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Imperfect resource substitution and  
optimal transition to clean technologies

N. Baris Vardar



**CORE**

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DISCUSSION PAPER

Center for Operations Research  
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Voie du Roman Pays, 34  
B-1348 Louvain-la-Neuve  
Belgium

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**Imperfect resource substitution and  
optimal transition to clean technologies**

N. Baris VARDAR<sup>1</sup>

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**Abstract**

Non-renewable and renewable resources are imperfect substitutes due to technical and geographical constraints. What is the role of imperfect substitution on the optimal transition path to the clean technologies? We address this question by characterizing the optimal growth path and resource use of an economy. We show that the economy initially starts with using the non-renewable and renewable resources simultaneously and gradually increases the share of renewable. The outcome can be either (i) the economy switches to a backstop at a certain date or (ii) the initial regime lasts forever. The results show that the economy converges to a steady state even if the backstop is too costly and a green, zero-carbon economy is the optimal final state in any case. We also present some simulation results to illustrate the shapes of the optimal paths. This analysis allows us to discuss the policy implications and question the existence of the Green Paradox.

**Keywords:** imperfect substitution, optimal transition, non-renewable resource, renewable resource, backstop, simultaneous use, switching, Green Paradox.

**JEL classification:** Q43, Q42, Q30, Q20.

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<sup>1</sup> Paris School of Economics, Université Paris 1 Panthéon-Sorbonne, F-75647 Paris, France; Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium. E-mail: baris.vardar@psemail.eu

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# 1 Introduction

Transition to clean technologies is a gradual and smooth process rather than an instantaneous decision of switch. There are technical and geographical constraints on the substitution possibilities of non-renewable and renewable resources, which make them imperfect substitutes. In this paper, we investigate the role of imperfect substitution on the energy transition.

Traditional approach in the literature is to consider non-renewable and renewable resources as perfect substitutes. The standard result is the increase of non-renewable price with the Hotelling (1931) rule. Consequently, the economy instantaneously switches to the backstop at a certain date when the non-renewable price equals to the price of backstop. Dasgupta and Heal (1974), Heal (1976), Hoel and Kverndokk (1996), Ploeg and Withagen (2011) and Smulders and Withagen (2012) are examples of this approach. These studies focus on the extraction path of non-renewable resources, but they are limited for analyzing the gradual transition to renewables.

Many studies extended this framework to analyze the simultaneous use of resources and the gradual transition. For example, Ploeg and Withagen (2012) considered convex costs for using the backstop. They showed the optimal transition of an economy from the only non-renewable use regime to the simultaneous use regime which then followed by the only backstop use regime. Tsur and Zemel (2003) considered decreasing costs for the backstop as a result of R&D, which led them to analyze the simultaneous use regime. Tahvonen (1997) focused on the stock pollution externalities, Chakravorty et al. (2006) and Chakravorty et al. (2012) included a carbon ceiling to investigate the same issue. Yet, all of these studies kept the assumption of perfect substitution between non-renewable and renewable resources.

What are the implications of relaxing the assumption of perfect substitution? And what is the role of imperfect substitution on the optimal transition path to the clean technologies? In this paper we address these questions.

These questions were partially answered by Acemoglu et al. (2012), Long (2013) and Michielsen (2011). Acemoglu et al. (2012) considered two inputs in the economy: clean and dirty, which are imperfect substitutes. This setting plays a key role in their analysis. They set up an endogenous growth model to investigate directed technical change and they showed that sustainable growth can be achieved if the degree of substitution is sufficiently high. Long (2013) investigated the Green Paradox under imperfect substitution between the clean and dirty fuels. In his work, he showed that higher degree of substitution may lead to the Green Paradox. He also showed the simultaneous use regime and the transition to only clean fuel use regime in the presence of a choke price for the fossil fuel. Similarly, Michielsen (2011) focused on the carbon leakages and considered a non-renewable resource together with the clean and dirty backstops as imperfect substitutes. He conducted static analysis whereas he also extended the model in two periods and showed some cases which contradict the Green Paradox.

In these studies, however, imperfect substitution is a feature of the model rather than the purpose of the study. The question on the role of imperfect substitution still remains open.

We depart from these studies by focusing only on the transition to clean resources and the role of imperfect substitution on this process. We present an optimal growth model which is consistent with the previous literature. We consider non-renewable and renewable resources which are imperfect substitutes. In addition, we consider a backstop which can be used in any kind of economic activity without an imperfection, hence a perfect substitute to the other two types. This setting allows us to investigate the role of imperfect substitution as well as to relate to the wide literature on switching to a backstop - a clean, efficient and everlasting resource.

Despite our framework is close to Ploeg and Withagen (2011), in line with Heal (1976) we assume that the non-renewable resource is limited economically instead of physically. This means that the extraction cost increases as the economy extracts more resource. In this context, we set the problem where the social planner maximizes the discounted value of intertemporal welfare which is determined by consumption and damages of non-renewable resource use to the environment. We obtain the optimal paths and establish the conditions for the resource use choice of the economy.

The results showed that the economy always starts with using the non-renewable and renewable resources simultaneously. It gradually increases the share of the renewable in production over time. Two scenarios appear: *(i)* the economy switches to the backstop at a certain date, *(ii)* the initial regime lasts forever.

In the initial regime, the optimal allocation of non-renewable and renewable resources depends on their prices and the degree of substitution. This allows us to define the *energy price index* which measures the marginal productivity of energy and feasibility of the backstop. Accordingly, this index determines the scenario that will be realized. The economy is more likely to switch to the backstop if it is cheaper and the degree of substitution between non-renewable and renewable resources is lower.

In the first scenario, the economy switches to the backstop when its price is equal to the energy price index. There is an *extraction ceiling* independent of the initial state which depends on the backstop price, the renewable price and the degree of substitution. It decreases when backstop is cheaper. Similarly, lower degree of substitution leads to a lower extraction ceiling. These results contradict the Green Paradox. In the contrary case, higher degree of substitution results as more cumulative extraction. This implies a Green Paradox and is consistent with the results of Long (2013). After the switch, the economy converges to a steady state determined by the price of backstop. The welfare in this steady state is higher if the price of backstop is lower.

In the second scenario, the price of backstop is so high that the condition for switch never holds. The economy gradually reduces the share of non-renewable resource over time. The role of imperfect substitution depends on the time period. Lower degree of substitution results as less non-renewable resource use in the short and medium run, but more in the long run. The economy converges to a steady state where it uses only renewable resource. This steady state is determined by the renewable

price and the degree of substitution. The welfare in this steady state is higher if the renewable is cheaper and the degree of substitution is higher.

These results show that even though the backstop is clean and unlimited, it might not always be the best option the economy has due to the presence of renewable resources we currently use today. The economy may well switch to the renewables and approach a clean production state. Sustainability can be achieved even if the backstop is too costly, and a green, zero-carbon economy is optimal final state in any scenario.

The remainder of the paper is organized as follows. Section 2 introduces the model and characterizes the general resolution. Section 3 establishes and investigates the two scenarios. Section 4 presents the results of simulations and illustrates the optimal transition path. Section 5 concludes.

## 2 The Model

We consider an economy where the social planner maximizes the discounted value of intertemporal welfare which is measured by consumption of the generic good and environmental damages. The time is continuous over infinite horizon.

$U(C)$  denotes the instantaneous utility from consumption ( $C$ ) and it is increasing and strictly concave in  $C$  ( $U_C(C) > 0$  and  $U_{CC}(C) < 0$ ).<sup>1</sup> We use the constant relative risk aversion (CRRA) utility function as follows:

$$U(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

where  $\frac{1}{\sigma}$  denotes the relative risk aversion coefficient and  $\sigma$  is the elasticity of intertemporal substitution.

The economy uses capital ( $K$ ) and energy ( $E$ ) to produce the generic good. Energy consists of non-renewable (dirty) ( $E_d$ ) and renewable (clean) ( $E_c$ ) resources and backstop ( $b$ ). Non-renewable and renewable resources are imperfect substitutes, and backstop is a perfect substitute to the other two.

The imperfect substitution of non-renewable and renewable resources is due to technical and geographical constraints. Renewable resources that we are using today, such as solar and wind power, have high installation and opportunity costs. For example, to harvest high amount of solar energy, the economy should allocate significant amount of land which could be used in various economic activities. There are also technical rigidities, for some industries it might take years to change and adopt the way the equipment works. The automotive industry could be such an example. Likewise, some resources require certain land and climate properties, the economy cannot allocate them in other geographies

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<sup>1</sup>For the sake of notational ease, in the rest of the text we will use the subscript for a function to denote its derivative respect to a variable or argument of the function. For example,  $f_1(\cdot)$  denotes the derivative of function  $f$  respect to its first argument and  $f_x(\cdot)$  denotes the derivative of function  $f$  respect to variable  $x$ . We suppress time indices of the endogenous variables throughout the text for the same reason.

even though their cost is much lower than the others. These reasons lead us to consider imperfect substitution between the two type of resources.

The function  $H(\cdot)$  captures the imperfect substitution and it is in CES form,  $H(E_d, E_c) = (\gamma_d E_d^{1-\frac{1}{\epsilon}} + \gamma_c E_c^{1-\frac{1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$  where  $\gamma_d > 0$  and  $\gamma_c > 0$  represent the weights of non-renewable and renewable in production respectively with  $\gamma_d + \gamma_c = 1$ .  $\epsilon$  denotes the elasticity of substitution. Since we assume imperfect substitution, it requires to assume  $\epsilon > 1$ .

The production function  $F(\cdot)$  is Cobb-Douglas,  $F(K, E) = K^\alpha E^\beta$ .  $\alpha > 0$  and  $\beta > 0$  are the output elasticities of capital and energy in production respectively, and  $\alpha + \beta \leq 1$ . Embedding the different types of resources for energy leads to the following form:

$$F(K, H(E_d, E_c) + b) = K^\alpha ((\gamma_d E_d^{1-\frac{1}{\epsilon}} + \gamma_c E_c^{1-\frac{1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} + b)^\beta$$

The marginal cost of renewable resource is  $\pi_c > 0$  and the marginal cost of backstop is  $p_b > 0$ . They are both inexhaustible and have constant marginal costs. We assume that the backstop has higher marginal cost compared to the renewable ( $\pi_c < p_b$ ). Hence, the difference between backstop and renewable comes from two facts: the backstop costs more than the renewable and the renewable is imperfect substitute to the non-renewable resource.

The extraction of non-renewable resource is costly. This cost depends on the level of cumulative extraction ( $Z$ ) where  $Z_t = Z_0 + \int_{s=0}^t E_{ds} ds$ . This approach, which was adopted by Heal (1976), Hoel and Kverndokk (1996), d'Autume (2012) and many others before, emphasizes that non-renewable resources are not limited by the nature but there are economic limitations in the long run. As the economy extracts more resource, the unit extraction cost will increase over time. This cost consists of both direct and indirect effects of cumulative extraction such as searching costs for new resources and technical innovation expenditures to harvest deeper deposits.  $G(Z)$  denotes the extraction cost and it is increasing and strictly convex in  $Z$  ( $G_Z(Z) > 0$  and  $G_{ZZ}(Z) > 0$ ). We consider the following specification:

$$G(Z) = \frac{\phi_g}{2} Z^2, \phi_g > 0$$

The extraction of non-renewable resource damages the environment, which in turn affects welfare. The welfare loss occurs due to direct and indirect effects of resource extraction such as accumulation of greenhouse gases in the atmosphere and reduction in production possibilities. These damages also depend on the cumulative extraction ( $Z$ ) and denoted as  $D(Z)$  which is increasing and strictly convex in  $Z$  ( $D_Z(Z) > 0$  and  $D_{ZZ}(Z) > 0$ ). The damage function is given by:

$$D(Z) = \frac{\phi_d}{2} Z^2, \phi_d > 0$$

## 2.1 Social Planner Problem and General Resolution

The social planner solves the following problem:

$$\max_{\{C_t, E_{dt}, E_{ct}, b_t\}} \int_{t=0}^{\infty} e^{-\rho t} (U(C_t) - D(Z_t)) dt$$

$$\dot{K}_t = F(K_t, H(E_{dt}, E_{ct}) + b_t) - G(Z_t)E_{dt} - \pi_c E_{ct} - p_b b_t - C_t \quad (1)$$

$$\dot{Z}_t = E_{dt} \quad (2)$$

$$C_t, E_{dt}, E_{ct}, b_t \geq 0 \text{ and } K_t, Z_t \geq 0 \quad \forall t$$

with  $K_0 > 0$  and  $Z_0 > 0$  are given.

The capital stock and cumulative extraction  $(K, Z)$  pair defines the state variables. The current value Hamiltonian function associated to this problem is:

$$\mathcal{H} = U(C) - D(Z) + \lambda (F(K, H(E_d, E_c) + b) - G(Z)E_d - \pi_c E_c - p_b b - C) - \mu E_d$$

where  $\lambda$  denotes the co-state variable associated to capital and is interpreted as the shadow value of capital. Similarly,  $\mu$  denotes the co-state variable associated to cumulative extraction and is interpreted as the shadow value of non-renewable resource.

The necessary conditions for an optimum are:

$$U_C(C) = \lambda \quad (3)$$

$$E_d \geq 0, \quad \lambda (F_2(K, H(E_d, E_c) + b) H_{E_d}(E_d, E_c) - G(Z)) - \mu \leq 0 \quad (4)$$

$$E_c \geq 0, \quad F_2(K, H(E_d, E_c) + b) H_{E_c}(E_d, E_c) - \pi_c \leq 0 \quad (5)$$

$$b \geq 0, \quad F_2(K, H(E_d, E_c) + b) \leq p_b \quad (6)$$

$$\dot{\lambda} = (\rho - F_1(K, H(E_d, E_c) + b)) \lambda \quad (7)$$

$$\dot{\mu} = \rho \mu - \lambda G_Z(Z) E_d - D_Z(Z) \quad (8)$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \lambda_t = 0 \quad (9)$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} \mu_t = 0 \quad (10)$$

together with the equations (1) and (2). In order to proceed on resolution, we define the price of non-renewable resource as follows:

$$\pi_d := G(Z) + \mu/\lambda \quad (11)$$

which consists of the extraction cost and the rent of resource in units of capital value. By taking the time derivative and using (7) and (8), we obtain the law of motion of  $\pi_d$  given by:

$$\dot{\pi}_d = F_1(K, H(E_d, E_c) + b) (\pi_d - G(Z)) - D_Z(Z)/U_C(C) \quad (12)$$

Equation (12) is the modified Hotelling rule. It shows that the price of non-renewable resource increases faster in the case of high resource rent (low extraction cost). The change in price is also determined by the marginal damage to the environment. Note that in the absence of extraction cost and environmental damages this expression reduces to the standard Hotelling rule.

Using (11) we can rewrite condition (4) as:

$$E_d \geq 0, \quad F_2(K, H(E_d, E_c) + b)H_{E_d}(E_d, E_c) - \pi_d \leq 0 \quad (4')$$

Conditions (4'), (5) and (6) are complementary slackness (c.s.) conditions and they show that a type of resource will be used if its marginal productivity is equal to its price. Accordingly, we can have many cases. We eliminate some of them since the backstop and the other two types are perfect substitutes, the economy will use either one of them at a given time. Therefore, we shall investigate the cases where the economy uses only non-renewable ( $E_d > 0, E_c = 0, b = 0$ ), only renewable ( $E_d = 0, E_c > 0, b = 0$ ) or it uses both simultaneously ( $E_d > 0, E_c > 0, b = 0$ ).

In fact, in this setup, an economy which uses the non-renewable resource will always use the renewable as well. Since they are imperfect substitutes, there will always be an optimal allocation for any given price pair  $(\pi_c, \pi_d)$ . This is what Proposition 1 states as follows:

**Proposition 1.** *The economy uses non-renewable and renewable resources always simultaneously.*

*Proof.* We claim if  $E_d > 0$ , i.e.  $F_2(\cdot)H_{E_d}(\cdot) = \pi_d > G(Z)$  and  $F_2(\cdot) < p_b$  hence backstop is not used ( $b = 0$ ),  $E_c > 0$  will hold true. We will prove the contrary case ( $E_d > 0, E_c = 0, b = 0$ ) is not possible.

$E_d > 0, E_c = 0, b = 0$  occurs if and only if  $F_2(\cdot)H_{E_d}(\cdot) = \pi_d$  and  $F_2(\cdot)H_{E_c}(\cdot) < \pi_c$ . But since  $H(\cdot)$  is in CES form and  $\lim_{E_c \rightarrow 0} H_{E_c}(\cdot) = +\infty, \forall (\pi_c, \pi_d)$  pairs where  $\pi_d < +\infty$  and  $\forall E_d > 0$  given,  $\exists E_c > 0$  that satisfies  $\frac{H_{E_c}(\cdot)}{H_{E_d}(\cdot)} = \frac{\pi_c}{\pi_d}$ . Therefore above condition can never hold. It would have been optimal not to use the renewable resource if its price is  $\pi_c \rightarrow +\infty$ , however, we consider the case  $0 < \pi_c < +\infty$ .  $\square$

The reason is indeed that these resources are imperfect substitutes and there are cases when a type of resource, however expensive, is rational to use. This result, albeit an implication of the assumptions on the production function, is consistent with the historical data on resource use.

Proposition 1 states that the economy initially starts with simultaneous use of the non-renewable and renewable resources. However, the future outcome depends on the backstop price, the renewable price and the degree of substitution.

### 3 The Stages of Optimal Transition

In this section, we present the stages of optimal transition to the clean technologies. We first characterize the initial regime and establish the conditions for the two scenarios. Then we investigate these scenarios in detail.

### Regime 1: Simultaneous Use of Non-renewable and Renewable Resources

( $E_d > 0$ ,  $E_c > 0$  and  $b = 0$ )

The economy initially starts in this regime in which it uses the non-renewable and renewable resources simultaneously. Due to conditions (4'), (5) and (6) the following should hold during this regime:

$$F_2(\cdot)H_{E_d}(\cdot) = \pi_d \quad (13)$$

$$F_2(\cdot)H_{E_c}(\cdot) = \pi_c \quad (14)$$

$$F_2(\cdot) < pb \quad (15)$$

The marginal productivity of non-renewable and renewable resources should be equal to their prices, and the marginal productivity of energy should be lower than the price of backstop.

Conditions (13) and (14) allow us to obtain the optimal amounts of non-renewable and renewable resources, that is, we get  $E_d^*(K, \pi_d, \pi_c)$  and  $E_c^*(K, \pi_d, \pi_c)$ . Using these results together with conditions (1 – 12) we find the differential equation system in  $(C, K, Z, \pi_d)$ . Optimal trajectories should satisfy the following:

$$\dot{C}/C = \sigma(F_1(K, H(E_d^*(\cdot), E_c^*(\cdot)))) - \rho \quad (16)$$

$$\dot{K} = F(K, H(E_d^*(\cdot), E_c^*(\cdot))) - G(Z)E_d^*(K, \pi_d, \pi_c) - \pi_c E_c^*(K, \pi_d, \pi_c) - C \quad (17)$$

$$\dot{Z} = E_d^*(K, \pi_d, \pi_c) \quad (18)$$

$$\dot{\pi}_d = F_1(K, H(E_d^*(\cdot), E_c^*(\cdot)))(\pi_d - G(Z)) - D_Z(Z)/U_C(C) \quad (19)$$

The economy will stay in regime 1 until it reaches the terminal conditions. Condition (15), the marginal productivity of energy, determines these terminal conditions. In fact, when computed at the optimum  $(E_d^*(\cdot), E_c^*(\cdot))$ , the marginal productivity of energy reduces to the following form:

$$F_2(K, H(E_d^*(\cdot), E_c^*(\cdot))) = (\gamma_c^\epsilon \pi_c^{1-\epsilon} + \gamma_d^\epsilon \pi_d^{1-\epsilon})^{\frac{1}{1-\epsilon}} \quad (20)$$

This property allows us to define the *energy price index* as follows:

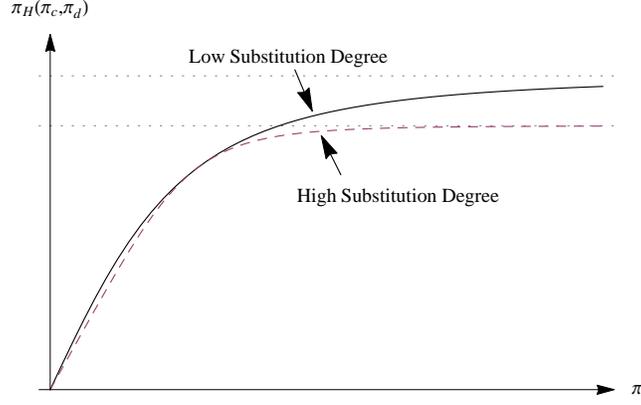
**Definition 1.** Let  $\pi_H$  be the energy price index given by:

$$\pi_H(\pi_c, \pi_d) = (\gamma_c^\epsilon \pi_c^{1-\epsilon} + \gamma_d^\epsilon \pi_d^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

The energy price index increases in the non-renewable price ( $\partial\pi_H/\partial\pi_d > 0$ ), increases in the renewable price ( $\partial\pi_H/\partial\pi_c > 0$ ) and decreases in the degree of substitution ( $\partial\pi_H/\partial\epsilon < 0$ ). It measures the marginal productivity of energy for a given menu of resource prices. It also determines the feasibility of a backstop at a given time. For a given menu of resource prices  $(\pi_c, \pi_d)$ , if the backstop price is higher than the energy price index ( $\pi_H(\pi_c, \pi_d) < pb$ ) then the backstop is not feasible. In case

of equality, the backstop is feasible and the economy will switch to it.

The energy price index has a limit for a given renewable price. As the non-renewable price increases it tends to a constant,  $\lim_{\pi_d \rightarrow +\infty} \pi_H(\pi_c, \pi_d) = \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}}$ . This maximum allows us to identify two scenarios: (i) the backstop will be feasible at a future date as the non-renewable resource price increases if  $p_b < \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}}$  or (ii) it will never be feasible if  $p_b \geq \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}}$ .



Note: The degree of substitution is  $\epsilon = 3$  when it we consider it low (the solid curve) and  $\epsilon = 10$  when we consider it high (the dashed curve).

Figure 1: The energy price index with respect to the non-renewable price

Fig. 1 shows the energy price index for a given renewable price and depicts how it changes with respect to the price of non-renewable resource. We see that its maximum changes according to the degree of substitution. The economy is more likely to switch to the backstop at a future date ( $\pi_H(\pi_c, \pi_d) = p_b$  will hold) in the case of lower degree of substitution. The same applies to higher renewable price, that is, more expensive renewable will increase the maximum and hence make the switch to the backstop more likely. However, if the degree of substitution is sufficiently high or the renewable resource is cheap enough, it might be optimal to gradually substitute the renewable over time instead of switching to the backstop. Obviously, the cheaper the backstop is, the more likely that the economy will switch to it in the future.

We also observe that the energy price index approaches its maximum slower in the case of low degree of substitution. This is a representation of imperfect substitution of resources. The economy cannot easily respond to changes in the prices when the degree of substitution is low. Therefore, it will still allocate considerable amount of non-renewable even if its price is relatively high. In contrast, when the degree of substitution is high, the economy will use high amount of non-renewable when it is relatively cheaper. As its price exceeds the price of renewable, the economy will quickly substitute the renewable.

As the non-renewable price increases, there will be two options for the social planner: (i) to stop using the non-renewable and renewable resources and switch to the backstop or (ii) to gradually reduce the share of non-renewable resource, and eventually approach to a state where it uses only renewable

resource.

The first scenario will be realized if the backstop price is lower than the maximum of energy price index ( $p_b < \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}}$ ) and the second scenario will be realized in the contrary case ( $p_b \geq \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}}$ ). Let us now discuss these scenarios in detail.

## Regime 2: Only Backstop Use

( $E_d = 0$ ,  $E_c = 0$  and  $b > 0$ )

In this regime the economy uses only backstop. The determinants of the time of switch are the resource prices, the extraction cost and the degree of substitution between non-renewable and renewable resources. The economy which initially started in regime 1 will switch to regime 2 at time  $T$  if the following conditions hold:

$$p_b < \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}} \quad (21)$$

$$\pi_{dT} = G(Z_T) \quad (22)$$

$$\pi_H(\pi_c, \pi_{dT}) = p_b \quad (23)$$

$$C^1(K_T, Z_T) = C^2(K_T) \quad (24)$$

Condition (21) is to ensure that at a future date the energy price index will be equal to the price of backstop, which was discussed in the previous subsection. Condition (22) is derived from the definition of non-renewable resource price (11). It means that the shadow value of non-renewable resource has to be zero, thus the resource should have no rent at the time of switch. Condition (23) is analogue to condition (6) and it means that the price of backstop has to be equal to the energy price index at the time of switch. In condition (24),  $C^1(\cdot)$  and  $C^2(\cdot)$  denote the optimal consumption as a function of the state variables in regime 1 and 2 respectively. This condition is to ensure that the state ( $K, Z$ ) and co-state ( $\lambda, \mu$ ) variables of the optimal control problem cannot jump, thus the economy will have a continuous path of consumption and capital over time.

Due to these conditions, there is a limit for the extraction of non-renewable resource. Using conditions (22, 23) we obtain the following expression for the *extraction ceiling*:

$$\bar{Z} = \left( \frac{\gamma_d^\epsilon}{\phi_g^{\epsilon-1} (p_b^{1-\epsilon} - \gamma_c^\epsilon \pi_c^{1-\epsilon})} \right)^{1/2(\epsilon-1)} \quad (25)$$

The extraction ceiling increases in the backstop price ( $\partial \bar{Z} / \partial p_b > 0$ ), decreases in the renewable price ( $\partial \bar{Z} / \partial \pi_c < 0$ ) and increases in the degree of substitution ( $\partial \bar{Z} / \partial \epsilon > 0$ ).

Accordingly, a decrease in the backstop price makes the economy switch earlier. As a result, the economy extracts less non-renewable resource. Similarly, the economy uses more renewable resource if the degree of substitution is lower. The total amount of non-renewable resource use will be lower at

the time of switch. These results contradict the Green Paradox.

In the case of high degree of substitution, the economy benefits from cheap non-renewable resource and uses it in higher proportion. The total amount of non-renewable resource extraction will increase, which implies a Green Paradox. This result is consistent with the findings of Long (2013).

As the economy switches to the backstop, the production function will be  $F(K, H(E_d = 0, E_c = 0) + b) = \tilde{F}(K, b) = K^\alpha b^\beta$ . It is clear that in this case the objective of the social planner reduces to a simple Ramsey optimal growth problem. Using condition (6) we obtain the optimal amount of backstop use,  $b^*(K)$ . Conditions (3, 6 and 7) allow us to obtain the differential equation system in  $(K, C)$ . Optimal trajectories should satisfy the following:

$$\dot{C}/C = \sigma(F_1(K, b^*(K)) - \rho) \quad (26)$$

$$\dot{K} = F(K, b^*(K)) - p_b b^*(K) - C \quad (27)$$

$$\dot{Z} = 0 \quad (28)$$

$$\dot{\pi}_d = 0 \quad (29)$$

This system has a stationary point  $(K^{ss}, C^{ss})$  which can be obtained by solving the following equations:

$$F_1(K^{ss}, b^*(K^{ss})) = \rho \quad (30)$$

$$C^{ss} = F(K^{ss}, b^*(K^{ss})) - p_b b^*(K^{ss}) \quad (31)$$

The system given in equations (26 – 27) has a unique trajectory which leads to the steady state  $(K^{ss}, C^{ss})$ . This unique trajectory allows us to find the optimal consumption rule in regime 2,  $C^2(K)$ , which we referred in condition (24).

**Proposition 2.** *If condition (21) holds, there exists a unique optimal path  $\{K_t, Z_t, C_t, \pi_{dt}\}_{t=0}^T$  starting from any initial state  $K_0 > 0, Z_0 > 0$  that satisfies conditions (22-24). The optimal path of non-renewable and renewable resource use is  $\{E_d^*(K_t, \pi_{dt}, \pi_c), E_c^*(K_t, \pi_{dt}, \pi_c)\}_{t=0}^T$ . The economy switches to the backstop at time  $T$ . After the switch, there exists a unique optimal path  $\{K_t, C_t\}_{t=T}^\infty$  starting from any switching state  $K_T > 0$  and converging to the steady state  $(K^{ss}, C^{ss})$ . Consequently, the optimal path of backstop use is  $\{b^*(K_t)\}_{t=T}^\infty$ .*

### Regime 2': Asymptotic Convergence to Only Renewable Use

$(E_d \rightarrow 0, E_c > 0$  and  $b = 0)$

In this regime, the economy gradually reduces the share of non-renewable resource and allocates more renewable resource over time. The backstop is not feasible because its price is too high and the renewable resource is already a good substitute. As the extraction cost increases, the economy substitutes the renewable resource instead of switching to the backstop. Eventually it converges

to a regime in which it uses only renewable. The economy which initially started in regime 1 will asymptotically converge to regime 2' if the following conditions hold:

$$p_b \geq \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}} \quad (32)$$

$$\lim_{t \rightarrow +\infty} \pi_{dt} - G(Z_t) = 0 \quad (33)$$

$$\lim_{t \rightarrow +\infty} \pi_H(\pi_c, \pi_{dt}) = \pi_c \gamma_c^{\frac{\epsilon}{1-\epsilon}} \quad (34)$$

$$\lim_{t \rightarrow +\infty} C^1(K_t, Z_t) - C^{2'}(K_t) = 0 \quad (35)$$

Condition (32) ensures that the backstop price is so high that it will never be feasible. Conditions (33 – 35) are the limit analogues of conditions (22 – 24) in regime 2, which were discussed in the previous subsection.

In order to find the properties of the regime that the economy converges, we apply a method which is similar to the previous subsection. We assume that the economy stops using non-renewable resource and uses only renewable. The production function will be  $F(K, H(E_d = 0, E_c)) = \hat{F}(K, E_c) = K^\alpha E_c^\beta \gamma_c^{\frac{\beta\epsilon}{\epsilon-1}}$ . Using condition (5), we obtain the optimal amount of renewable resource use,  $E_c^*(K)$ . Finally, conditions (3, 5 and 7) allow us to obtain the differential equation system in  $(K, C)$ . Optimal trajectories of the asymptotic regime 2' should satisfy the following:

$$\dot{C}/C = \sigma(F_1(K, H(0, E_c^*(K))) - \rho) \quad (36)$$

$$\dot{K} = F(K, H(0, E_c^*(K))) - \pi_c E_c^*(K) - C \quad (37)$$

$$\dot{Z} = 0 \quad (38)$$

$$\dot{\pi}_d = 0 \quad (39)$$

As the extraction of non-renewable resource approaches zero, the dynamics of endogenous variables in equations (16 – 19) will approach the above differential equation system. This system has a stationary point  $(K^{ss'}, C^{ss'})$  which can be obtained by solving the following equations:

$$F_1(K^{ss'}, H(0, E_c^*(K^{ss'}))) = \rho \quad (40)$$

$$C^{ss'} = F(K^{ss'}, H(0, E_c^*(K^{ss'}))) - \pi_c E_c^*(K^{ss'}) \quad (41)$$

The system given in equations (36 – 37) has a unique trajectory which leads to the steady state  $(K^{ss'}, C^{ss'})$ . This unique trajectory allows us to find the optimal consumption rule in regime 2',  $C^{2'}(K)$ , which we referred in condition (35).

**Proposition 3.** *If condition (32) holds, the economy will gradually reduce the share of non-renewable resource and eventually converge to a regime where it uses only renewable. There exists a unique optimal path  $\{K_t, Z_t, C_t, \pi_{dt}\}_{t=0}^{\infty}$  starting from any initial state  $K_0 > 0, Z_0 > 0$  that satisfies conditions*

(33-35). This path converges to the steady state  $(K^{ss'}, C^{ss'})$ . The optimal path of non-renewable and renewable resource use is  $\{E_d^*(K_t, \pi_{dt}, \pi_c), E_c^*(K_t, \pi_{dt}, \pi_c)\}_{t=0}^{\infty}$ . Consequently, the optimal path of renewable use converges to  $\{E_c^*(K_t)\}_{t=0}^{\infty}$ .

In summary, we characterized the optimal trajectories of the initial regime and the two scenarios. We established the conditions of whether the economy will switch to the backstop or it will substitute the renewable resource over time. The degree of substitution, the renewable price and the backstop price determine the outcome scenario. For a given backstop price, if the degree of substitution is sufficiently high and the renewable resource is cheap enough, it is optimal to substitute the renewable over time. Otherwise, the economy switches to the backstop at a certain date

In both scenarios, the economy converges to a steady state where it uses only backstop or renewable resource. This result shows that sustainability can be achieved even if the backstop is too costly, and a green, zero-carbon economy is optimal final state in any scenario.

## 4 Numerical Analysis

In this section, we present some simulation results to illustrate the optimal transition paths in both scenarios which we discussed in the previous section. The aim is to provide a numerical example of the optimal transition paths when the assumption of perfect substitution is relaxed.

We use a calibration which is similar to Ploeg and Withagen (2011) and Acemoglu et al. (2012). The elasticity of capital in production is set to  $\alpha = 0.2$ , the elasticity of energy in production to  $\beta = 0.1$ . Since we conduct long run analysis, it is convenient to set equal weights to non-renewable and renewable resources in production, i.e.  $\gamma_d = \gamma_c = 0.5$ . We set the discount rate to  $\rho = 0.02$  and the elasticity of intertemporal substitution to  $\sigma = 0.5$ , hence the relative risk aversion coefficient to be  $\frac{1}{\sigma} = 2$ . The renewable resource price is  $\pi_c = 3$  and the backstop price is  $p_b = 6$ . We set the parameter of marginal cost of extraction  $\phi_g = 0.1$  and the parameter of marginal damages to the environment  $\phi_d = 0.0002$ . Finally, the degree of substitution is  $\epsilon = 3$  when we consider it low, and  $\epsilon = 10$  when we consider it high.

In the following subsections, we first illustrate the scenario in which the economy switches to the backstop. Then, we illustrate the scenario of gradual transition to the renewable resource.

### 4.1 Transition with Switch to the Backstop

This subsection provides a numerical illustration of Proposition 2. We first analyze the phase diagram in the state variables  $(K, Z)$  in fig. 2.

Fig. 2 shows that the economy starting from any initial state accumulates capital and extracts the non-renewable resource until it reaches to the extraction ceiling. At the ceiling, it switches to the backstop and converges to the steady state.

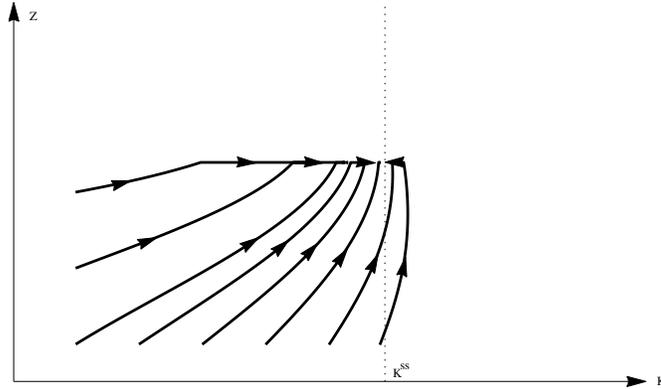


Figure 2: The phase diagram for  $(K, Z)$ : Switch to the backstop

We see that, for a given initial capital stock, if the initial cumulative extraction is higher (higher extraction cost) the slope of optimal path is lower. The economy accumulates same amount of capital by using less non-renewable resource, which means that it uses more renewable over time. Similarly, for a given initial cumulative extraction, if the initial capital stock is lower the economy allocates more renewable in production.

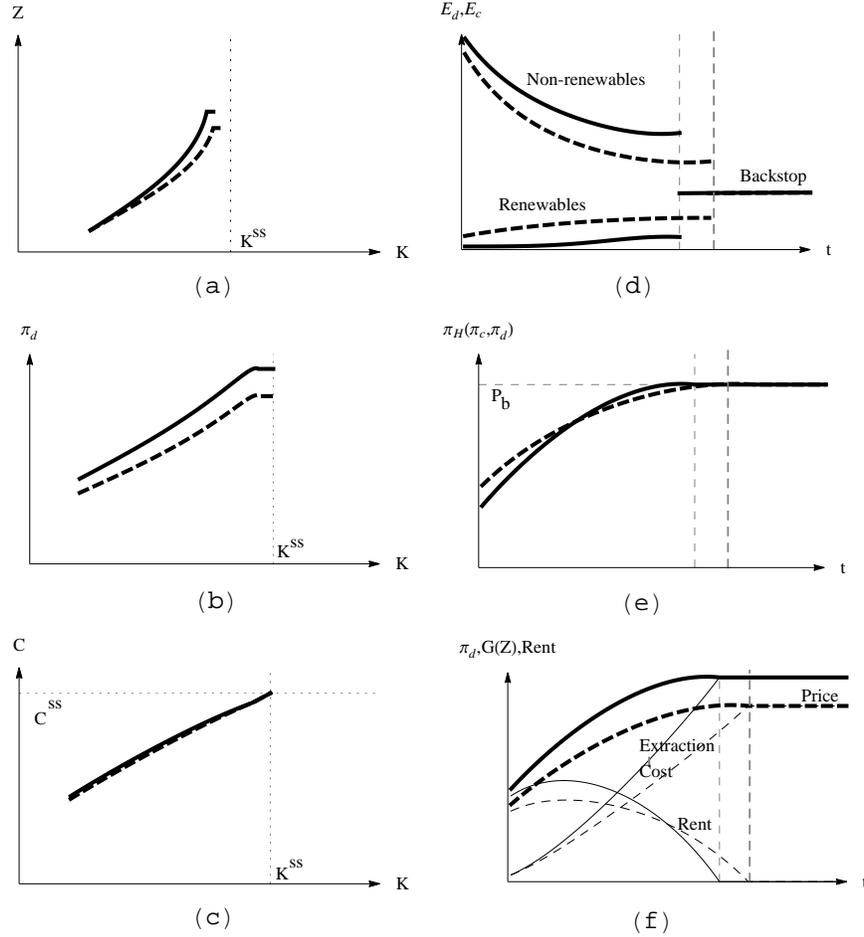
We also observe that the economy over-accumulates capital if the initial capital stock is close to the steady state and the initial cumulative extraction is low. After the switch, it reduces the capital stock and converges to the steady state. The economy benefits from being already developed and having a cheap resource and it transforms the resource into capital for higher future consumption.

Let us now analyze the optimal paths of the key endogenous variables for a given initial state.

We first focus on the properties of the optimal paths in fig. 3. In the initial regime, the economy accumulates capital by using non-renewable and renewable resources simultaneously, and it gradually increases the share of renewable resource over time (fig. 3(a,d)). The price of non-renewable resource is increasing as a result of higher extraction cost, which increases the energy price index until it reaches to the price of backstop. When the energy price index equals to the price of backstop, the economy stops using the two types of resources and switches to regime 2 (fig. 3(b,e,f)). Note that the non-renewable resource has no rent and its price is equal to the extraction cost at the time of switch. After the switch, the economy uses only backstop and converges to the steady state of regime 2 (fig. 3(c,d)).

Fig. 3 allows us to discuss the role of imperfect substitution on these paths as well. In the case of low degree of substitution the extraction ceiling is lower, hence the economy extracts less non-renewable resource in total. It accumulates same amount of capital by using less non-renewable resource, which means that the share of renewable resource is higher over time (fig. 3(a,d)).

The price of non-renewable resource is also lower and increases slower when the degree of substitution is low. In addition, the extraction cost has the same profile as a result of less non-renewable use,



Note: The solid lines represent the case of high degree of substitution and the dashed lines represent the case of low degree of substitution.

Figure 3: Optimal transition with switch to the backstop and the role of imperfect substitution

however, the rent of non-renewable resource is first lower then higher. This means that a decrease in the degree of substitution increases the future value of non-renewable resource. We also observe that the optimal price to stop using the non-renewable resource is lower (fig. 3(b,f)). Moreover, the energy price index is initially higher in the case of low degree of substitution, but it increases slower and equals to the price of backstop much later. Therefore the economy postpones the switch (fig. 3(d,e)). After the switch, it converges to the same steady state since the degree of substitution does not play any role in regime 2 (fig. 3(c)).

In summary, the economy extracts less non-renewable resource in the case of lower degree of substitution. The share of renewable resource is higher over time, however, the economy switches to the backstop later than the case of higher degree of substitution.

## 4.2 Gradual Transition to the Renewable Resource

This subsection provides a numerical illustration of Proposition 3. We analyze the optimal paths and discuss the differences in the two scenarios by using the phase diagram in the state variables  $(K, Z)$  in fig. 4.

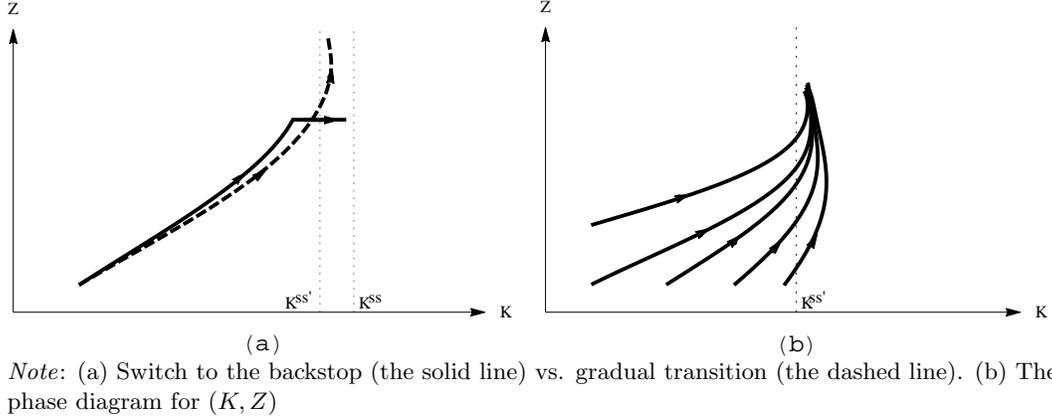
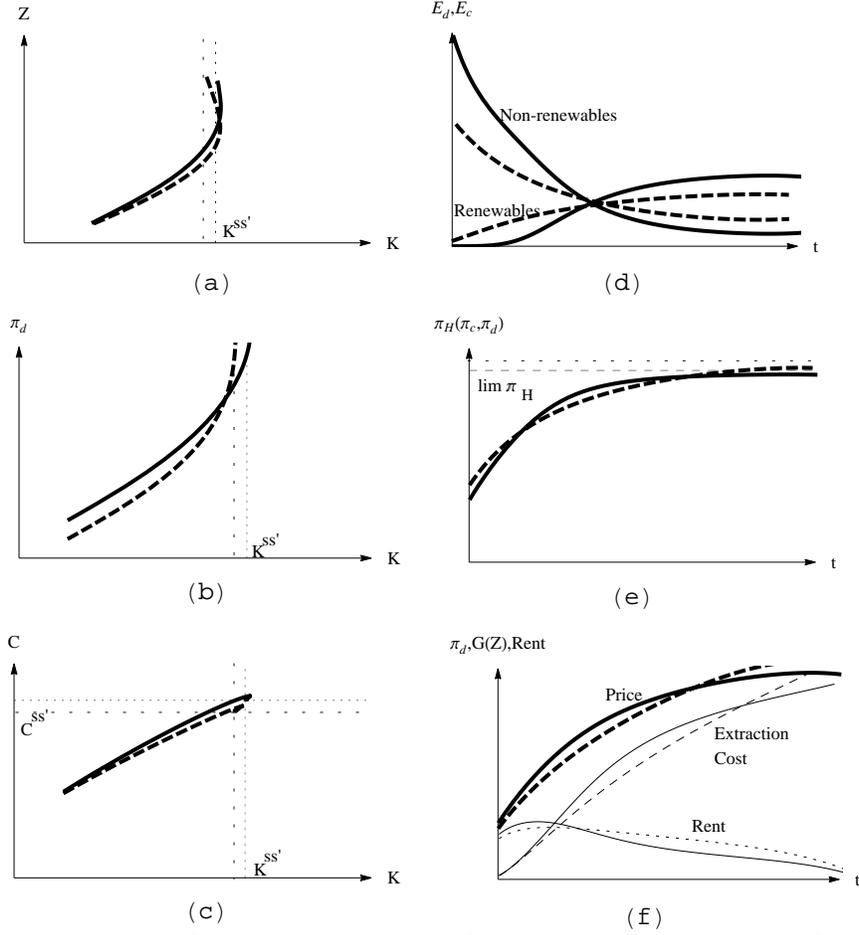


Figure 4: Gradual transition to the renewable resource

Does the optimal paths differ whether the economy has a feasible backstop in the future? Fig. (4,a) addresses this question by illustrating the comparison of the two scenarios. We see that the slope of the optimal path is lower when the backstop is not feasible in the future. This means that the economy uses less non-renewable resource. Since the economy does not switch to the backstop, it continues to extract the non-renewable resource and converges to a steady state where the capital stock is lower than the scenario of switch. The welfare is lower as a result of lower level of consumption and higher damages to the environment. Therefore, the economy is worse-off in the gradual transition scenario compared to the scenario in which it switches to the backstop.

Fig. (4,b) shows the phase diagram in the state variables  $(K, Z)$ . The economy extracts more non-renewable resource if the initial capital stock is higher and the initial cumulative extraction is lower (lower extraction cost). We see that the economy always over-accumulates capital and reduces it while converging to the steady state. It benefits from the non-renewable resource when it is cheap and transforms it into capital for higher future consumption.

The optimal paths in this scenario have similar properties to the switch scenario which we discussed in the previous subsection. Fig. 5 illustrates these paths for a given initial state. Considering that the backstop is not feasible, the economy always uses the two types of resources simultaneously. We see that the economy gradually reduces the share of non-renewable resource, and the share of renewable exceeds the share of non-renewable when the two resource prices are equal. Eventually the economy converges to the steady state of regime 2' in which it uses only renewable (fig. 5(c,d)). The use of non-renewable resource approaches zero as the difference between its price and extraction cost, thus



Note: The solid lines represent the case of high degree of substitution and the dashed lines represent the case of low degree of substitution.

Figure 5: Gradual transition to the renewable resource and the role of imperfect substitution

its rent, vanishes. In addition, the continuous increase in the price of non-renewable increases the energy price index and it approaches its limit (fig. 5(b,d,e,f)).

Finally, let us discuss the role of imperfect substitution in this scenario. Fig. 5 depicts the comparison of low and high degree of substitution. In the case of low degree of substitution, the economy converges to a steady state where the capital stock is lower, which means the consumption is lower as well. The total amount of non-renewable resource extraction is higher and therefore the damages to the environment is higher. As a result of lower consumption and higher environmental damages, the welfare is lower when the degree of substitution is low (fig. 5(a,c)).

The effect on the resource allocation differs according to the time period. When the degree of substitution is low, the economy uses more renewable in the short and medium run, but less in the long run. This is a consequence of imperfect substitution. Since the economy cannot easily substitute the renewable resource, it has to use the non-renewable even though its relative price is too high. In

the contrary case, high degree of substitution allows the economy to benefit from the price differences and easily substitute the renewable as it becomes the cheaper resource (fig. 5(d)).

We observe similar differences for the price of non-renewable resource, the extraction cost and the energy price index. In the case of low degree of substitution, the price of non-renewable resource and the extraction cost are lower in the short and medium run. But as the economy continues to extract the non-renewable resource, they increase more rapidly and thus they are higher in the long run. Moreover, the energy price index is higher in the short run and lower in the medium run. Since the limit of energy price index is higher, it approaches a higher value in the long run. We also see that lower degree of substitution leads to higher rent of non-renewable resource in the future (fig. 5(b,e,f)).

In summary, the economy extracts the non-renewable resource slower when the backstop is not feasible in the future. Since it does not switch to the backstop, it continues the extraction and converges to the steady state in which it uses only renewable. The welfare in this steady state is lower compared to the one where the economy uses only backstop.

The role of imperfect substitution depends on the time period. In the short run, the economy extracts less non-renewable resource when the degree of substitution is lower. In the long run, however, it substitutes less renewable resource due to lower substitution possibility, thus it uses more non-renewable resource. The welfare in the final state is lower in the case of lower degree of substitution.

## 5 Conclusion

This paper shows the implications of imperfect substitution between non-renewable and renewable resources on the optimal transition path to clean technologies.

The results show that the economy uses non-renewable and renewable resources always simultaneously. If the backstop is cheap enough and the renewable resource is not a good substitute for the non-renewable, the economy switches to the backstop at a certain date. Otherwise, it gradually reduces the share of non-renewable and converges to a state in which it uses only renewable resource. This means that a green, zero-carbon economy is optimal final state in any scenario.

We show that in the first scenario where the backstop is feasible, there is an extraction ceiling at the time of switch which is independent of the initial state. It is determined by the backstop price, the renewable price and the degree of substitution. A policy which reduces the backstop price decreases the extraction ceiling. Similarly, a policy which restricts the use of non-renewable resource, hence decrease the degree of substitution, has the same effect. These results contradict the Green Paradox. If the degree of substitution increases, however, the economy extracts more non-renewable resource when it is cheaper than the renewable. In this case, the extraction ceiling will increase and this implies a Green Paradox.

In the second scenario where the backstop price is too high, the economy substitutes the renewable resource for the non-renewable over time. The role of imperfect substitution depends on the time

period. A policy which increases the degree of substitution decreases the renewable use in the short run, but increases in the long run. Accordingly, the welfare in the final state increases if the degree of substitution is higher.

Further research includes the extension of the model by considering multiple non-renewable and renewable resources and incorporating the carbon emission and accumulation dynamics. This would allow us to estimate the model parameters and conduct more realistic simulations. The energy price index is also an interesting analytical result which can be used as an empirical tool to assess energy productivity and feasibility of a backstop.

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