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Why not both?

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DISCUSSION PAPER

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**Do we go shopping downtown or in the 'burbs'?
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Abstract

We combine spatial and monopolistic competition to study market interactions between downtown retailers and an outlying shopping mall. Consumers shop at either marketplace or at both, and buy each variety in volume. The market solution stems from the interplay between the market expansion effect generated by consumers seeking more opportunities, and the competition effect. Firms' profits increase (decrease) with the entry of local competitors when the former (latter) dominates. Downtown retailers swiftly vanish when the mall is large. A predatory but efficient mall need not be regulated, whereas the regulator must restrict the size of a mall accommodating downtown retailers.

Keywords: shopping behavior, retailers, shopping mall, spatial competition, monopolistic competition.

JEL Classification: D43, L81, R10

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1 Introduction

In North Decatur, Georgia (United States), local residents have recently come out against the construction of a Walmart supermarket. Common criticisms were that Walmart’s presence eliminates more jobs than it creates; lowers a community’s quality of life by eroding local tax bases; and, through aggressively low pricing, kills small or family-owned businesses. Such an attitude toward large discount stores has a long history in the United States and may be encountered both in urban and rural communities (Courser, 2005). But this is only one facet of the problem. Competition between small downtown retailers and out-of-town shopping malls has important implications for the residential structure of cities and the quality of urban life. In particular, constructing large shopping malls in suburbia has exacerbated the extent of urban sprawl and contributed to the hollowing out of city centers. According to Fogelson (2005), “the decentralization of the department store is one of the main reasons that the central business district, once the mecca for shoppers, does less than 5 percent of the retail trade of metropolitan areas everywhere but in New York, New Orleans, and San Francisco.”

Those issues have been tackled mainly from the urban planning viewpoint, an approach that typically disregards the competition and welfare aspects. In this paper, we aim to develop a new framework that combines spatial competition (Hotelling, 1929) and monopolistic competition (Dixit and Stiglitz, 1977). In our framework, the city has two shopping locations: one is downtown (shopping street) and is composed of many retailers; the other is set up at the city outskirts (shopping mall) and is run by a developer.¹ Each downtown retailer is too small to affect the market outcome and treats its competitors’ strategies as a given. In contrast, the mall developer is a big player who manipulates the market. Consumers are exogenously dispersed between the two marketplaces and bear specific travel costs to acquire the varieties available in each one.

Though consumers’ shopping behavior has several determinants (Teller, 2008), we focus on two first-order forces affecting consumer attitudes: *love for variety*, and *travelling costs*. The former expresses consumers’ desire to access a broad range of choices; the latter reflects the economic importance of shopping costs in consumers’ budget. For example, American households spend almost the same amount of their overall travel time to go to work and to go shopping, i.e., 23.6 percent versus 21.8 percent (Couture et al., 2012). By combining spatial and monopolistic competition, we are able to disentangle the various effects triggered by these two opposing forces. Although price difference is a major determinant of consumer behavior, our initial setting assumes that prices are the same in the two shopping places. This allows us to capture in a simple way a wide range of interactions between these places. Furthermore, we show later that our approach displays enough versatility to account for price differences as well as various types of asymmetries between the two

¹The reasons why retailers are agglomerated at the city center are not addressed in this paper. However, it is well known that, when travel costs are not too high, being clustered in a few places allows firms to benefit from agglomeration economies that overshadow the market crowding effect (Wolinsky, 1983; Stahl, 1987; Fujita and Thisse, 2013).

marketplaces.

One-stop shopping is cheaper and thus more appealing to consumers than two-stop shopping. However, since consumers love variety, shopping behavior depends not only on the travel costs but also on the range of goods available in each shopping place. This has an important consequence: unlike standard models of spatial competition, *market areas overlap*. The extent of overlapping hinges on the number of firms located in each shopping place and the travel costs. Moreover, both the individual consumption of a variety and the product range consumed also change with the consumer's shopping behavior. These are important considerations that have been neglected in the industrial organization literature where every consumer typically buys one unit of a variety.

Our main findings may be summarized as follows. First, the market outcome stems from the interplay between two opposing forces. The first, the market expansion effect, is generated by consumers seeking more opportunities. As a result, the entry of competitors generates a *network effect* that makes a shopping location more attractive.² The second force amounts to a market competition effect: consumption of a variety obtained from each shop decreases as the size of the marketplace increases, and as the number of marketplaces visited by the consumer goes up. When the network effect is sufficiently strong, we show that entry in a marketplace is *profit-increasing*.³ Equally important is the fact that consumers are attracted by marketplaces that are close by because of travel costs. Combining the distance and network effects implies that consumers' shopping behavior is driven by gravitational forces whose intensity increases with the size of a marketplace and decreases as the distance grows between consumers and the marketplace.

As predicted by the North Decatur anti-Walmart activists, the number of downtown retailers shrinks as the outlying shopping mall grows. Less expected, perhaps, is that the city commercial center swiftly vanishes when the size of the out-of-town shopping mall is sufficiently large. This result highlights the main forces at work in our setting and, therefore, deserves more explanation. Establishing new stores at the city outskirts diverts consumers from visiting downtown retailers. This in turn leads to a contraction of the central commercial district through the exit of retailers, which makes this marketplace even less attractive. The overall effect is to further reduce the number of customers, which cuts down the number of retailers once more. When the relative size of the downtown shopping area is small enough, this keeps going on until no firm operates in the city center.

Second, we confirm the well-documented fact that the disappearance of downtown retailers is more likely to arise in small or poor cities. In a large or rich city, the mall developer must build a large capacity to attract all customers. To be precise, the mall size needed to drive downtown retailers out of business may be too big for a predatory strategy to be profitable. In this event, the developer chooses to accommodate the presence of downtown retailers. This situation may also

²This is reminiscent of Chou and Shy (1990) who analyze endogenous network effects without network externalities, but the sources of such effects are very different.

³Note that Schulz and Stahl (1996) and Chen and Riordan (2008) obtain profit-increasing competition in different, but related, models.

arise because local governments adopt policies aimed at maintaining the city center’s attractiveness through the supply of various types of urban amenities. On the contrary, large-scale economies in logistics have allowed outlying shopping malls to become more attractive through lower prices.

Third, the question whether mall developers should be regulated or not is a priori unclear. Yet speculation on this issue has never been in short supply, and this is one of the main questions that local decision makers would like addressed. Our analysis suggests a qualified answer. Regulation is needed when the mall developer is not efficient enough to capture the whole market: the regulator always selects a smaller size for the mall than the size that would emerge under free competition. In contrast, when the mall developer is very efficient, the regulator should not intervene: the market outcome is socially optimal despite the disappearance of downtown retailers. However, we also show that a majority of consumers are opposed to an outlying shopping mall, even when the opening of such a mall is socially desirable. This provides a rationale for the widespread hostile attitude of citizen groups and city councils toward big-box stores coming to town.

Last, we consider four extensions of the baseline model. In the first one, downtown retailers and mall stores are heterogeneous, charge different prices, and offer different amenities to their customers. We show how these asymmetries affect the behavior and size of the marketplaces. In the second extension, we stress that a large and/or rich city is more likely to keep a lively downtown than a small and/or poor city. In the third extension, we show that urban sprawl raises the competitive advantage of out-of-town malls at the expense of downtown retailers. Our last extension studies the impact of the mall location.

Related literature. In the literature on consumers’ search, it is typically assumed that visiting a new shop generates the same given cost as any other one (McMillan and Rothschild 1994). Wolinsky (1983) is a noticeable exception in which search costs vary with the location of shops. Schulz and Stahl (1996) consider a market for differentiated products in which imperfectly informed agents engage in costly search for the best variety-price combination, while Armstrong and Vickers (2010) allow for two-stop shopping. Unlike us, these various authors assume that the volumes and product ranges consumed are independent of firms’ strategies and individual shopping behavior. Moreover, they do not consider competition between marketplaces of different size and different organizational structure. Smith and Hay (2005) study competition between two spatially separate marketplaces but there are three major differences between their approach and ours. First, their consumers are one-stop shoppers. Second, the individual demand for a variety is perfectly inelastic. Third, their marketplaces have the same organizational structure. Our paper, because it combines a large number of small businesses and one big developer, also contributes to the recent literature that focuses on competition between “large” and “small” firms, namely firms that differ in nature, not just in type (Shimomura and Thisse, 2012).

Finally, our model is related to the literature on vertical relations where a typical question is how different types of contracts between upstream and downstream firms affect the market structure and social welfare. We address a similar issue but our approach has features that distinguish it

from the existing literature: the upstream firm (the mall developer) produces a scarce input (slots for shops) and chooses both the number of downstream firms (the mall stores) that will get a slot and its price. Since the slot price may be interpreted as a fixed cost of downstream firms, the mall developer does not affect directly these firms' price and outputs. However, as in Chemla (2003), it does so indirectly through the number of operating downstream firms. It thus seems fair to say that our model ties together different strands of literature.

The paper is organized as follows. The model is described in Section 2, focusing on a shopping mall. However, we show that most of our analysis still holds for the more technically involved case of a supermarket such as Walmart. In Section 3, we study how downtown retailers are affected by the size of the shopping mall, and how mall stores are affected by the size of the downtown shopping district. In Section 4, the market outcome is determined as the perfect Nash equilibrium of a sequential game involving a mall developer, who chooses the number of slots he sells/rents to stores; mall stores, which buy/rent a slot in the shopping mall; and a large number of retailers, which locate in the city center. In Section 5, we investigate the welfare implications of regulating shopping malls and provide a political economy analysis of big retailers' entry. Section 6 studies extensions of the baseline model, namely, cost and quality asymmetries between downtown retailers and mall stores; the impact of city size and population density; and the location of the shopping mall. Section 7 concludes. In particular, we show how our setting can be reinterpreted to describe consumers having heterogeneous tastes for different shopping environments.

2 The Model and Preliminary Results

2.1 Consumers and sellers

Consider a one-dimensional city populated with a unit mass of consumers distributed with a uniform density over $[0, 1]$. The cases of a variable city size and a non-uniform population density are discussed in Section 6. Let $x \in [0, 1]$ denote a consumer's location and her distance to the SST. The city has two marketplaces. The first one is a *shopping street* (SST) located in the central business district at $x = 0$. The second one is a *shopping mall* (SM) located at $x = 1$. The SM is located at the city limit because such a location allows offering the customers various facilities, e.g. parking, which can hardly be provided at the city center where the price of land is much higher. In what follows, the SST and the SM are referred to as *shopping or marketplaces*.

There are two goods, that is, a horizontally differentiated good and an outside good. The differentiated good is supplied by a large number (formally, a continuum) of profit-maximizing firms; each one is free to choose in which marketplace to locate its shop. We use the term *retailers* to refer to the firms located in the SST, while those accommodated by the SM are called *stores*. We refer to the SST and SM as the city's *shopping centers*. Each firm supplies a single variety and each variety is supplied by a single firm. Hence, varieties available in the two shopping places are

differentiated. The outside good is supplied by perfectly competitive firms located at $x = 0$ and $x = 1$; it is chosen as the numeraire.

Consumers. Consumers share the same CES preferences over the differentiated good, which are nested in a quasi-linear utility:⁴

$$U \equiv \frac{1}{\rho} \ln \left(\mathcal{I}(SST) \int_0^n q_i^\rho di + \mathcal{I}(SM) \int_0^N Q_j^\rho dj \right) + A \quad (1)$$

where n (N) is the mass of varieties supplied in the SST (SM), q_i (Q_j) the consumption of the i th variety (j th variety) variety available in the SST (SM), $0 < \rho < 1$, and A the consumption of the numeraire. The unit of the differentiated good is chosen for the coefficient of the logarithm to be equal to $1/\rho$. In (1), $\mathcal{I}(k)$ is the indicator function defined as follows:

$$\mathcal{I}(k) \equiv \begin{cases} 1 & \text{when the consumer visits } k \in \{SST, SM\} \\ 0 & \text{otherwise.} \end{cases}$$

Because preferences are symmetric, the utility of consuming a variety available in the SST is the same as the utility of a variety supplied in the SM. Given (1), it is readily verified that a consumer buys a strictly positive amount of the differentiated good, for otherwise its utility equals $-\infty$. As a consequence, every consumer visits at least one shopping location. Since consumers have a love for variety, they have an incentive to visit the two of them. However, travelling to a shopping place requires is costly. In standard models of spatial competition, travel costs are proportional to the distance covered and the quantities shipped. This approach is not suitable to study shopping places because, once a consumer is in the STT or the SM, it does not have to pay additional costs to visit all the sellers established therein. By implication, *there are scope economies in shopping*. They are captured by assuming that consumers bear travel costs independent of the quantities of goods they purchase, but linear in the distance to the shopping place. In other words, individual outlays on transportation are lump-sum (Stahl, 1982, 1987). Let $\tau > 0$ be the travel cost per unit distance. Hence, a consumer residing at x faces the budget constraint:

$$\mathcal{I}(SST) \left(\int_0^n p_i q_i di + \tau x \right) + \mathcal{I}(SM) \left(\int_0^N P_j Q_j dj + \tau(1 - x) \right) + A \leq Y \quad (2)$$

where p_i (P_j) the price of the i th (j th) variety supplied in the SST (SM). We assume that the income Y is sufficiently high for the consumption of the numeraire to be positive in equilibrium.

If a consumer visits the SST (SM) only, its inverse demand for a variety i (j) is given by

$$p_i = \frac{e_0 q_i^{\rho-1}}{A_0(\mathbf{q})} \quad \left(P_j = \frac{e_1 Q_j^{\rho-1}}{A_1(\mathbf{Q})} \right) \quad (3)$$

⁴The partial equilibrium approach used here is more in the spirit of industrial organization. An urban economics approach would favor a general equilibrium setting in which shopping expenditure equals income. At the expenses of a more technical analysis, our setting can be modified to account for income effects.

whereas its respective inverse demands are given by

$$p_i = \frac{(e_0 + e_1)q_i^{\rho-1}}{A(\mathbf{q}, \mathbf{Q})} \quad P_j = \frac{(e_0 + e_1)Q_j^{\rho-1}}{A(\mathbf{q}, \mathbf{Q})} \quad (4)$$

when it visits both marketplaces.

In these expressions, $A_0(\mathbf{q})$, $A_1(\mathbf{Q})$ and $A(\mathbf{q}, \mathbf{Q})$ are market aggregates given by

$$A_0(\mathbf{q}) \equiv \int_0^n q_k^\rho dk \quad \left(A_1(\mathbf{Q}) \equiv \int_0^N Q_l^\rho dl \right)$$

$$A(\mathbf{q}, \mathbf{Q}) \equiv \int_0^n q_k^\rho dk + \int_0^N Q_l^\rho dl$$

while e_0 (e_1) is the consumer's expenditure in the SST (SM). It follows from (1) and (2) that $e_0 = 1$ and $e_1 = 0$ if the consumer patronizes the SST only; $e_0 = 0$ and $e_1 = 1$ if it visits the SM only; and $e_0 + e_1 = 1$ if it goes to both places. Thus, regardless of its shopping behavior, the consumer spends one unit of the numeraire on the differentiated good, but the values of e_0 and e_1 depend on the consumer's shopping behavior.

Sellers. Up to Section 6, we assume that both retailers and stores share the same marginal production cost c . Because a seller is negligible to the market, its own price choice has no impact on the market aggregates A_0 , A_1 and A in (3) and (4). For the same reason, a seller's price choice also has no impact either on the number of customers visiting the shopping place in which the seller is set up. Therefore, each seller faces an isoelastic demand, which implies that the profit-maximizing prices are the same across all varieties and given by

$$p^* = P^* = \frac{c}{\rho}. \quad (5)$$

Consequently, as in Dixit and Stiglitz (1977), profit-maximizing prices are unaffected by the number of competitors and market size. Admittedly, these properties are very restrictive. Our line of defence involves the following arguments. First, our model can easily be extended to cope with discount retailing. For example, if stores are more efficient than retailers, i.e. $c_1 < c_0$, varieties are cheaper in the SM than in the SST but the price ratio $p^*/P^* > 1$ remains independent of n and N . Everything else being equal, this makes the SM relatively more attractive than the SST. We return to this issue in Section 6.1. Second, as argued in the concluding section, most of our results remain valid for a wider class of preferences that accounts for price competition between sellers. We have chosen to work with the CES because it allows us to obtain simple closed-form solutions as well as to disentangle size and distance effects from price effects.

Third, the out-of-town shopping place is established by a profit-maximizing developer who runs a *shopping mall* - the developer choosing the number of independent shops that will be included in the mall and a renting price. Alternatively, the developer could run a *supermarket*, which means

that the developer selects the number of stores and also the price at which they sell their product.⁵ Thus, a supermarket may be viewed as a vertically integrated firm, whereas a shopping mall can be described by a vertical relationship involving one upstream producer and a myriad of downstream shop-keepers. If the developer operates a supermarket, the symmetry of preferences implies that all varieties are sold at the same profit-maximizing price \bar{P} . Therefore, we can redo the same analysis by replacing P^* with \bar{P} . How the supermarket chooses \bar{P} is immaterial for our purpose. Last, in Section 6.1, we show that consumers may have a preference for one type of marketplace over the other.

Although we assume that retailers act noncooperatively, we recognize that small-business associations aim to exchange their political influence for governmental policies that compensate for their weakness in the marketplace. In the same vein, coalitions of local merchants and community leaders work to improve local public spaces, which takes the concrete form of urban amenities and pedestrian areas. These issues are discussed in the extensions of our baseline model.

2.2 Shopping behavior

Consumer shopping behavior is driven by utility maximization. Since both retailers and stores charge the same price while the degree of product differentiation is the same in the two shopping areas, the number of consumers drawn by a marketplace hinges only upon the mass of varieties it supplies relatively to the mass of varieties provided by the other shopping place. However, as will be shown below, the individual utility level varies with the number of varieties supplied in each shopping place, n and N , as well as with the total number of varieties, $n + N$. To simplify notation, we choose the unit of the numeraire for c/ρ to be equal to 1.

Three shopping patterns may arise.

(i) $\mathcal{I}(SST) = \mathcal{I}(SM) = 1$. The consumer shops at both marketplaces. Using (4) and (5), it is readily verified that such a consumer buys the same quantity of each of the $n + N$ varieties:

$$q = Q = \frac{1}{n + N}. \quad (6)$$

Plugging (6) into (1), we obtain the indirect utility of a consumer visiting the two shopping places:

$$V = \frac{\ln(n + N)}{\sigma - 1} - \tau + (I - 1) \quad (7)$$

where $\sigma \equiv 1/(1 - \rho)$ is the elasticity of substitution across varieties. Note that (7) is independent of the consumer location x because $\tau x + \tau(1 - x) = \tau$.

(ii) $\mathcal{I}(SST) = 1$ and $\mathcal{I}(SM) = 0$. In this event, the consumer prefers a one-stop shopping at

⁵In this case, the pricing rule (5) still holds for the retailers, but not for the stores because the developer internalizes the cannibalization effect generated by the sale of substitutes.

the SST. It follows from (3) and (5) that its demand for a variety is

$$q = \frac{1}{n} \quad (8)$$

while its indirect utility level is equal to

$$V_0(x) = \frac{\ln n}{\sigma - 1} - \tau x + (I - 1) \quad (9)$$

which decreases with x .

(iii) $\mathcal{I}(SST) = 0$ and $\mathcal{I}(SM) = 1$. When the consumer prefers shopping at the SM only, its demand is given by

$$Q = \frac{1}{N}. \quad (10)$$

Such a consumer enjoys an indirect utility level given by

$$V_1(x) = \frac{\ln N}{\sigma - 1} - \tau(1 - x) + (I - 1) \quad (11)$$

which increases with x . Although the developer does not provide directly varieties, it faces the issue of cannibalization that characterizes multi-product firms: the larger the number of stores in the SM, the smaller the quantity sold by each store.

Observe that, regardless of the consumer's shopping behavior, its utility level increases at a decreasing rate with the number of varieties available in the center(s) it chooses to visit. Figure 1 depicts the utility level reached by consumers according to their location. Consumers located near the middle of the segment are worse-off than those located close to the shopping places because they bear higher travel costs. They partially compensate their locational disadvantage by consuming the entire range of varieties.

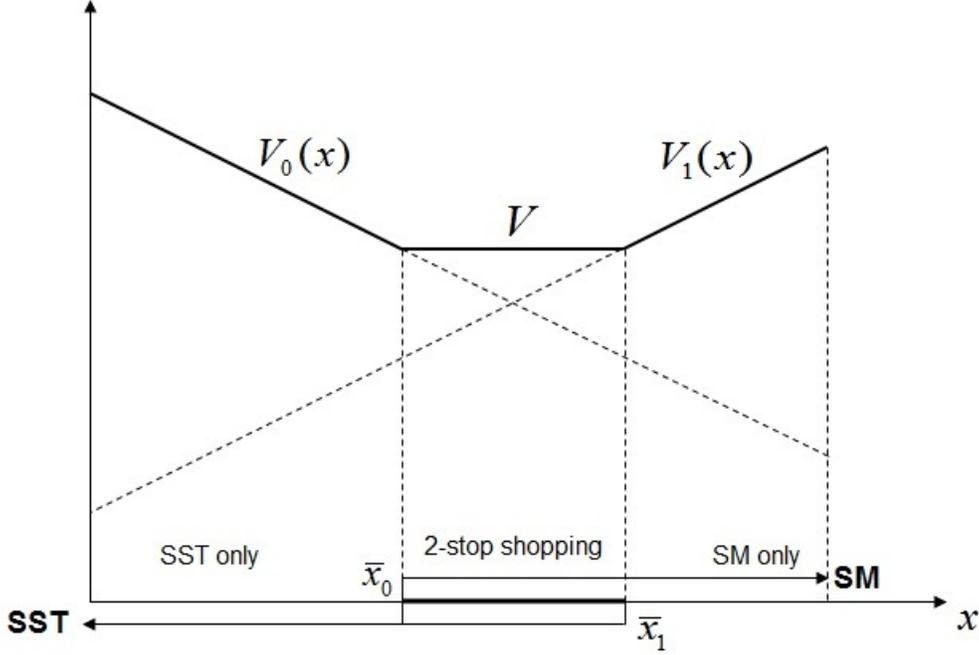


Figure 1. Utilities and shopping behavior

Throughout the paper, we will use the following notation. Let $\nu \equiv N/n$ be the relative size of the SM with respect to the SST and

$$T \equiv \exp[(\sigma - 1)\tau] > 1.$$

A high (low) value of T means that travel costs are high (low) or varieties are good (poor) substitutes. The parameter T thus captures the travel and product differentiation effects, which are critical in our approach to shopping behavior. Both marketplaces become more attractive when either of these parameters decreases. For a given value of τ , more differentiated varieties make both the SST and SM more attractive, while for a given σ , lowering travel costs makes it easier to visit both marketplaces.

The solution to $V_0(x) = V_1(x)$ is given by

$$\bar{x} = \frac{1}{2} \left(1 - \frac{\ln \nu}{\ln T} \right)$$

so that more consumers visit the SST than the SM if the size of the former exceeds that of the latter. This expression shows that, for a given T , the two marketplaces cannot be too dissimilar in size, for otherwise all consumers would patronize the bigger marketplace only.

Our approach to shopping behavior implies the existence of three groups of consumers.

(i) A consumer located at x visits both the SST and the SM if it has a higher utility when it patronizes both places rather than a single one:

$$V \geq \max \{V_0(x), V_1(x)\} \quad (12)$$

which is equivalent to $\bar{x}_0 \leq x \leq \bar{x}_1$ where

$$\bar{x}_0 = \max \left\{ 0, 1 - \frac{\ln(1 + \nu)}{\ln T} \right\} \quad (13)$$

$$\bar{x}_1 = \min \left\{ 1, \frac{\ln(1 + 1/\nu)}{\ln T} \right\} \quad (14)$$

the second term in (13) and (14) being the solution to, respectively, $V = V_0(x)$ and $V = V_1(x)$. For a positive mass of consumers to be two-stop shoppers, it must be that $V > V_0(\bar{x}) = V_1(\bar{x})$.

(ii) A consumer at x prefers shopping at the SST if

$$V_0(x) > \max \{V, V_1(x)\}$$

which is equivalent to $0 \leq x < \min \{\bar{x}_0, \bar{x}\}$. This interval is nonempty if and only if the consumer at $x = 0$ shops at the SST only, which amounts to

$$\nu < T - 1.$$

In other words, the size advantage of the SM is not sufficiently large for the consumers located near the SST to bear the additional travel costs to visit the SM. In this case, the consumer indifferent between patronizing the sole SST or both shopping places, is located at $x = \min \{\bar{x}_0, \bar{x}\}$.

(iii) Similarly, a consumer at x patronizes only the SM if

$$V_1(x) > \max \{V, V_0(x)\}$$

which is equivalent to $\max \{\bar{x}_1, \bar{x}\} < x \leq 1$. This interval is nonempty if and only if the consumer at $x = 1$ shops at the SM only, which amounts to

$$\nu > \frac{1}{T - 1}.$$

For our purpose, the most interesting market configuration involves the existence of the three groups of consumers, which requires $0 < \bar{x}_0 < \bar{x} < \bar{x}_1 < 1$. For two-stop shopping to arise, (12) must hold at \bar{x} , which is not satisfied in general when $T > 4$. We also rule out the case in which all consumers visit the two marketplaces, i.e. $\bar{x}_0 = 0$ and $\bar{x}_1 = 1$. Using (13) and (14), it is readily verified that these two equalities never hold simultaneously when $T > 2$. Therefore, in what follows we focus on the special, but relevant, case in which $2 < T < 4$. We will discuss in the concluding section what our main results become when $T \leq 2$ and $T \geq 4$.

One of the main distinctive features of our model then lies in the existence of a *contention*

area $[\bar{x}_0, \bar{x}_1]$ formed by the two-stop shoppers (see Figure 1 for an illustration). This area widens (shrinks) as T decreases (increases) because both marketplaces become more (less) attractive. Consumers living in the contention area split their expenditure between the SST and the SM according to the wallet shares $1/(1 + \nu)$ and $\nu/(1 + \nu)$. When ν rises, the SST (SM) has fewer (more) customers because both \bar{x}_0 and \bar{x}_1 move to the left (right), and thus fewer (more) consumers shop downtown. In addition, consumers located in the contention area spend less (more) in the SST (SM). Thus, unlike standard models of spatial competition, we have two - instead of one - marginal consumers and a wallet share effect that varies with the relative size of the SST and SM.

For any given n and N , it follows immediately from (5), (6), (8), and (10) that the operating profits of a retailer are given by

$$\pi_R(n, N) = (1 - \rho) \left(\frac{\bar{x}_0}{n} + \frac{\bar{x}_1 - \bar{x}_0}{n + N} \right) \quad (15)$$

whereas those made by a store are such that

$$\pi_S(n, N) = (1 - \rho) \left(\frac{\bar{x}_1}{N} + \frac{\bar{x}_1 - \bar{x}_0}{n + N} \right). \quad (16)$$

Therefore, the profit functions are *symmetric*:

$$\pi_S(n, N) = \pi_R(N, n). \quad (17)$$

3 Competition between Shopping Centers

In this section, we study how competition between the two shopping places is affected by an exogenous change in the size of one marketplace, here the shopping mall. The equilibrium markup being constant, for any given n , the operating profits of a store are proportional to its demand. Since \bar{x}_0 and \bar{x}_1 are homogeneous of degree 0 in (n, N) , this can be achieved by working with $\nu = N/n$ only. Alternatively, (17) implies that the same analysis can be done in terms of $\mu \equiv 1/\nu = n/N$ to study the impact of a change in the STT size.

Consider a fixed number n of retailers and an SM whose relative size is ν . It is then readily verified that total expenditures in the SM increase with ν . However, this does not imply that the quantity sold by each store also rises with ν because more firms compete within the SM.

A store demand is given by

$$D_S(\nu) = \frac{1}{n} \left(\frac{1 - \bar{x}_1}{\nu} + \frac{\bar{x}_1 - \bar{x}_0}{1 + \nu} \right). \quad (18)$$

The number n of retailers being exogenous, (18) implies that a store demand depends only upon the SM's relative size. To see how D_S varies with ν , consider a marginal increase $d\nu$ in the

mass of stores:

$$n dD_S = - \left(\frac{1 - \bar{x}_1}{\nu^2} + \frac{\bar{x}_1 - \bar{x}_0}{(1 + \nu)^2} \right) d\nu - \frac{1}{1 + \nu} d\bar{x}_0 - \left(\frac{1}{\nu} - \frac{1}{1 + \nu} \right) d\bar{x}_1. \quad (19)$$

The first term of this expression stands for the “competition effect” within the SM when the market boundaries \bar{x}_0 and \bar{x}_1 are unchanged. This term, which stems from the deeper fragmentation of demand associated with a larger number of local varieties, is always negative. The second and third terms of (19) represent the “market expansion effect” generated by a bigger SM. When more varieties are offered in the SM, some consumers located close to the SST choose to visit both marketplaces instead of shopping at the SST only (\bar{x}_0 decreases), whereas more consumers established in the vicinity of the SM now choose to visit the sole SM (\bar{x}_1 decreases). To be precise, using (13) and (14), it is easy to see that $d\bar{x}_0$ and $d\bar{x}_1$ are both negative. As a result, the second and third terms of (19) are always positive.

Which effect dominates depends on the relative size of the two shopping areas as well as on the value of T . This interaction distinguishes our model from the existing literature in which individual demands are perfectly inelastic (Schulz and Stahl, 1996; Gehrig, 1998; Smith and Hay, 2005). Here increasing the number of firms in a shopping center affects a seller’s demand through the number of customers *and* their individual consumption.

Three cases may arise according to the value of ν .

1. Consider the case where the SM is small, and thus $0 < \bar{x}_0 < \bar{x}_1 = 1$. The demand faced by a store is given by

$$D_S(\nu) = \frac{1}{n} \frac{1 - \bar{x}_0}{1 + \nu}.$$

Differentiating this expression with respect to ν and using (13), we obtain

$$n \frac{dD_S}{d\nu} = \frac{1 - \ln(1 + \nu)}{(1 + \nu)^2 \ln T} > 0$$

because $\nu < 1/(T - 1) < 1$. In this case, the market expansion effect dominates the competition effect. This is because the boundary \bar{x}_0 is very sensitive to an increase in ν , which makes the market expansion effect stronger, whereas there are only a small number of consumers visiting the SM, and thus the competition effect is weak. Hence, when the SM is small, the market expansion effect is sufficiently strong to shift upwards the demand for the varieties supplied by the incumbents. As a consequence, these stores earn higher profits when they face more competition within the SM. In other words, *there is profit-increasing competition*.

2. We now come to the configuration $0 < \bar{x}_0 < \bar{x}_1 < 1$, which arises when the SM is larger but not too large. Two subcases may arise. In the first one, we have $T \leq 2.39$. We show in Appendix 1 that the market expansion effect still dominates the competition effect for all $1/(T - 1) < \nu < T - 1$. The intuition behind this result is that under a high degree of product differentiation and/or low

travel costs, both the boundaries \bar{x}_0 and \bar{x}_1 are very sensitive to an increase in ν , thus generating a strong market expansion effect.

In the second subcase, T exceeds 2.39. In other words, varieties are less differentiated and/or travel costs are higher. We show in Appendix 1 that there exists a threshold $\bar{\nu} \in [1/(T-1), T-1]$ such that the market expansion effect keeps dominating the competition effect provided that $\nu < \bar{\nu}$. Otherwise, raising the number of stores makes the incumbents worse off. Indeed, the range of varieties provided by the SM being larger, the marginal utility of new varieties is lower. We fall back here on the standard result of profit-decreasing competition.

3. It remains to consider the case in which the SM is very large. Since $\bar{x}_0 = 0 < \bar{x}_1 < 1$, the demand faced by a store located in the SM is now given by

$$D_S(\nu) = \frac{1}{n} \left(\frac{1 - \bar{x}_1}{\nu} + \frac{\bar{x}_1}{1 + \nu} \right). \quad (20)$$

Again, two subcases may arise according to the value of T . First, when $T \leq 2.06$, $\hat{\nu} > T - 1$ exists such that the market expansion effect dominates the competition effect for all values of $\nu \in]T - 1, \hat{\nu}[$. Second, when $T > 2.06$, the competition effects becomes stronger than the market expansion effect for all $\nu > T - 1$ (see Appendix 1).

The next proposition summarizes the above results.

Proposition 1. *Assume that n is fixed. Then, if the shopping mall is not too big relatively to the number of retailers established at the SST, a store's profit increases with the number of competitors. In contrast, the profits earned by a store decrease with N when the shopping mall is big enough.*

In other words, a store's profit function $\pi_S(n, N)$ is unimodal in N . In contrast, raising the number of stores in the SM decreases total expenditure in the SST, thus implying that adding stores to the SM is always detrimental to the retailers. Therefore, a retailer's profit function $\pi_R(n, N)$ decreases with N . It follows from (17) that $\pi_R(n, N)$ is unimodal in n while $\pi_S(n, N)$ decreases with n when N is fixed.

The standard thought experiment of spatial competition theory is to study the impact of better transportation facilities on the market outcome. Because of symmetry, we restrict ourselves to the case in which the SST is smaller than the SM ($n < N$). When $n < N/(T-1)$, the SST attracts more customers purchasing the whole array of varieties, while the SM retains the same number of customers. As a consequence, retailers benefit from lower travel costs. The pattern changes when n exceeds $N/(T-1)$. Owing to the better accessibility from everywhere in the city, both shopping places gain customers. However, because $n < N$, those who patronized the SST but now shop in both places spend more in the SM than in the SST. In contrast, those who used to buy from the SM only keep spending more in the SM. Appendix 2 shows that, in this event, lower travel costs make the retailers worse off. Therefore, unlike standard models of spatial competition with one-stop shopping, the relationship between retailers' profits and travel costs is non-monotonic.

The above analysis has highlighted the market effects at work in the process of competition between spatially separated shopping places. Working with exogenous numbers of retailers and stores can be justified on the ground that the sizes of the SST and SM may be determined through different institutional mechanisms. For example, local policies may restrict retailing by allocating land to offices, housing, and transport facilities, whereas laws or antitrust authorities may impose limits on the size of shopping malls.

4 The Size of Shopping Centers

In this section, we determine the equilibrium size of the shopping places, which have different organizational forms. The turnover of small retailers being high, free entry and exit prevails in the SST. In contrast, the SM is built by a profit-maximizing *developer* who charges a fee to the stores that settle therein. Thus, the developer internalizes the benefits associated with size, whereas retailers maximize their own profits while neglecting the importance to act on an aggregate level.

Independent profit-maximizing stores are free to enter by paying a per slot price to the developer. Because each store is small (precisely, of measure zero), it is natural to assume that the developer has the whole bargaining power over the per slot price, and thus chooses ϕ to maximize its profits given by

$$\Pi \equiv \phi N - B(N^D) \tag{21}$$

where $B(N^D)$ denotes the developer's cost of building an SM of size N^D , while N is the number of stores choosing to set up in the SM and pay the fee ϕ . Though admittedly simple, this type of contract is sufficient for us to show our main results. To be sure, more sophisticated contracts between the developer and stores, which would allow the developer to act as a supermarket, could be considered. However, investigating such issues would take us far from the main objective of this paper.

It is historically well documented that downtown retailers have been active long before the appearance of suburban malls (Cohen, 1996; Fogelson, 2005). Thus, we assume that, *prior to the entry of a developer, the SST hosts $n^e \equiv (1 - \rho)/f$ retailers*, that is, the number of firms prevailing under free entry in the absence of a shopping mall. The ensuing market process is then described by the following three-stage game. The developer, stores and retailers are involved in a strategic environment involving one “large” player and a continuum of “small” players. The timing of the game is as follows. In the first stage, the developer chooses the size N^D of the SM and the per slot price ϕ he charges to the stores that set up in the SM. When $N^D = 0$, the developer does not enter. In the second stage, out of a large number of potential firms (formally, a continuum), some decide to buy/rent a slot in the SM when $N^D > 0$, some others choose to stay in the SST, whereas the remaining firms are out of business. Last, in the third stage, given the SST and SM sizes, retailers and stores compete in price.

Several reasons justify this staging. First, the developer necessarily commits to a certain size when it builds an SM. Second, the actual number of stores in the SM depends on how much they have to pay for a slot. Third, the developer understands that the decisions N^D and ϕ are closely linked. For example, building a small capacity and charging a low fee are inconsistent because the SM cannot accommodate the large number of stores attracted by a low fee. As a result, we find it reasonable to assume that the developer chooses N^D and ϕ simultaneously at the first stage. Last, retailers and stores move together because they display a similar flexibility in their investment and price decisions. Their payoffs are given respectively by (15) and (16). Since Hotelling (1929), the sequence between the location and price stages is standard in spatial competition models.

We seek a subgame perfect Nash equilibrium and solve the game by backward induction. We have seen that the equilibrium prices chosen in the third stage are given by (5). In what follows, we describe the equilibrium outcome of the first and second stages.

4.1 The size of the SST under free entry

Retailers bear an exogenous entry cost f , which may be a fixed production cost or a lump-sum tax to be paid to the city government to set up in the SST. For any given N , a free-entry equilibrium $n^*(N)$ arises when the zero-profit condition

$$\pi_R(n, N) = f \tag{22}$$

holds. Assume that N^D is large enough for stores to enter the SM until their operating profits equal the per slot price ϕ . Under these circumstances, the number $N^*(n)$ of stores in the SM satisfies the zero-profit condition

$$\pi_S(n, N) = \phi. \tag{23}$$

In other words, the number of stores is determined as if firms were to operate under monopolistic competition while facing the fixed cost ϕ set by the developer. The functions $\pi_R(n, N)$ and $\pi_S(n, N)$ being homogeneous of degree -1 , $\pi_R(\mu, 1)$ and $\pi_S(1, \nu)$ are homogeneous of degree 0. Since the functions $\pi_S(1, \cdot)$ and $\pi_R(\cdot, 1)$ are identical, $N^*(n)$ is the mirror image of $n^*(N)$. An equilibrium of a second-stage subgame is thus an intersection point of the two curves $n^*(N)$ and $N^*(n)$.

Because of the presence of network effects, there exist several free-entry equilibria. First, as shown by Figure 2, there are two equilibria such that $n^*(N)$ is strictly positive. However, the zero-profit condition admits solutions that are unstable to small perturbations: the entry (exit) of an arbitrarily small mass of retailers may trigger a profit hike (drop), hence the entry (exit) of new (existing) retailers. In such a context, it is conventional to refine the set of equilibria by requiring stability. A free-entry equilibrium is *stable* if the entry of additional sellers reduces profits below zero:

$$\frac{\partial \pi_R}{\partial n}(n^*, N) < 0.$$

How does the equilibrium number of retailers $n^*(N)$ vary with the size of the shopping mall? Clearly, we have

$$\frac{\partial \pi_R}{\partial N} + \frac{\partial \pi_R}{\partial n} \frac{dn^*}{dN} = 0.$$

The first term of this expression is negative because total expenditure in the SST always decreases with N (see Section 3). Therefore, $dn^*/dN < 0$ (> 0) if and only if $n^*(N)$ is a stable (unstable) equilibrium. In other words, the equilibrium number of retailers decreases (increases) with the size of the shopping mall when the free-entry equilibrium is stable (unstable).⁶

Using (13) and (14), it is readily verified that no consumer patronizes the SST when N is arbitrarily large. Furthermore, we know that $\pi_S(1, \nu)$ is unimodal in ν . Therefore, the set of free-entry equilibria may be described as follows.

Proposition 2. *There is a positive threshold \bar{N} such that (i) if $N < \bar{N}$, there exist two free-entry equilibria, $n^* > n^{**}$, where the former is stable and the latter unstable and (ii) if $N \geq \bar{N}$, the SST involves no retailers.*

Note that, whenever $N > 0$, $n^* = 0$ is also a stable free-entry equilibrium because the SST is too small for the operating profits made by a few sellers to cover their entry cost. Thus, for $N < \bar{N}$, there exist three equilibria, two are stable and one is unstable, a configuration which is typical of models with network effects. The pattern of free-entry equilibria as a function of N is described in Figure 2.

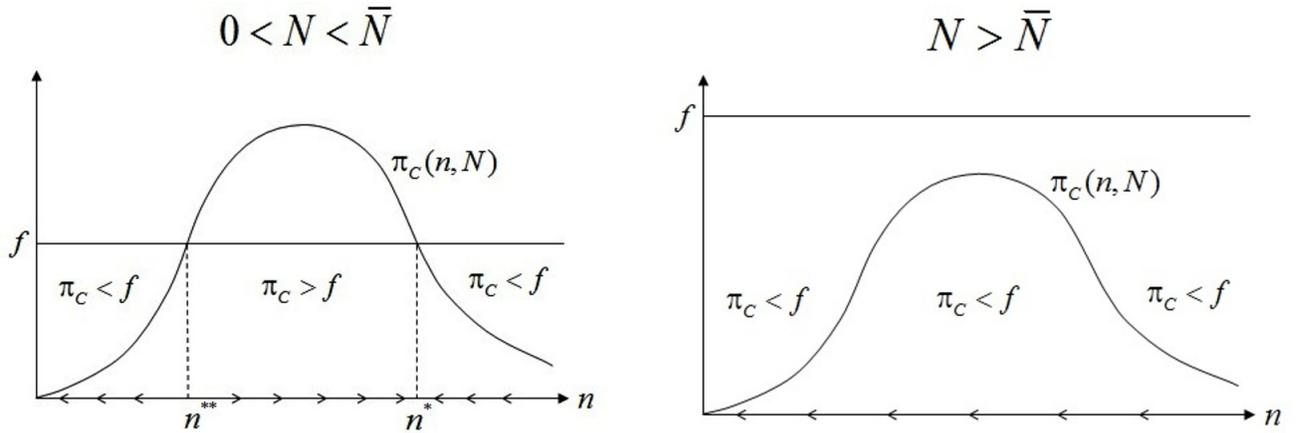


Figure 2. Free entry equilibria

Two more comments are in order. First, when its size is large ($N \geq \bar{N}$), the SM is sufficiently attractive to hollow out the city center. In other words, the SM is so big that the marginal utility of the additional varieties that the retailers could supply is lower than the travel costs consumers,

⁶The existence of the unstable free entry equilibrium is an artefact stemming from the assumption of a continuum of retailers. If a retailer were of a very small but positive size ϵ , the unstable equilibrium would cease to be a Nash equilibrium of the entry game. In this case, an increase in the mass of active firms by ϵ would render profits positive, and thus non-entrant retailers would find it profitable to enter.

even those located in the vicinity of the city center, must bear to go to the SST. By analogy with the concept of limit price, we refer to \bar{N} as the *limit size* of the shopping mall. As a result, when $N < \bar{N}$, the SM never attracts the whole city population and the SST survives as a marketplace. In other words, for all consumers to visit the SM, it must be that all downtown retailers are out of business.

Second, an SM having the limit size triggers the complete and sudden disappearance of the SST. For $N < \bar{N}$, Proposition 2 implies that $n^*(N)$ decreases with N . However, $n^*(N)$ does not decrease smoothly to 0. Indeed, $n^*(N)$ experiences a downward jump to $n^*(\bar{N}) = 0$. The intuition behind this result is as follows. When $N = \bar{N}$, the remaining retailers make negative profits, and thus some of them must exit the market. This in turn makes the SST less attractive to consumers so that fewer of them visit the SST. This unravelling process keeps going on until no retailers are in business. For such a process to be sustainable, the relative size of the SST must be sufficiently small, which explains why it occurs when the SM is sufficiently large.

Since $N^*(n)$ and $n^*(N)$ are mirror images, Proposition 2 can be restated in terms of n and $\mu = 1/\nu$, that is, $\bar{n} > 0$ exists such that there is no SM if the size of the SST exceeds \bar{n} . As a result, *for downtown retailers and a shopping mall to coexist within the same city, one shopping place cannot be much larger than the other.*

4.2 Competition between stores and retailers

We now determine the equilibrium outcome (n^*, N^*) of the second-stage subgame generated by (N^D, ϕ) . Because characterizing the equilibria for all subgames is long and tedious, we find it convenient to prove an intermediate result that rules out a priori some pairs (n, N) .

Claim 1. *At any perfect Nash equilibrium such that $N^D > 0$, there is no idle capacity. Furthermore, the equilibrium per slot price is equal to $\pi_S(n^*, N^*)$.*

(i) Assume that $N^D > N^*$. Then, by slightly reducing N^D , the developer saves on building costs without reducing his revenue. Indeed, N^* solves the equation $\pi_S(n^*, N) = \phi$, which is independent of N^D . (ii) If $\phi < \pi_S(n^*, N^*)$ and $N^* = N^D$, the developer could increase his profit by charging a higher fee because stores earn positive profits. If $\phi > \pi_S(n^*, N^*)$, stores would make negative profits.

We have seen that, for any $N < \bar{N}$, there exist three free-entry equilibria. We follow the literature and disregard the unstable outcome. Thus, we end up with two stable equilibria. Are they equally likely? As said above, downtown retailers were active long before the appearance of suburban malls. Therefore, it seems natural to expect the SST to shrink from n^e until it gets into the basin of attraction of the second stable equilibrium $n^* = N/\nu^* < n^e$ where the melting of the SST would stop. In other words, the stable equilibrium $n^* = N/\nu^*$ is more likely to emerge than $n^* = 0$ as a free-entry equilibrium. Thus, we find this equilibrium implausible and exclude it.

Alternatively, we can appeal to the concept of coalition-proof Nash equilibrium to justify this

choice. Because a retailer making zero-profits prefers to be in than out of business, the only coalition-proof Nash equilibrium is the stable free-entry equilibrium involving $n^* > 0$ sellers. Indeed, the incumbent retailers are not willing to exit the market as long as they make zero profits, whereas the entry of a positive mass of retailers would result in negative profits. The other two equilibria are not coalition-proof because a positive mass of retailers such that the size of the SST is n^* would choose to enter the SST. Nevertheless, we recognize that a big shock can trigger the sudden disappearance of the SST in a small city (n jumps down from n^e to $n^* = 0$).

Based on this argument and Claim 1, we find it legitimate to restrict the analysis to equilibria (n^*, N^*) of the second-stage subgame such that (i) $N^* = N^D$, (ii) $\pi_S(n^*, N^*) = \phi$, (iii) if $N^* < \bar{N}$, then $n^* > 0$. Such equilibria are called *plausible*. In the next claim, we provide a full characterization of plausible equilibria.

Claim 2. *Assume that $N^D > 0$ and $\pi_S(n^*(N^D), N^D) = \phi$. Then, $(n^*(N^D), N^D)$ is a plausible equilibrium. Furthermore, it is unique. If $N^D < \bar{N}$, then both shopping places are active. Otherwise, only the SM is active.*

(i) Retailers have no incentives to entry/exit for $n^*(N^D)$ is a stable free-entry equilibrium. Furthermore, the capacity of the SM prevents further entry in the SM, while stores have no incentives to exit since their profits are non-negative. Thus, $(n^*(N^D), N^D)$ is a plausible equilibrium. Since N^D is given, uniqueness follows from the fact that $n^*(\cdot)$ is a single-valued mapping. (ii) We know from Proposition 2 that $n^*(N^D) > 0$ if and only if $N^D < \bar{N}$.

Among other things, Claim 2 allows describing the emergence of the plausible equilibrium through an auction undertaken by the developer. A large number of stores compete by bidding to get a slot from the range supplied by the developer. In this auction, each store understands that the highest bid it can offer depends on the number of stores that will get a slot as well as on the number of retailers that will locate in the SST. This makes the auction much more complex than in standard models because the individual surplus depends on the decisions made by the other players. In other words, to find its maximum bid each store must guess what the equilibrium values of N and n will be. Such settings are typically plagued with the existence of multiple equilibria. Because the plausible equilibrium is unique, the auction ends up here with a single number of stores, which is equal to $N^*(n^*)$.

4.3 The size of the shopping mall

It remains to determine the equilibrium size and fee chosen by the developer at the first stage of the game. The value of N stems from the collective decisions made by stores in the second stage of the game. It then follows from Claim 2 that the developer always chooses $N^D = N$ and $\phi = \pi_S(n^*(N), N)$. As a result, his profit function (21) may be rewritten as follows:

$$\Pi(n^*(N), N) = N\pi_S(n^*(N), N) - B(N). \quad (24)$$

For $N < \bar{N}$, the function $n^*(N)$ is defined implicitly by the free-entry and stability conditions in the STT:

$$\pi_R(n, N) = f \quad \frac{\partial \pi_R}{\partial n} \leq 0$$

whereas we have $n^*(N) = 0$ for $N \geq \bar{N}$. Since $n^*(N)$ exhibits a downward jump at $N = \bar{N}$ where $n^* = 0$, the developer's profit function has an upward jump at $N = \bar{N}$. Indeed, because stores' profits decrease with the number of SST retailers, we have $\pi_S(n^*(N), \bar{N}) < \pi_S(0, \bar{N})$, which would allow the developer to increase the per slot price ϕ by an increment $\Delta\phi > 0$ when $N = \bar{N}$. Furthermore, the developer never chooses a size exceeding \bar{N} because his profits are equal to

$$\Pi = 1 - \rho - B(N)$$

which always decreases with N .

Because the function $\Pi(n^*(N), N)$ is bounded above on $]0, \bar{N}[$, the developer's maximum profit is given by

$$\Pi^* = \max \left\{ 0, \sup_{N \in]0, \bar{N}[} \Pi, (1 - \rho) - B(\bar{N}) \right\}.$$

If $\Pi^* = 0$, the developer does not launch an SM and consumers will shop at the SST only. If $\Pi^* = \sup \Pi > 0$, the developer opens a shopping mall and accommodates retailers located in the central business district. Finally, if $\Pi^* = (1 - \rho) - B(\bar{N})$, the developer chooses to launch an SM having the limit size \bar{N} , and thus triggers the exit of all retailers. Hence, a subgame perfect Nash equilibrium of our three-stage game always exists. When the equilibrium size N^* of the SM is smaller than its limit size, we say that the developer *accommodates* the presence of retailers; otherwise, the market involves a *predatory* developer.

In what follows, we assume that the building cost is given by

$$B(N) \equiv F + \begin{cases} mN & \text{if } 0 < N < N^E \\ M(N - N^E) + mN^E & \text{if } N^E \leq N. \end{cases} \quad (25)$$

where F , m , M , and N^E are positive parameters such that $m < M$ and $F < (M - m)N^E$. The latter inequality implies that (25) displays increasing (decreasing) returns for N smaller (larger) than N^E . It is readily verified that $F < (M - m)N^E$ implies that the *efficient size* of the SM, which minimizes the average building cost, is equal to N^E . When $F > (M - m)N^E$, the efficient size of the SM is arbitrarily large.

The developer's revenue is equal to $N\pi_S(n^*(N), N) = \pi_S(\mu^*(N), 1)$. Since $\pi_S(\mu, 1)$ decreases with μ and $\mu^*(N)$ decreases with N , the revenue function always increases over $[0, \bar{N}]$. As a result, the developer always chooses its limit size when $N^E > \bar{N}$ and secures the whole market. If $N^E \leq \bar{N}$

and M is sufficiently large, the developer chooses his efficient size and accommodates the presence of retailers. By changing the value of M , the profit-maximizing size of the SM can increase from 0 to the limit size \bar{N} , whereas the equilibrium size of the SST shrinks from n^e to 0. Note also that the fixed cost F cannot be very large for the developer to enter.

5 Should the Size of the Shopping Mall Be Regulated?

Several countries have passed laws restricting the size or entry of big-boxes because the presence of small businesses would allow consumers to benefit from a wider array of varieties and services. It is, therefore, worth studying how the total number of varieties available in the city, $\mathbf{N} \equiv N + n^*(N)$, varies with the size N of the SM. Since the free-entry equilibrium $n^*(N)$ decreases with N , it is a priori unclear whether \mathbf{N} increases or decreases with N . Yet, it turns out to be possible to characterize the behavior of \mathbf{N} in a precise way. The argument goes as follows.

The function $\pi_R(n; N)$ being homogenous of degree -1 , the zero-profit condition $\pi_R(n; N) = f$ can be rewritten as follows:

$$\pi(\mu) \equiv \pi_R(\mu, 1) = fN \quad (26)$$

where $\mu = n/N$. Since $\pi_R(n, N) = \pi_R(\mu, 1)/N$, Proposition 1 implies that the function $\pi_R(\mu; 1)$ is unimodal in μ . Multiplying both sides of (26) by $1 + \mu$, we obtain:

$$\frac{1 + \mu}{f} \pi(\mu) = N + n^*(N). \quad (27)$$

We show in Appendix 3 that this expression is unimodal in μ with a unique maximizer at $\mu_0 > 0$. Hence, \mathbf{N} increases (decreases) with N if and only if (27) is decreasing (increasing) at $\mu^*(N)$. Since $\mu^*(N)$ decreases with N , \mathbf{N} increases with N if and only if $N \leq N_0 \equiv \pi(\mu_0)/f$.

The following proposition is a summary.

Proposition 3. *The total number of varieties available in the city, first, increases and, then, decreases with the size of the SM.*

Thus, the mass of varieties reaches its maximum when the city accommodates both an SST and an SM. In other words, the entry of an SM need not be detrimental to product diversity: the city involves more varieties as long as the size of the SM is not large enough for \mathbf{N} to be smaller than n^e . This suggests that the presence of an SM could well be beneficial to consumers.

Such an argument is incomplete, however, because it overlooks the fact that consumers must bear specific travel costs to visit the shopping places. Owing to the exit of downtown retailers, the consumers in $[0, \bar{x}_0]$ have access to a narrower array of varieties. The consumers residing in $[\bar{x}_0, y]$, where $y \geq \bar{x}_0$ is the solution to

$$(1 - \rho) \ln n^e - \tau y = (1 - \rho) \ln N - \tau(1 - y) \quad (28)$$

now consume the whole range of varieties but they bear a higher travel cost. Only the consumers located in the interval $[y, 1]$, if any, are better off when an SM of size N is launched. Therefore, when N is sufficiently small, it follows from (28) that y is close or equal to 1, and thus the majority of consumers is worse off when the SM is launched.

Does this explain why citizen groups and/or local governments often object to the entry of a big-box in their area? For the majority of consumers to agree with the launching of an SM, it must be that the consumer located at $x = 1/2$ is better off after entry. Therefore, the following condition must hold:

$$(1 - \rho) \ln \mathbf{N}(N) - \tau \geq (1 - \rho) \ln n^e - \tau/2$$

which amounts to

$$\mathbf{N}(N) \geq \sqrt{T}n^e. \tag{29}$$

Figure 3, which depicts the plot of the maximum product range as a function of T , shows that this inequality never holds. Thus, regardless of the value of N smaller than the limit size, *a majority of consumers vote against the entry of an SM*. That a majority of consumers is against the entry of the SM when \mathbf{N} is maximized may come as a surprise since $\mathbf{N}(N_0)$ exceeds n^e . This is because the additional travel cost the consumer at $x = 1/2$ must bear to consume the varieties supplied by the SM is not compensated by the variety increase. Of course, this result depends on the lower bound $T > 2$. When T takes on sufficiently low values, (29) will be satisfied and the launching of an SM at the city outskirts will get the support of a majority of consumers. In other words, people's attitude toward the entry of a shopping mall influenced by the quality of the transportation system.

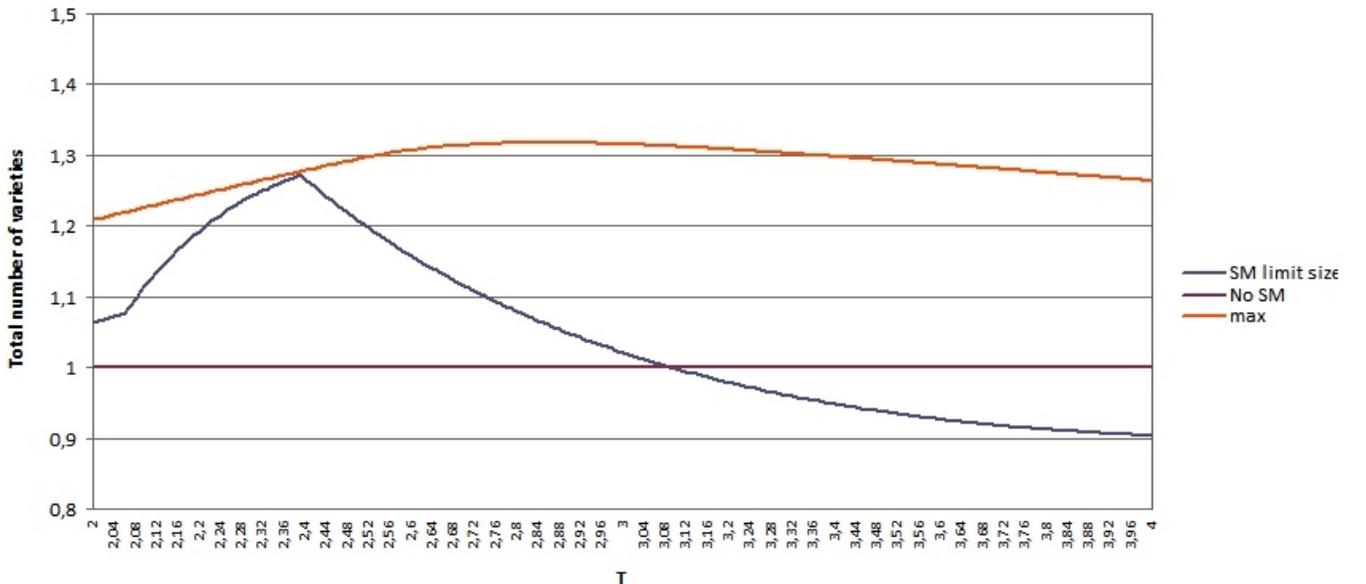


Figure 3. Maximum product range as a function of T

This political economy argument could explain why in several countries or jurisdictions the size of shopping malls is regulated by public authorities. Is this decision justified on efficiency grounds? To answer this question, we consider a welfare-maximizing regulator who chooses the size N of the SM that prevents the central business district to become a ghost town. In contrast, the number of retailers is unregulated and determined by free entry and exit.

Since preferences are quasi-linear, the social welfare function is defined as total consumer surplus plus firms' profits:

$$W = \int_0^1 V(x)dx + n [\pi_R(n, N) - f] + N [\pi_S(n, N) - \phi] + \Pi(N). \quad (30)$$

The regulator seeks the SM size that maximizes (30) subject to $n = n^*(N)$ and $N < \bar{N}$. The former constraint is the free-entry condition at the SST. The latter means that the city center must remain a shopping place whose size is endogenously determined by entry and exit.

Using (7) – (11), (30) can be rewritten as follows (up to a constant):

$$W = \left[(1 - \rho) \ln (N + n^*(N)) + \frac{\tau}{2} (\bar{x}_0^2 + (1 - \bar{x}_1)^2) \right] + [N\pi_S(n^*(N), N) - B(N)] \quad (31)$$

where \bar{x}_0 and \bar{x}_1 are the marginal consumers' locations given by (13) – (14).

It is straightforward to show that the first term in (31) decreases with N for all $N < \bar{N}$, whereas the second is given by (24). Since the equilibrium size N^* maximizes the developer's profits under $n^*(N)$, we get the following result.

Proposition 4. *Let N^* and N^o be the size of the SM at the market equilibrium and social optimum, respectively. Then, we have $N^o \leq N^*$. Furthermore, when $0 < N^* < \bar{N}$, a welfare-maximizing regulator always chooses $N^o < N^*$.*

Therefore, the regulator chooses a size for the SM *smaller* than the size that would emerge under unleashed competition. In other words, when the market outcome involves an accommodating developer, regulating the size of the SM is welfare-enhancing. However, *banning the entry of an SM is generally not optimal.*

Should a predatory developer be regulated too? The answer depends on the developer's efficiency. To illustrate, consider the building cost function (25) and set $m = 0.1$ and $N^E = 0.3$ as well as $\sigma = 2$, $\tau = 0.564$ and $f = 0.5$. Numerical calculations show that the developer adopts a predatory behavior if and only if the marginal cost of an additional store M is not too high ($M \leq 4.71$), while it is optimal to regulate the size of the SM when M is not too low ($M \geq 0.357$). Hence, when M is very small, the developer is so efficient that the planner chooses not to regulate the SM, which chooses its limit size ($N^o = \bar{N}$). The disappearance of the SST is compensated by the very large number of stores hosted by the SM. In contrast, when $M = 3$, the SM is much less efficient. This leads the regulator to intervene and to choose the size $N^o = 0.5$, and thus the SST has a size $n^*(N^o) \approx 1.57$. In sum, *a very efficient developer must not be regulated.* Otherwise, the

regulator increases social welfare by choosing a size for the SM smaller than its equilibrium size. However, in both cases, a majority of consumers remains hostile to the SM's entry. This provides a neat illustration of the possible discrepancy between the efficient and voting outcomes.

In France, the Royer Law imposes restrictions on department stores whose creation has to be approved by a local board composed of shop-owners, consumer representatives, and locally elected politicians. Between 1974 and 1998, the local boards approved only about 40 percent of the applications. Bertrand and Kramarz (2002) have showed that the enforcement of this law has had a negative impact on job creation. In other words, the entry of a new SM, which may be desirable in terms of job creation, is opposed by a majority of board members. This seems to be in accordance with the above results.

6 Heterogeneous Marketplaces and the City

In this section, we discuss the following extensions of the baseline model: (i) the presence of cost and quality asymmetries between retailers and stores; (ii) a change in market/city size; (iii) a non-uniform distribution of consumers across the city; (iv) the choice of the SM location by the developer.

6.1 Heterogeneous shopping places

We have argued in Section 2 that various types of asymmetries between the two shopping places may be taken into account in our setting. To illustrate, we consider the following special, but relevant, case: retailers are less efficient than stores but supply better quality varieties. In other words, the retailers share the marginal cost c , whereas stores have the marginal cost $C = \lambda c$, where $\lambda < 1$. The parameter λ captures the *efficiency heterogeneity* between stores and retailers, which may stem from additional facilities supplied by the developer to his tenants. In the same vein, being designed, developed and managed as a single unit, a supermarket benefits from scope economies and is often able to buy inputs in bulk at prices lower than retailers.

Consumers' preferences are rewritten as follows:

$$U \equiv \frac{1}{\rho} \ln \left(\mathcal{I}(SST) \alpha \int_0^n q_i^\rho di + \mathcal{I}(SM) \int_0^N Q_j^\rho dj \right) + A$$

where the parameter α captures the *quality heterogeneity* between the two types of varieties. In what follows, we focus on the case where $\alpha > 1$.

Under these circumstances, the prices charged by the retailers and stores are, respectively, given by

$$p^* = \frac{c}{\rho} \quad P^* = \frac{\lambda c}{\rho}$$

and thus the price ratio is equal to $\lambda < 1$. Retailers' and stores' profit functions are now given by

$$\pi_R(n, N) = (1 - \rho) \left(\frac{\bar{x}_0}{n} + \frac{\bar{x}_1 - \bar{x}_0}{n + kN} \right) \quad (32)$$

$$\pi_S(n, N) = k(1 - \rho) \left(\frac{1 - \bar{x}_1}{kN} + \frac{\bar{x}_1 - \bar{x}_0}{n + kN} \right) \quad (33)$$

where $k \equiv \lambda^{-(\sigma-1)}\alpha^{-\sigma}$. Note that k is larger (smaller) than 1 when only efficiency (quality) heterogeneity matters. When both kinds of heterogeneity are present, k can be either smaller or larger than 1, depending on which of the two types of heterogeneity dominates. When $k > 1$, everything work as if the number of stores were larger and given by kN instead of N . This in turn makes the SM more attractive, and thus expenditure in the SM also gets higher and equal to $k > 1$. When $k < 1$, the argument is reversed. Thus, the two types of heterogeneity between the centers are formally equivalent.

It is easily verified that the marginal consumers are now given by

$$\bar{x}_0 = \max \left\{ 0, 1 - \frac{\ln(1 + kN/n)}{\ln T} \right\} \quad (34)$$

$$\bar{x}_1 = \min \left\{ 1, \frac{\ln(1 + n/kN)}{\ln T} \right\} \quad (35)$$

where N is again replaced with kN .

The expressions (32)-(35) show that the profit functions $\pi_R(n, N)$ and $\pi_S(n, N)$ are the same as those obtained when $k = 1$ up to replacing N with kN and multiplying π_S by k . Therefore, the results that hold under $k > 1$ are qualitatively the same as those obtained in the foregoing two sections. The main difference is that “effective” number of stores, kN , exceeds the “actual” number, N . In particular, selling at a lower price translates into a smaller limit size for the SM, which is less costly for the developer to implement.

Other heterogeneities such as a higher degree of product differentiation in the SST than in the SM, which supplies more standardized goods, and the presence of urban amenities available at the city center, can similarly be taken into account as long as the price ratio remains constant.

6.2 City size

Owing to the presence of network effects, it is worth studying how an exogenous shock on the city's population size L affects the market outcome. Consumers' preferences are still given by (1), while retailers' and stores' operating profits are given by $L\pi_l(n, N)$ with $l \in \{R, S\}$, the functions π_l being given by (15) – (16).

We may tackle this problem from two different angles. In the first one, we study the impact of a growing city on the size $n^*(N; L)$ of the SST when N is given and determined through different

institutional mechanisms. Therefore, the analysis of Section 4.1 is applicable.

Free entry in the SST and homogeneity of π_R implies

$$\pi_R\left(\frac{n}{L}, \frac{N}{L}\right) = f \quad (36)$$

which yields

$$n^*(N; L) = L n^*\left(\frac{N}{L}; 1\right). \quad (37)$$

Furthermore, $n^*(N; 1)$ being decreasing in N , raising L amounts to decreasing N . It then follows from (37) that, when the SM has a given size N , *a population hike triggers the entry of a more than proportionate number of downtown retailers*. The intuition for this a priori unexpected result is as follows. When there is no SM, the size of the SST is proportional to the population size: $n^e(L) = L(1 - \rho)/f$. Since N/L decreases with L , things work as if N were decreasing in Section 4.1. Thus, for any given n , profits of downtown retailers are shifted upward, which invites additional entry.

In the second one, we study the impact of a growing city on the developers' limit size. It follows from (36) that

$$\pi(\mu) = \frac{Nf}{L}. \quad (38)$$

Since $\pi(\mu)$ is independent of L , the limit size $\bar{N}(L)$ of the SM is given by

$$\bar{N}(L) = \frac{L}{f}\pi(\bar{\mu})$$

which means that the limit size of the SM is proportional to the city's population size: $\bar{N}(L) = L\bar{N}(1)$. In other words, *in a bigger city it is harder for the developer to secure the whole market*.

These results together support the idea that a larger city is more likely to retain downtown retailers than a smaller (or poorer) city. To some extent, this explains why large and rich cities, such as New York, San Francisco, Paris or Milan, have maintained a vibrant downtown, Los Angeles being a prominent counter-example. The hollowing-out of urban centers characterizes mainly small and medium size cities, especially those which do not have historical amenities generated by monuments, buildings, parks, and other urban infrastructure from past eras that are aesthetically pleasing to people.

6.3 Population density

In monocentric cities, population is concentrated around the central business district (Fujita and Thisse, 2013). Let $g(x)$ and $G(x)$ be, respectively, the population density and cumulative distribution functions. Observe, first, that the marginal consumers \bar{x}_0 and \bar{x}_1 given by (13) and (14)

are independent of G . Hence, the fraction of people who choose one-stop shopping at the SST (SM) equals $G(\bar{x}_0) (1 - G(\bar{x}_1))$, whereas $G(\bar{x}_1) - G(\bar{x}_0)$ is the share of consumers who shop in both marketplaces. Further, the equilibrium price of each variety and the quantities bought by consumers are also independent of G . Hence, retailers' operating profits are given by

$$\pi_R(n, N) = (1 - \rho) \left[\frac{G(\bar{x}_0)}{n} + \frac{G(\bar{x}_1) - G(\bar{x}_0)}{n + N} \right]. \quad (39)$$

The analysis of Sections 3 and 4 still holds for a large class of distributions $G(x)$. Basically, for most of our analysis, what we need are the following two conditions: $\pi_R(0, N) = \pi_R(\infty, N) = 0$ for $N > 0$. To show how this works, consider a *negative exponential population density*, which is known to provide a good approximation of city population densities (Anas et al., 1998):

$$g(x) \equiv \frac{\alpha e^{-\alpha x}}{1 - e^{-\alpha}} \quad G(x) = \frac{1 - e^{-\alpha x}}{1 - e^{-\alpha}} \quad (40)$$

where $\alpha > 0$ measures the population skewness towards the SST. When $\alpha = 0$, the density is uniform.

Simulations show that (i) for each T there exists a finite positive threshold value $\bar{\alpha}(T)$, such that π_R is unimodal in n if and only if $\alpha \leq \bar{\alpha}(T)$, (ii) π_S is unimodal in N ; and (iii) when $\alpha \leq \bar{\alpha}(T)$, the developer's revenue increases in N . This has the following implications. First, when the population is not too much concentrated toward the SST, \bar{N} exists such that for each $N \in]0, \bar{N}[$ there is a unique plausible free-entry equilibrium; otherwise, several plausible equilibria may exist. Second, if α is not too large, neither the market expansion effect nor the market crowding effect always dominates the other. Third, under the same condition on α , the market outcome is described by a perfect Nash equilibrium whose structure is similar to that studied in Section 4. Thus, *when the skewness of the population distribution is not too high, the key results obtained in Sections 3 and 4 remain qualitatively the same.*

6.4 The location of the shopping mall

It remains to discuss what our analysis becomes when the SM is built at $x_{SM} \in [0, 1[$. For the reasons mentioned in Section 5.1, the SST is located at $x = 0$.

To see how consumers' shopping behavior is affected by a change in the SM location, we go back to the indirect utilities given in Section 2.2. It is straightforward to show that (7), (9) and (11) become

$$V_0(x, x_{SM}) = (1 - \rho) \ln n - \tau x$$

$$V_1(x, x_{SM}) = (1 - \rho) \ln N - \tau |x - x_{SM}|$$

$$V(x, x_{SM}) = (1 - \rho) \ln(n + N) - \tau \max\{x, x_{SM}\}.$$

The key difference with the case where $x_{SM} = 1$ is that the welfare of consumers patronizing both shopping places now depends on their locations (see Figure 4 for an illustration). To be precise, V decreases with the distance x to the SM for those consumers who are located to the right of x_{SM} . Moreover, V strictly increases when the SM gets closer to the SST. In other words, the closer SM to the SST, the larger the number of two-stop shoppers.

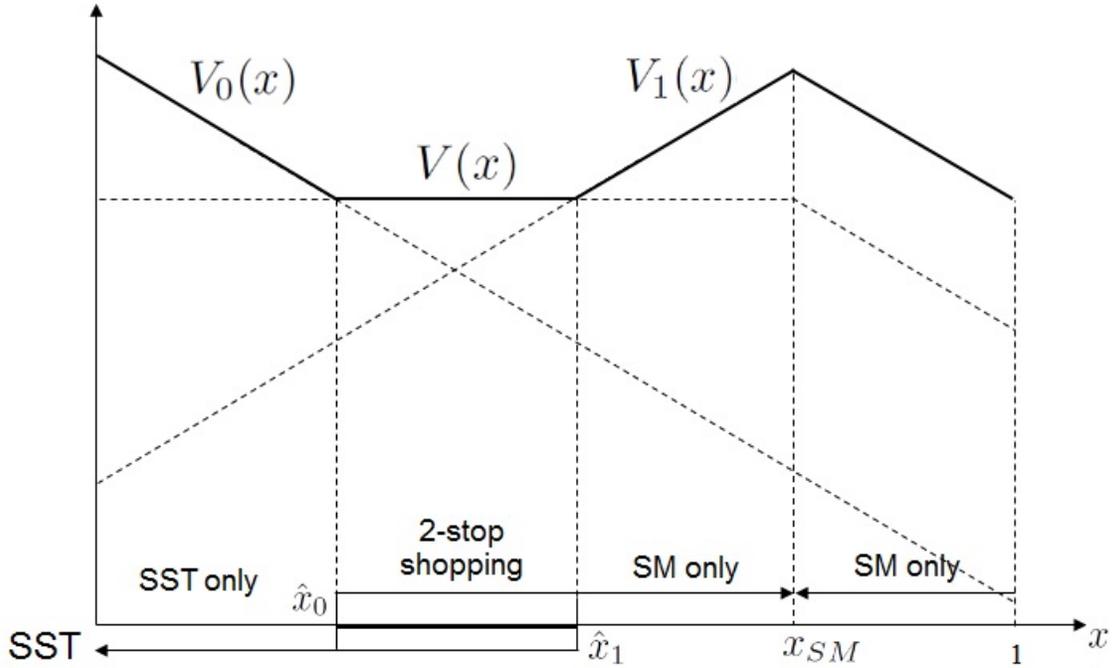


Figure 4. Shopping behavior under variable location of SM

It is readily verified that

$$x_{SM} \geq \frac{\ln(1 + 1/\nu)}{\ln T}$$

implies the existence of two marginal consumers located \hat{x}_0 and \hat{x}_1 , which solve, respectively, the equations $V_0 = V$ and $V_1 = V$:

$$\hat{x}_0 = \max \left\{ 0, x_{SM} - \frac{\ln(1 + \nu)}{\ln T} \right\} \quad \hat{x}_1 = \min \left\{ 1, \frac{\ln(1 + 1/\nu)}{\ln T} \right\}$$

which are identical to (13) and (14) when $x_{SM} = 1$.

As in Section 2.2, a consumer located at x visits the SST only (both shopping places, or the SM only) if and only if $x < \hat{x}_0$ ($\hat{x}_0 \leq x \leq \hat{x}_1$, or $x > \hat{x}_1$). Note that \hat{x}_1 is independent of the SM location, while \hat{x}_0 moves together with x_{SM} . Thus, if

$$x_{SM} \leq \frac{\ln(1 + \nu)}{\ln T} \quad (41)$$

then $\hat{x}_0 = 0$, which implies that no consumer chooses to visit only the SST. Furthermore, if

$$x_{SM} < \frac{\ln(1 + 1/\nu)}{\ln T} \quad (42)$$

then all consumers located to the right of \hat{x}_0 are two-stop shoppers. As a consequence, there is a downward jump in the stores' revenue at $x_{SM} = \hat{x}_1$.

In sum, if

$$x_{SM} \leq \min \left\{ \frac{\ln(1 + \nu)}{\ln T}, \frac{\ln(1 + 1/\nu)}{\ln T} \right\}$$

then all consumers are two-stop shoppers. Thus, choosing a location closer to the SST leaves stores' revenues, hence the developer's revenue, unchanged. Since the developer's building cost function $B(x_{SM}, N^D)$ is likely to decrease with x_{SM} because the land rent at a location closer to the city center is much higher (Fujita and Thisse, 2013), choosing a location in the vicinity of the SST reduces the developer's profits. Thus, the developer is incentivized to choose a location for the SM distant from the SST. How far are the two shopping places depends on the parameters of the market. To a certain extent, our analysis highlights why shopping malls or supermarkets have chosen to set up at the city outskirts once travel costs became sufficiently low through the widespread use of cars (Fogelson, 2005).

7 Concluding Remarks

We have developed a model of competition between two spatially separated shopping areas having different organizational forms and showed how the interaction of love for variety and travel costs affects consumers' shopping behavior. The standard competition effect is supplemented by a market expansion effect, which stems from the higher attractiveness of a shopping area offering a wider range of products. Our results have been obtained for the case in which $2 < T < 4$. What do they become when this condition is not satisfied? Simulations suggest that they remain qualitatively the same for a broader domain of parameter values. However, there is another domain in which multiple interior stable free-entry equilibria arise for both $T < 2$ and $T > 4$. The reason is that the function $\pi(\mu)$ may become bimodal, which implies that $n^*(N)$ exhibits two discontinuities. Using the same argument as in Section 4.2, we may conclude that the expected outcome is given the largest stable free-entry equilibrium, the size of which is smaller than n^e . Thus, dealing with the domains $T < 2$ and $T > 4$ requires longer developments, but does not affect the nature of our results. Moreover, whether consumers are one-stop or two-stop shoppers still depends on the relative size of the two shopping places.

Relaxing the assumption of CES preferences implies that consumers' expenditures on the differentiated good and sellers' markups are variable and determined by the number of varieties available in the two marketplaces. Typically, consumers expenditures in a shopping area increases with the number of varieties supplied therein, while sellers' markups decrease with the number of local competitors. As a consequence, the analysis of the competition and market expansion effects becomes much more involved. Nevertheless, the specific functional forms of π_R and π_S are not needed for the main findings of Sections 3 and 4 to hold. We need only these functions to display a few regularities we now summarize. First, π_R must be unimodal in n to get the pattern of free-entry equilibria described in Proposition 2. Second, π_R must be unimodal in n and π_S in N for the market expansion effect to overcome (be dominated by) the competition effect when the shopping area is small (large). Last, the behavior of the developer described in subsection 4.3 hinges on the fact that the developer's revenue increases in the SM size.

Our paper generalizes the law of retail gravitation proposed by Reilly (1931), which has been extensively studied in spatial interaction theory. According to this law, two cities attract consumers living in an intermediate place in direct proportion of the populations of the two cities and in inverse proportion to the square of the distances to these two cities. We show that consumers close to one city need not visit both of them, while those located in intermediate places split their expenditures between the two cities according to a rule more general than Reilly's. In the same vein, our model is closely related to the gravity equation used in trade theory (Anderson and van Wincoop, 2004), by providing microspatial foundations to this equation.

Finally, in the wake of Hotelling (1929), we can reinterpret our model to describe a population of consumers heterogeneous in their attitude toward the organizational form used to supply a differentiated good - think of bakeries, breweries, or bookstores. In the first one, the good is produced and sold by a large firm; in the second, it is provided by many small firms. Though consumers like variety, they may also have a strong preference for one organizational form over the other. Our results then show how the two organizational forms interact to shape the structure of the industry.

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Appendix

Appendix 1.

(i) Assume that $T \leq 2.39$. Using (19), it is readily verified that $dD_S/d\nu > 0$ for all $\nu \in]1/(T-1), T-1[$ if and only if

$$\theta(\nu) > \ln T$$

where

$$\theta(\nu) \equiv \ln \left(1 + \frac{1}{\nu} \right) + \frac{\nu^2(1 - \ln(1 + \nu)) + 1}{1 + 2\nu}.$$

The function θ decreases with ν , whereas $\theta\left(\frac{1}{T-1}\right) > \ln T$ for all $2 < T < 4$. Therefore, $dD_S/d\nu > 0$ for $0 < \bar{x}_0 < \bar{x}_0 < 1$ if and only if

$$\theta(T-1) \geq \ln T \tag{A.1}$$

or, equivalently,

$$A(T) \equiv 1 + (T-1)^2(1 - \ln T) - (2T-1) \ln(T-1) \geq 0. \tag{A.2}$$

The function A decreases with T on $]2, 4[$. Since $A(2) > 0$ and $A(4) < 0$, the equation $A(T) = 0$ has a unique solution in $]2, 4[$. Solving numerically this equation shows $T = 2.39$ so that that (A.2) holds if and only if $T \leq 2.39$.

(ii) Assume now that $T > 2.39$. Then, there exists a threshold $\bar{\nu} \in]1/(T-1), T-1[$ such that $dD/d\nu > 0$ if and only if $\nu < \bar{\nu}$. Indeed, we have just seen that (A.1) does not hold if and only if

$T > 2.39$. In this case, the equation $\theta(\nu) = \ln T$ has a unique solution $\bar{\nu} \in]1/(T-1), T-1[$ and (A.2) does not hold if and only if ν exceeds $\bar{\nu}$. Q.E.D.

(iii) When $T > 2.06$, the competition effects becomes stronger than the market expansion effect for all $\nu > T-1$. Indeed, differentiating (20) with respect to ν yields:

$$n \frac{dD_S}{d\nu} = -\frac{1-\bar{x}_1}{\nu^2} - \frac{\bar{x}_1}{(1+\nu)^2} - \frac{1}{\nu} \frac{d\bar{x}_1}{d\nu} \frac{1}{1+\nu}.$$

This expression is negative if and only if

$$\lambda(\nu) < \ln T \tag{A.3}$$

where

$$\lambda(\nu) \equiv \frac{(1+2\nu) \ln\left(1+\frac{1}{\nu}\right) + 1}{(1+\nu)^2}.$$

The function λ decreases with ν for all $\nu > 1$ because the denominator is increasing, whereas the numerator is positive and decreasing. As a result, (A.3) holds for all $\nu \geq T-1$ if and only if

$$\lambda(T-1) < \ln T$$

or, equivalently,

$$H(T) \equiv (T-1)^2 \ln T + (2T-1) \ln(T-1) - 1 > 0.$$

The function $H(T)$ is increasing in T , negative at $T=2$ and positive at $T=4$. Therefore, $H(T) = 0$ has a single solution in $]2, 4[$. We find numerically that this solution is given by $T_0 = 2.06$. Hence (A.3) holds for all $\nu > T-1$ if and only if $T > 2.06$.

(iv) Assume that $T \leq 2.06$. Then, there exists $\hat{\nu} > T-1$ such that the market expansion effect dominates the competition effect if and only if $\nu < \hat{\nu}$. Indeed, we have just seen that (A.3) does not hold for all $\nu > T-1$ under $T \leq 2.06$. As $\lambda(\nu)$ is a decreasing function and $\lambda(\infty) = 0$, there exists $\hat{\nu} > T-1$ such that (A.3) holds only for $\nu < \hat{\nu}$. Q.E.D.

Appendix 2.

(i) $n < N/(T-1)$. Then,

$$\bar{x}_0 = 0 \quad \bar{x}_1 = \frac{\ln(1+n/N)}{\ln T}.$$

Hence, using (15), retailers' operating profit boils down to

$$\pi_R = \frac{1-\rho}{\ln T} \frac{\ln(1+n/N)}{n+N}$$

which increases when τ decreases.

(ii) $N/(T-1) \leq n \leq N$. In this case,

$$\bar{x}_0 = 1 - \frac{\ln(1 + N/n)}{\ln T} \quad \bar{x}_1 = \frac{\ln(1 + n/N)}{\ln T}.$$

Hence,

$$\pi_R = (1 - \rho) \left[\frac{1}{n} - \frac{1}{n + N} + \frac{1}{\ln T} \frac{(1 - N/n) \ln(1 + N/n) - \ln(N/n)}{n + N} \right].$$

When $n < N$, we have

$$\left(1 - \frac{N}{n}\right) \ln\left(1 + \frac{N}{n}\right) - \ln \frac{N}{n} < 0$$

and thus π_R decreases when τ decreases. Q.E.D.

Appendix 3. Since $\pi(\mu)$ is increasing over the interval $]0, 1/(T - 1)[$, there is no stable free-entry equilibrium such that $\mu < 1/(T - 1)$. Consider now the interval $]1/(T - 1), T - 1[$ and show that $\mathbf{N}(\mu)$ is increasing. Up to a positive coefficient independent of μ , we have

$$\mathbf{N}(\mu) = \frac{\ln T + (\mu - 1) \ln(1 + \mu) + \ln \mu}{\mu}.$$

Differentiating this expression yields

$$\mathbf{N}'(\mu) = \frac{\Lambda(\mu)}{\mu^2}$$

where

$$\Lambda(\mu) \equiv \frac{\mu^2 + 1}{\mu + 1} - \ln T + \ln(1 + \mu) - \ln \mu.$$

To show that $\Lambda(\mu)$ is positive over $]1/(T - 1), T - 1[$, we differentiate $\Lambda(\mu)$ and get

$$\Lambda'(\mu) \equiv \frac{(\mu - 1)(\mu^2 + 3\mu + 1)}{\mu(\mu + 1)^2}$$

so that $\mu = 1$ is a minimizer of $\Lambda(\mu)$. Hence, $\Lambda(\mu) > 0$ if $\Lambda(1) > 0$. Since $T < 4$, we have

$$\Lambda(1) = 1 + \ln 2 - \ln T > 1 - \ln 2 > 0.$$

Last, we examine the behavior of $\mathbf{N}(\mu)$ for $\mu \geq T - 1$. Up to a positive coefficient independent of μ , we have

$$\mathbf{N}(\mu) = \frac{(1 + \mu) \ln T - \ln(1 + \mu) + \ln \mu}{\mu}.$$

Differentiating this expression, we obtain

$$\mathbf{N}'(\mu) = \frac{K(\mu)}{\mu^2}.$$

where

$$K(\mu) \equiv \frac{1}{1+\mu} - \ln T + \ln(1 + 1/\mu).$$

Clearly, $K(\mu)$ decreases with μ . Furthermore, $K(\mu)$, whence $\mathbf{N}'(\mu)$, is negative under sufficiently large values of μ . As a result, $\mathbf{N}(\mu)$ is unimodal and has a unique maximizer $\mu_0 \geq T - 1$. Q.E.D.

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