings function, *Journal of Public Economic Theory* **13**, 957-971.
- [12] Jacobs, B., D. Schindler and H. Yang, H., 2012, Optimal taxation of risky human capital" *Scandinavian Journal of Economics*, forthcoming.
- [13] Mendolicchio, C., D. Paolini, and T. Pietra, 2012, Income taxes, subsidies to education, and investments in human capital, *Journal of Public Economic Theory* (forthcoming).
- [14] Mendolicchio, C., D. Paolini, and T. Pietra, 2010, Investments in education in a random matching model, DSE WP 702.
- [15] Moretti, E., 2004, Human capital externalities in cities, in *Handbook of Urban and Regional Economics*, North Holland-Elsevier.
- [16] Roy, A., 1951, Some thoughts on the distribution of earnings, *Oxford Economic Papers*, **3**, 135-146.

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