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Arrow's theorem of the deductible: moral hazard and stop-loss in health insurance

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Abstract

We show that the logic of Arrow's theorem of the deductible, i.e. that it is optimal to focus insurance coverage on the states with largest expenditures, remains at work in a model with ex post moral hazard. The optimal insurance contract takes the form of a system of "implicit deductibles", i.e. it results in the same indemnities as a contract with full insurance above a variable deductible positively related to the elasticity of medical expenditures with respect to the insurance rate. In a model with an explicit stop-loss arrangement, i.e. with a predefined ceiling on the annual expenses of the insured, this stop-loss takes the form of a deductible, i.e. there is no reimbursement for expenses below the stop-loss amount. One motivation to have some insurance below the deductible arises if regular health care expenditures in a situation of standard health have a negative effect on the probability of getting into a state with large medical expenses.

Keywords: optimal health insurance, deductible stop-loss, moral hazard.

JEL Classification: I13

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1 Introduction

One of the most elegant results in the theory of optimal insurance is Arrow's so-called "theorem of the deductible": "If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100% coverage above a deductible minimum" (Arrow, 1963). In his seminal article, Arrow assumed that the loading factor is proportional to total (expected) reimbursements and that the buyer maximizes expected utility. However, these assumptions are not essential for the basic result. The optimal insurance policy features a positive deductible as soon as the loading increases with total reimbursements (see, e.g., Zweifel et al., 2009). Moreover, Gollier and Schlesinger (1996) have shown that a deductible insurance policy second-degree stochastically dominates any other feasible insurance policy, and that deductibles should therefore be preferred by all risk-averse agents even if they are not expected utility-maximizers. The robustness of Arrow's result reflects its simple logic: since it is better for the consumer to insure expenditures when disposable income is low rather than high, insurance funds should always be spent on the highest expenditures.

In its original form, Arrow's theorem does not apply under moral hazard. This explains why, despite its strong intuitive appeal, it did not play an important role in later developments of the theory of optimal health insurance. With full insurance above a deductible, the *ex post* marginal cost to the insured of additional expenses beyond the deductible is zero, leading to *ex post* over-consumption. Following another lead in Arrow (1963), the literature has focused on this moral hazard problem and has analysed how introducing coinsurance, i.e. partial reimbursement of expenses, may lower the incentives for overconsumption. The optimal level of coinsurance should then strike a balance between the welfare loss of moral hazard, calling for a larger out-of-pocket share for the insured, and the welfare gain of risk sharing, calling for a more generous reimbursement (Pauly, 1968; Zeckhauser, 1970).[1]

Most models in the literature have assumed a linear insurance scheme

[1] An extensive survey of the literature on optimal health insurance, including more references, can be found in Cutler and Zeckhauser (2000) and McGuire (2012). Both chapters also comment on the variety of medical insurance policies in the real world.

with a fixed coinsurance rate. Note that this linear structure is an assumption, not a result of the theory. The simple logic of Arrow's theorem cannot be recovered in this approach. Moreover, the assumption of a fixed coinsurance rate does not fit insurance policies in the real world which often have nonlinear features, such as explicit deductibles or a (possibly income-dependent) stop-loss, i.e. a maximum imposed on total out-of-pocket payments of the consumer. The authoritative RAND-experiment (Manning et al., 1987) introduced in its experimental policies partial first-dollar insurance and a stop-loss, although the researchers were well aware that this would make it more difficult to compare their results to the existing literature.[2] As another example, our home country, Belgium, has a social insurance system with a highly differentiated structure of co-payments and with an income-dependent stop loss, the so-called maximum billing system.[3] The theoretical results derived from a model with a constant coinsurance rate may be misleading when one wants to analyse these more complex real-world systems. However, formulating a more general theoretical model has been considered difficult and non-rewarding. Commenting on Blomqvist (1997), who solves through optimal control theory a model of non-linear health insurance, Cutler and Zeckhauser (2000, pp. 586-587) conclude: "Alas, this is a complicated problem, whose algebra is not particularly revealing".

In this paper, which builds upon the analysis in Drèze (2002), we derive the optimal insurance policy in a general model with a discrete number of states of health and we show that *Arrow's theorem of the deductible remains relevant in a setting with moral hazard.* In section 2 we introduce our model and we derive the original Arrow-result in a simple first-best setting. In section 3 we introduce *ex post* moral hazard. We find the usual trade-off between moral hazard and risk sharing, but we also show how the logic of Arrow's theorem of the deductible is still at work in this more general model. The optimality results can be interpreted in terms of an *implicit* deductible property, namely: Arrow's theorem holds over subsets of cases characterised

[2] The authors are crystal-clear about their position: "We make no apologies for this intentional noncomparability; a constant coinsurance rate, while convenient for obtaining comparative statics results, is not an insurance policy that theory suggests would be optimal, assuming risk aversion. Indeed, an optimal policy would almost certainly contain a stop-loss feature, exactly as the experimental plans did" (Manning et al., 1987, referring to Arrow).

[3] These features are partly motivated by redistributive considerations – altogether absent from the present paper.

by similar elasticities of medical expenses with respect to insurance rates, with elasticity-related deductibles; under a single common elasticity, Arrow's theorem holds, but the deductible increases with that elasticity (which plays the same role as the loading factor). Linear coinsurance schemes are suboptimal, as – conditional on the demand elasticity – insurance has to be more generous for larger expenditures. We compare our approach to the one of Blomqvist (1987). In section 4 we analyse a system (not considered in Drèze (2002)) with an explicit stop-loss, i.e. with a predefined ceiling on the annual expenses of the insured, and we show how Arrow's result survives the introduction of ex post moral hazard, i.e. ex post moral hazard does not offer an argument to introduce partial first-dollar insurance (as in the RAND experiment and in Belgium) and demand elasticities become irrelevant. However, Section 5 suggests that some first-dollar insurance can be rationalized in a setting with *ex ante* moral hazard. We relate our findings to the literature on "willingness to pay for safety" (Dehez and Drèze, 1982), to existing models on optimal insurance for prevention (Ellis and Manning, 2007) and to the recent literature stressing the importance of taking into account cross-price effects in a setting with more health care commodities (Goldman and Philipson, 2007). Section 6 concludes.

2 First best: Arrow's theorem in a simple model

In its simplest form, a medical insurance problem concerns an individual facing uncertainty about her future health condition. There are S states of health indexed $s = 1, ..., S$ with probabilities p_s. Individuals have conditional preferences between vectors $(M_s, C_s) \in \Re^2_+$, where $M_s \geqslant 0$ and $C_s \geqslant 0$ stand respectively for medical expenditures and for disposable wealth (or expenditures on consumption exclusive of medical expenditures) in state s. In general these preferences could be represented by state-dependent utility functions $U_s(M_s, C_s)$. To simplify the analysis we assume, in line with much of the related literature, that preferences are separable between medical expenditure and consumption and that preferences over disposable wealth are state-independent, i.e. $U_s(M_s, C_s) = f_s(M_s) + g(C_s)$.[4] The function $f_s(M_s)$ captures both the effect of medical expenditures on health and the effect of

[4] The general model is analysed in Drèze (2002).

health on utility.[5] We assume f_s and g to be continuously differentiable and strictly concave, i.e. $f_s' > 0$, $f_s'' < 0$, $g' > 0$, $g'' < 0$. We also assume that resources are state-independent, i.e. $W_s = W_t = W$ for all $s, t = 1, ..., S$. Under these assumptions, preferences over S-vectors of medical expenditures and disposable wealth are represented by the expected utility

$$V(M, C) = \sum_s p_s \left[f_s(M_s) + g(C_s) \right]. \tag{1}$$

The individual may buy medical insurance $\alpha_s M_s$, $0 \leqslant \alpha_s \leqslant 1$ at a premium

$$\pi = (1 + \lambda) \sum_s p_s \alpha_s M_s, \tag{2}$$

where λ is a state-independent loading factor and α_s is a state-specific insurance rate (with $1 - \alpha_s$ as the coinsurance rate). The assumption that the insurance rate α_s can be state-specific seems to suggest that the state s is observable. This is in general not a realistic assumption. We will return to this issue later on.

Let us now consider optimal health insurance in a first-best setting without moral hazard. This means that the individual decisions about medical expenditures in state s take into account their impact on the premium π. The optimal policy is then found by solving the problem

$$\max_{\alpha_1, ..., \alpha_S, M_1, ..., M_S} V(M, C) = \sum_s p_s \left[f_s(M_s) + g(W - \pi - (1 - \alpha_s) M_s) \right] \tag{3}$$

subject to eq. (2). The first-order conditions are

$$\frac{dV}{dM_s} = p_s \left[f_s' - (1 - \alpha_s) g_s' \right] - (1 + \lambda) p_s \alpha_s \sum_t p_t g_t' = 0, \tag{4}$$

$$\frac{dV}{d\alpha_s} = p_s M_s \left[g_s' - (1 + \lambda) \sum_t p_t g_t' \right] \leqslant 0, \qquad \alpha_s \frac{dV}{d\alpha_s} = 0. \tag{5}$$

[5] Our model can be interpreted as a shortcut for $U_s(M_s, C_s) = v(H_s) + g(C_s)$, with H_s indicating health in state s, influenced by health care expenditures, i.e. $H_s = h_s(M_s)$. Our assumptions of separability of preferences and state-independence of $g(.)$ remove the potential effect of health on the marginal utility of income. It is well known that a non-zero cross-effect complicates all the results on optimal insurance and that the empirical information at this moment does not allow us to make strong statements about the (variable) signs of cross-effects.

Simplifying these first-order conditions immediately yields

$$\text{for all } s = 1, ..., S,\ f_s' = g_s' \tag{6}$$

and

$$\text{either } \alpha_s = 0 \text{ or } g_s' = (1+\lambda)\sum_t p_t g_t' := (1+\lambda)\overline{g}'. \tag{7}$$

Eq. (6) shows that medical expenditures are set optimally, with marginal benefits equal to marginal costs in each state s.

Eq. (7) is more interesting. Since $(1+\lambda)\overline{g}'$ is independent of s, g_s' (and therefore $(1-\alpha_s)M_s$) will be the same for all states s with $\alpha_s > 0$. Define the deductible $D := (1-\alpha_s)M_s$ and write g_D' for the marginal utility of wealth at $C = W - \pi - D$. We can then rewrite eq. (7) as

$$\alpha_s = \max(0, \frac{M_s - D}{M_s}), \qquad g_D' = (1+\lambda)\overline{g}'. \tag{8}$$

This is precisely Arrow's *theorem of the deductible*. The marginal utility of wealth must be the same in all states for which $\alpha_s > 0$; if medical expenditures are smaller than D, expenses are fully borne by the insured. Note that, if the loading factor $\lambda = 0$, we get full insurance ($g_s' = \overline{g}'$ for all s). Note also that this deductible policy can easily be implemented, even if the state s is not observable.

It is readily verified that, under DARA preferences,[6] in the optimum D *is increasing in* W and λ but decreasing in risk aversion, as measured for instance by the Arrow-Pratt coefficient of relative risk aversion.[7]

3 Second best: Ex post moral hazard and implicit deductibles

While the logic of Arrow's theorem of the deductible in the case of first-best is well known, we will now show that this logic remains at work in a second-best context with ex post moral hazard. In this setting it takes the form of an "implicit deductible" property.

[6] DARA: decreasing absolute risk aversion.

[7] The impact of an increase in risk on the optimal level of the deductible is analysed in Eeckhoudt et al. (1991).

We speak of "ex post-moral hazard" when the treatment is chosen by the insured after observing the state, thus without regard for the impact of M_s on the premium π. In state s, (s)he therefore solves the problem

$$\max_{M_s} f_s(M_s) + g(W - \pi - (1 - \alpha_s)M_s)$$

yielding the first-order condition

$$f'_s = g'_s(1 - \alpha_s). \tag{9}$$

Condition (9) immediately reveals the "overconsumption" feature induced by the insurance policy: instead of obtaining a marginal rate of substitution between medical expenditures M_s and consumption expenditures C_s equal to unity (as in eq. (6)), we now obtain a marginal rate of substitution equal to $1 - \alpha_s$, which is smaller than 1 in all states where $\alpha_s > 0$. The higher is the insured fraction α_s, the higher is overconsumption.[8] We write medical expenditures as a function $M_s(\alpha_s)$ of the insurance rate and we define the elasticity of medical expenditure with respect to the insurance rate as $\eta_s = \frac{\alpha_s}{M_s}\frac{dM_s}{d\alpha_s}$.

The optimal insurance problem now becomes

$$\max_{\alpha_1,\ldots,\alpha_S} \Lambda = \sum_s p_s \left[f_s(M_s(\alpha_s)) + g(W - \pi - (1 - \alpha_s)M_s(\alpha_s)) \right]$$

subject to

$$\pi = (1 + \lambda) \sum_s p_s \alpha_s M_s(\alpha_s).$$

This yields the first-order conditions

$$\frac{\partial \Lambda}{\partial \alpha_s} = p_s \left[f'_s \frac{dM_s}{d\alpha_s} + g'_s \left(M_s - (1 - \alpha_s) \frac{dM_s}{d\alpha_s} \right) \right]$$

$$-(1 + \lambda) p_s \left(M_s + \alpha_s \frac{dM_s}{d\alpha_s} \right) \sum_t p_t g'_t \leqslant 0, \tag{10}$$

$$\alpha_s \frac{\partial \Lambda}{\partial \alpha_s} = 0. \tag{11}$$

Using eq. (9) and the definition of η_s, we can simplify eq. (10) as

$$\frac{\partial \Lambda}{\partial \alpha_s} = p_s M_s \left[g'_s - \overline{g}' (1 + \lambda)(1 + \eta_s) \right] \tag{12}$$

[8] The first-best result obtains if $\alpha_s = 0$. If $\alpha_s = 1$, one gets $f'_s = 0$.

Combing (11) and (12) we immediately derive the characteristics of the optimal insurance policy:

$$\text{either } \alpha_s = 0 \text{ or } g'_s = (1+\lambda)\overline{g}'(1+\eta_s) \tag{13}$$

It is instructive to compare eqs. (13) and (7). If the demand elasticities in the different health states are equal, i.e. $\eta_s = \overline{\eta}$ for all s, we are back to the deductible result of Arrow's theorem, but with the loading factor $(1+\lambda)$ blown up by the moral hazard factor $(1+\overline{\eta})$. Not surprisingly, the deductible will therefore be larger than in the first-best. More generally, the solution is characterized by an "implicit deductible" property, where the deductible increases with η_s. We formulate this result as

Proposition 1 *If resources are state-independent, preferences are separable with state-independent consumption preferences and the probabilities of the different states cannot be influenced by the consumer, the optimal insurance contract results in the same indemnities as a contract with 100% insurance above a variable deductible positively related to η_s, the elasticity of medical expenditures with respect to the insurance rate α_s.*

It is important to interpret Proposition 1 correctly. Consider the special case with $\eta_s = \overline{\eta}$ for all s. This case is not devoid of interest. Indeed, the empirical information on the differences between the demand elasticities in different health states is limited, and in many cases the best one has is a global estimate which can be interpreted as an "average" $\overline{\eta}$. One could then apply (13) with this common $\overline{\eta}$. This will in general be suboptimal, but there is a saving grace: the uncertainty about $\overline{\eta}$ is borne by the insurer, not the insured; and the insurer is compensated for bearing uncertainty through the loading factor λ. This strongly suggests that a deductible (or a stop-loss arrangement) should be an important feature in any optimal insurance policy. Note, however, that (13) characterises a second-best insurance policy implemented through the variable insurance rates $\alpha_s = (M_s - D)/M_s$, not through the explicit announcement of a deductible D. Indeed, if the indemnities $\alpha_s M_s$ were formulated as $M_s - D$, then the first-order conditions (9) should be replaced by $f'_s(M_s) = 0$, reflecting the fact that the insured perceives a marginal cost of medical expenditures equal to 0 in that case. Moreover, one can argue that health states are costly to verify and that the assumption of state-specific insurance rates is therefore unrealistic.[9] This need not always be true. One could for instance think about a model with two states: a "good" health state in which only ambulatory care is needed and a "bad" health state with a hospitalization and intensive follow-up treatment. These states are readily verifiable and our proposition 1 then gives an immediate justification for the feature present in many real world systems of a higher insurance rate for hospital expenditures. Yet, in general we are ready to admit that the rule (13) has limited applicability. In the next section we will therefore analyse a setting with an explicit stop-loss arrangement.

The fact that the results in this section do not lend themselves easily to implementation, does not mean that our qualitative findings are devoid of practical implications. Let us summarize the most important ones. First, our results confirm the intuition that insurance rates should (*ceteris paribus*) be inversely related to the elasticity of health care expenditures with respect to the insurance rate and positively related to risk aversion. More importantly, they also validate the practice of (*ceteris paribus*) higher *insurance rates* (not only indemnities) for major medical expenses. Note that, if $\eta_s = \eta_t$, it follows from eq. (13) that $(1-\alpha_s)M_s = (1-\alpha_t)M_t$ – and therefore $\alpha_s > \alpha_t$ if $M_s > M_t$. This is an important qualitative finding, which obviously cannot be recovered in a linear model with a fixed insurance rate.

Second, our results suggest an easy empirical procedure for the ex post-evaluation of existing systems of health insurance on the basis of information about individual out-of-pocket payments. This information is often available. If interindividual differences in risk aversion are not too large and if demand elasticities in the different health states can be assumed to be equal, an optimal insurance scheme should put an income-dependent ceiling on these out-of-pocket payments in different states. More generally, out-of-pocket payments should be linked in a straightforward way to demand elasticities. One could either use the available information about demand elasticities to check the optimality of the existing scheme, or derive the "implicit" demand elasticities which would make the existing scheme optimal and check if they show a reasonable pattern.

[9]Moreover, if a sufficiently refined classification of health states were verifiable, there would be no need to specify the indemnity through the insurance rate. One could as well define a lump-sum indemnity, specific to state s: this would immediately solve the moral hazard-problem.

Third, our results strongly suggest that the common assumption of a constant insurance rate $\alpha_s = \alpha$, identical in all states, is suboptimal. The optimal medical insurance scheme will in general be nonlinear. This suggests a comparison with Blomqvist (1997). Blomqvist assumes that a random state-of-the-world variable represents exogenous shocks to the consumer's health status and is not observable to the insurer: the amount to be paid to the consumers can only depend on their health care expenditures. The qualitative results he derives from the resulting optimal control problem are analogous to our findings in (13). More specifically, he emphatically rejects the optimality of a linear scheme with a fixed insurance rate and shows that there should be more generous insurance for larger expenditures, conditional on the demand elasticities. Our vector of insurance rates $(\alpha_1, ..., \alpha_S)$ can be seen as a discrete approximation of his non-linear scheme; this is especially obvious when considering his numerical illustration, in which he implements a discrete version of his general model.

4 Third best: Ex post moral hazard under an explicit deductible

The previous analysis strongly suggests that some stop-loss feature should be part of the optimal insurance policy, even in a setting with ex post moral hazard: this simply reflects the original intuition of Arrow's theorem that it is optimal to focus insurance on the states with the largest expenditures. Moreover, as noted before, stop-loss arrangements are indeed present in many contracts and countries and played an important role in the RAND-experiment. However, as we explained in the previous section, the second best-insurance scheme with state-specific α_s cannot be implemented as such. We therefore turn now to what could be called a "third best"-policy, in which an explicit stop loss arrangement is introduced into the health insurance contract. We will show that such a stop-loss arrangement should take the form of a deductible, i.e. there should be no insurance for expenses below the stop-loss amount.

When the insurance policy refers explicitly to an upper bound D on the medical expenses borne by the insured, then (s)he will choose ex post M_s such that $f_s'(M_s) = 0$ whenever $M_s \geqslant D$ – instead of $f_s' = (1 - \alpha_s)g_s'$ as in eq. (9). Therefore, overconsumption will increase. This has implications for the structure of the insurance rates α_s in the states with $M_s < D$.

With an explicit stop-loss, the optimal policy problem becomes

$$\max_{\alpha_s, D} \Lambda = \sum_{M_s < D} p_s \left[f_s(M_s(\alpha_s)) + g(W - \pi - (1-\alpha_s)M_s(\alpha_s)) \right]$$
$$+ \sum_{M_s \geqslant D} p_s \left[f_s(M_s) + g\left(W - \pi - D\right) \right] \tag{14}$$

under the constraints

$$\pi = (1+\lambda) \left[\sum_{M_s < D} p_s \alpha_s M_s(\alpha_s) + \sum_{M_s \geqslant D} p_s (M_s - D) \right] \tag{15}$$

$$f'_s = (1-\alpha_s) g'_s \text{ if } M_s < D, \qquad f'_s = 0 \text{ if } M_s \geqslant D. \tag{16}$$

The first-order conditions for α_s (for the states with $M_s < D$) are identical to those that were derived in the second best-setting of the previous section – see eqs. (10), (11) and (12), leading to the conclusion (13), which is repeated here for convenience:

$$\text{either } \alpha_s = 0 \text{ or } g'_s = (1+\lambda)\overline{g}'(1+\eta_s). \tag{17}$$

In differentiating Λ w.r.t. D, attention must be paid to the fact that the two sums defining Λ are defined with reference to D. If (and only if) there exists some s^* such that $M_{s^*} = D$, then raising D (infinitesimally) will transfer s^* from the second sum to the first.[10] Note that the cost to the agent of $M_{s^*} = D$ is the same as would result from $\alpha_{s^*} = 0$. We shall evaluate $\frac{\partial \Lambda}{\partial D}$ at unchanged M_{s^*} and justify that procedure on the basis of our conclusion. Accordingly:

$$\frac{\partial \Lambda}{\partial D} = - \sum_{M_s \geqslant D} p_s \left[g'_s - (1+\lambda) \sum_t p_t g'_t \right] \leqslant 0, \qquad D \frac{\partial \Lambda}{\partial D} = 0. \tag{18}$$

The argument of g is constant over all s such that $M_s \geqslant D$, namely $W - \pi - D$. Write, as before, g'_D for $g'(W - \pi - D)$. Then (18) entails

$$\text{either } D = 0 \text{ or } g'_D = \overline{g}'(1+\lambda). \tag{19}$$

Eq. (19) gives a clear rule for fixing the optimal value of D. Note that, if medical expenses are very large in some states, $\overline{g}'$ and therefore g'_D and D

[10] Lowering D infinitesimally will not trigger a transfer because $M_s < D$ in the first sum.

may also be very large.[11] Yet, this does not detract from the principle that an optimal insurance plan should include a stop-loss arrangement.

Combining (17) and (19), we obtain

$$\text{if } \alpha_s D > 0, \text{ then } g'_s = g'_D(1+\eta_s) > g'_D.$$

With $g(.)$ concave, $g'_s > g'_D$ implies $W - \pi - (1-\alpha_s)M_s < W - \pi - D$, and therefore $M_s > D$. This contradicts the condition $M_s < D$ defining the first sum. Accordingly, either $\alpha_s = 0$ or $D = 0$. Thus, if $D > 0$, then $\alpha_s = 0$, so that Arrow's theorem holds – without the condition that η_s be independent of s. Also, if $\alpha_{s^*} = 0$, the assumption of unchanged M_{s^*} underlying (18) is verified.

We can summarize these results as

Proposition 2 *If resources are state-independent, preferences are separable with state-independent consumption preferences and the probabilities of the different states cannot be influenced by the consumer, an optimal stop-loss insurance policy takes the form of a deductible, i.e. there is no reimbursement for expenses below the stop-loss amount and full reimbursement of the excess of expenses over the deductible.*

Proposition 2 is a striking illustration of the strength of the logic underlying Arrow's theorem of the deductible. Even in a situation with ex post moral hazard, it is optimal to spend insurance funds in the states with the largest expenditures. It is not optimal to have insurance below the deductible, even in a setting with ex post moral hazard. Additional arguments are needed to justify the kind of first-dollar insurance arrangements that were included in the RAND-experiment or that we observe in the Belgian maximum billing-system.

5 Ex ante moral hazard

It has already been suggested in the literature that a deductible is not necessarily optimal in health insurance contracts as soon as we take into account

[11] This is in line with the empirical results of Manning and Marquis (1996), who find that an optimal plan with a stop-loss would imply a very high value for the latter and, indeed, claim that they "were unable to find a plausible estimate of the optimal stop-loss within the range of the Health Insurance Experiment data" (p. 631).

the preventive value of some health services (e.g. Bardey and Lesur, 2005). In this section we will further explore this argument. We distinguish two possible cases. In subsection 5.1, we follow the literature (Ellis and Manning, 2007; Zweifel et al., 2009) and model *identifiable* preventive actions that are taken before the health state realizes. One can think about lifestyle variables (such as smoking, drinking, dieting or physical activity) or about general medical screening. We will show that such preventive actions should in general be subsidized. In subsection 5.2, we assume that prevention is linked to medical expenditures in relatively healthy states and that it is impossible to distinguish the curative and the preventive component, for instance in regular visits to the GP.[12] This justifies some insurance below the deductible.

In both cases we retain the model of the previous section, i.e. a model with an explicit deductible D. In order to bring out the effect of prevention with a maximum of clarity, we rely on a simplified version of our model. There are only two states of health, s and t, where s denotes a state of "standard" health, whereas t corresponds to a disease calling for an expensive therapy. As in the example given before, the "good" health state could be one in which only ambulatory care is needed, while the "bad" health state would require hospitalization and intensive follow-up treatment. As we have shown in the previous section, under socially efficient health insurance, the high cost M_t will be largely covered, i.e. the expenses for the patient will be limited to the deductible D. Moreover, if we do not take into account the effect of prevention, we found that in the optimum insurance contract $\alpha_s = 0$.

5.1 General preventive behaviour

We denote the costs incurred for prevention by x. This preventive behaviour lowers the probability that the agent ends up in the expensive bad health state t, i.e. $\frac{dp_t}{dx} < 0$ and $\frac{d^2p_t}{dx^2} > 0$. In this subsection we assume (i) that the insured assesses correctly the impact of x on p_t, and (ii) that x can be subsidized as part of the insurance contract. Call the subsidy rate β.

[12] A different approach to prevention has been worked out in Eeckhoudt et al. (2008). They compare (i) a strategy in which patients apply preventive measures before knowing if they have the disease and (ii) a "wait and treat" strategy, in which patients are treated only if they contract the disease.

The optimization problem then becomes

$$\max_{\alpha_s, D, \beta} \Lambda = (1 - p_t(x)) \left[f_s(M_s) + g(W - (1-\beta)x - \pi - (1-\alpha_s)M_s) \right]$$
$$+ p_t(x) \left[f_t(M_t) + g(W - (1-\beta)x - \pi - D) \right] \tag{20}$$

subject to

$$\pi = (1 + \lambda) \left[(1 - p_t(x))\alpha_s M_s + p_t(x)(M_t - D) + \beta x \right].$$

Of course, the first order condition on D just becomes a simplified version of eq. (18)):

$$\frac{\partial \Lambda}{\partial D} = -p_t g_t' + p_t \overline{g}' (1 + \lambda) \leqslant 0, \qquad D \frac{\partial \Lambda}{\partial D} = 0 \tag{21}$$

and the same is true for the first order condition (16) on α_s – provided we neglect the possible effect of α_s on x.

We focus here on the effect of prevention. An agent that is insured with a contract as specified in the previous section (i.e. with $\alpha_s = 0$ and $D > 0$) will decide about x without taking into account the effect on the premium π. This leads to the following condition:

$$\frac{\partial \Lambda}{\partial x}|_{\pi} = \frac{dp_t}{dx} \left[f_t + g_t - (f_s + g_s) \right] - \left[(1 - p_t) g_s' + p_t g_t' \right] (1 - \beta) = 0. \tag{22}$$

Note that, although the agent does not take into account the effect on the premium, he will still invest in prevention because of the utility gain in moving from state t to state s. In fact, condition (22) is well known in the literature on prevention and admits the same interpretation as the "willingness-to-pay for safety" in the literature on the value of life (see, e.g., Dehez and Drèze, 1982). Indeed, it can be rewritten as

$$-\left(\frac{dp_t}{dx} \right)^{-1} = \frac{dx}{dp_t} = \frac{(f_s + g_s) - (f_t + g_t)}{\left[(1 - p_t) g_s' + p_t g_t' \right] (1 - \beta)}. \tag{23}$$

The "willingness-to-pay for a lower probability of ending up in the expensive bad health state" through extra prevention dx is equal to the ratio of (i) the associated benefit in utility terms $(f_s + g_s) - (f_t + g_t)$, and (ii) the net marginal utility cost of x, i.e. the expected marginal cost of one additional unit of x, taking into account the subsidy rate β. It follows from eqs. (22) and (23) that $\frac{dx}{d\beta} > 0$.[13]

[13] The impact of risk aversion and of prudence on optimal prevention has been analysed by Dionne and Eeckhoudt (1985) and Eeckhoudt and Gollier (2005) respectively.

Let us now look at the socially optimal value of x, i.e. taking into account the effect on the premium. This results in the following first-order condition

$$\frac{\partial \Lambda}{\partial x}|_{\pi} - \overline{g}'(1+\lambda)\left[\beta + p_t'(M_t - D - \alpha_s M_s)\right] = 0. \tag{24}$$

The additional term in this expression captures the effect of changes in x on the premium π (evaluated through $\overline{g}'$). Under the assumption that individuals choose x so as to satisfy eq. (22), we can immediately derive an explicit solution for the optimal β :

$$\beta = p_t'(x)\left[\alpha_s M_s - (M_t - D)\right]. \tag{25}$$

The subsidy rate β should obviously be zero if $p_t'(x) = 0$, i.e. if prevention is not effective. It will be positive if $M_t - D > \alpha_s M_s$. Note that this will always be the case if $\alpha_s = 0$ as per a straight deductible scheme. Hence, we can conclude that it is optimal to subsidize x. This result is close to that of Ellis and Manning (2007).[14]

The treatment of prevention in this section does not offer an immediate argument to move away from the straight Arrow-deductible result. Subsidizing preventive behaviour x can rather be seen as a complementary measure. To give an example: subsidizing cancer screening will lower the premium by lowering p_t.

5.2 Treatment as prevention

It is often the case that regular doctor visits lead to an earlier diagnosis and therefore improve the prospects of the patient, i.e. lower the probability p_t. Consulting a GP as soon as some symptoms are discovered may lead to early detection of the threat of t and treatment of the disease at an early stage may help avoiding to have to go to the emergency department of the hospital, or may help avoiding more severe complications and hence larger costs. The preventive aspect of these regular doctor visits cannot be distinguished from the curative aspect, however. They are both included in the expenditures M_s. Let us therefore now turn to a model in which $p_t = p_t(M_s)$ with $\frac{dp_t}{dM_s} < 0$.

The policy problem can then be formulated as follows

$$\max_{\alpha_s, D} \Lambda = (1 - p_t(M_s))\left[f_s(M_s(\alpha_s)) + g(W - \pi - (1-\alpha_s)M_s(\alpha_s))\right]$$

[14] Our expressions (22) and (24) are directly comparable to eqs. (12) and (13) in Ellis an Manning (2007, p. 1138).

$$+p_t(M_s))\left[f_t(M_t)+g(W-\pi-D)\right]$$

subject to

$$\pi=(1+\lambda)\left[(1-p_t(M_s))\alpha_s M_s(\alpha_s)+p_t(M_s)(M_t-D)\right].$$

The first-order condition for D remains as in eq. (21). However, the condition on M_s (or α_s) should now take into account the dependence of $p_t(.)$ on M_s.

We follow the same procedure as in the previous subsection. We first consider the decisions taken by an insured patient, who disregards the impact of $M_t - D$ on the premium π. The – private – first order condition on M_s is then given by

$$\frac{\partial \Lambda}{\partial M_s}|_{\pi}=(1-p_t)\left[f_s'-g_s'(1-\alpha_s)\right]+\frac{dp_t}{dM_s}\left[f_t+g_t-(f_s+g_s)\right]=0. \quad (26)$$

The first term in this condition is well-known from the previous sections - see (9) or (16). The second term already appeared in the previous subsection (see eq. (22). This term will be positive if $f_t + g_t$ is smaller than $f_s + g_s$, which motivates the prevention. Therefore eq. (26) implies that $f_s' < g_s'(1-\alpha_s)$, meaning that expenditures M_s will be larger than in the situation without prevention. Eq. (26) again admits an interpretation in terms of marginal benefits and marginal costs, similar to eq. (23), but with an adjusted definition of the marginal cost: this now becomes $g_s'(1-\alpha_s)$ net of the direct marginal benefit f_s'.

The first order condition (26) may be compared with the condition defining a socially efficient level of M_s, i.e. taking into account the implications of M_s for the premium π. This condition for social optimality is given by (compare with eq. (24)):

$$\frac{\partial \Lambda}{\partial M_s}=\frac{\partial \Lambda}{\partial M_s}|_{\pi}-\overline{g}'\,(1+\lambda)\left[(1-p_t)\alpha_s+\frac{dp_t}{dM_s}\,(M_t-D-\alpha_s M_s)\right]=0. \quad (27)$$

The last term in this expression reflects the additional incentive for preventive care linked to the associated reduction in π.

Just as we did for β, we can solve condition (27) explicitly for α_s, under the assumption that the insured selects M_s such that $\frac{\partial \Lambda}{\partial M_s}|_{\pi}=0$. This yields

$$\alpha_s=\frac{\eta_{p_s M_s}}{1+\eta_{p_s M_s}}\frac{(M_t-D)}{M_s} \quad (28)$$

where we used the obvious property that $\frac{dp_s}{dM_s} = -\frac{dp_t}{dM_s}$ and defined $\eta_{p_s M_s} = \frac{M_s dp_s}{p_s dM_s} > 0$, the elasticity of p_s with respect to M_s. This optimality condition directly implies that α_s should be larger than 0, unless $\eta_{p_s M_s} = 0$, i.e. unless there is no prevention effect. We therefore find a justification for some insurance of the low "standard" medical expenses M_s below the deductible – a departure from our result in section 4. Note that nothing guarantees that α_s, as defined in (28), satisfies $\alpha_s \leqslant 1$. It can be optimal to subsidize M_s if $(M_t - D)$ and $\eta_{p_s M_s}$ are relatively large – and this holds even if $\lambda > 0$. This result is due to the fact that prevention helps containing insurance costs and this remains justified when λ is high: the deterrent to insurance is offset by the lower probability of the expensive therapy. In the more realistic case (with $0 < \alpha_s < 1$) – as would obtain for instance if the elasticity $\eta_{p_s M_s}$ is small enough – condition (28) provides a clear guideline for setting the optimal α_s.

While the analysis in this section was cast in terms of prevention, it is closely related to the insights that are put forward by Goldman and Philipson (2007) in their model with many health care commodities. They argue that the optimal structure of cost-sharing should take account of the complementarity and substitution relationships between these different commodities; for instance subsidising medicines can be justified if the resulting increase in pharmaceutical consumption (including improved medication adherence) lowers hospital expenditures. The elasticity $\eta_{p_s M_s}$ in our analysis plays the same role as the cross-price elasticities in the Goldman-Philipson (2007)-model. In both cases, one finds an argument for a lower level of patient cost-sharing for small health expenses if this decreases the probability of larger expenditures. Our formulation in terms of probabilities seems at least as natural as the one of Goldman and Philipson (2007).

Proposition 3 *If resources are state-independent and preferences are separable with state-independent consumption preferences, the desirability of preventive behaviour (lowering the probability of the expensive health states) justifies some insurance below the deductible (i.e.* $\alpha_s > 0$*) if health care expenditures in a state of standard health have a negative effect on the probability of getting into a state with large medical expenses, but the preventive component of these expenditures cannot be identified as such.*

6 Conclusion

We have shown that the logic of Arrow's theorem of the deductible, i.e. that it is optimal to focus insurance on the states with largest expenditures, remains at work in a model with ex post moral hazard. The optimal insurance contract in a situation with ex post moral hazard takes the form of a system of "implicit deductibles", i.e. it results in the same indemnities as a contract with 100% insurance above a variable deductible positively related to the elasticity of medical expenditures with respect to the insurance rate. This optimal scheme can seldom be implemented as such. We therefore turned to an insurance scheme with an explicit stop-loss and showed that the common practice of first-dollar insurance is not optimal in this standard model: there should be no reimbursement for expenses below the stop-loss amount. Again, the logic of Arrow's theorem remains fully relevant.

Additional arguments are needed to justify the common practice of first-dollar insurance. In this respect we introduced the possibility of preventive benefits and showed that some insurance below the deductible is optimal if health care expenditures in relatively healthy states have a negative effect on the probability of getting into a state with large medical expenses, as will be the case e.g. for regular visits to a general practitioner. Other possible arguments, not developed in this paper, could relate to the existence of externalities not apt to be taken into account by the insured, for instance risks of contagion, or the possibility that patients (and doctors) are poorly informed about the effectiveness of different treatments and should be guided in the direction of optimal treatment choices by a clever design of cost-sharing (Chernew et al., 2007; Pauly and Blavin, 2008). A more thorough analysis of the latter argument calls for the explicit modelling of specific health care services, a topic that lies outside the scope of this paper.

We worked within a model with a discrete number of (mutually exclusive) health states. This makes it possible to derive transparent results easily – more easily in any case than with the optimal control approach explored by Blomqvist (1997). In fact, we have shown that the optimal health insurance policy will in general be nonlinear, and that the most popular modelling strategy, assuming a linear insurance scheme with a fixed coinsurance rate, may yield misleading results. Moreover, despite the restrictions of our model, it still allows us to recover the results on prevention of Ellis and Manning (2007) and the basic intuition of the importance of offset-effects as argued by Goldman and Philipson (2007).

The optimality of some stop-loss arrangement seems a quite robust result – it directly follows from the equally robust intuition that it is better for the consumer to insure expenses when disposable income is low rather than high. This immediately suggests the important issue of the time dimension of insurance, which was left open in this paper. In practice most stop-loss arrangements are based on a fixed time period, usually one year. In theory, however, optimal insurance should take a life-time perspective – possibly implemented through some form of "cumulative averaging." Exploring the implications of this, e.g. for the optimal compensations for the chronically ill, is a topic for further research. Moreover, in this paper we focused on the optimal design of the health insurance contract from the perspective of an individual insured. In public health insurance schemes redistributive considerations may play an important role – the logic of Arrow's theorem then suggests the introduction of an income-dependent stop loss. Yet, a full analysis of such public health insurance scheme would require the explicit specification of a social welfare function and a careful consideration of the relationship between health insurance and other redistributive instruments, mainly the nonlinear income tax. This is also left for further research.

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