

2013/73



Jumping the hurdles for collaboration: fairness in
operations pooling in the absence of transfer payments

Alejandro Lamas and Philippe Chevalier



CORE

The word "CORE" is written in a large, bold, black sans-serif font. A thin, light blue curved line starts above the 'C', arches over the 'O' and 'R', and ends below the 'E'.

DISCUSSION PAPER

Center for Operations Research
and Econometrics

Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium

<http://www.uclouvain.be/core>

CORE DISCUSSION PAPER
2013/73

**Jumping the hurdles for collaboration: fairness in operations pooling
in the absence of transfer payments**

Alejandro LAMAS¹ and Philippe CHEVALIER²

January 2014

Abstract

Independent firms may be interested in collaborative alliances in order to reduce their costs and risks, among others benefits. Particularly in operations, different firms can gain from economies of scale by pooling their production resources. Even though there may be a significant reduction of the overall cost, the success of the partnership may generally depend on the fairness of the agreement. With this in mind, the firms can utilize transfer payments with the aim to achieve a balanced allocation of benefits. The implementation of such payments, however, could be difficult in practice, because of the presence of legal constraints or additional contracts that make such agreements more complicated. Therefore, the partners should jointly plan operations as to deal with the trade-off between the efficiency and the fairness. We propose a novel methodology to guide joint operations based on the Rawls' theory of justice, such that we prioritize improving the firm that tends to benefit less from the collaboration. When comparing the quality of such approach with other notions of fairness, we pay particular attention to the effect of the uncertainty on the collaboration. We prove that agreements based on basic notions of fairness have a positive impact on the risk reduction of the firms. Furthermore, our numerical results show that the firms can reduce even more their risk when the agreement is based on more advanced notions of fairness. In particular, our proposed methodology outperforms other approaches of collaboration in terms of risk, while the efficiency is not significantly damaged.

Keywords: supply chain collaboration, fairness, lot sizing problem, group decisions, bargaining.

¹ Université catholique de Louvain, Louvain School of Management and CORE, B-1348 Louvain-la-Neuve, Belgium. E-mail: alejandro.lamas@uclouvain.be

² Université catholique de Louvain, Louvain School of Management and CORE, B-1348 Louvain-la-Neuve, Belgium. E-mail: philippe.chevalier@uclouvain.be

The authors are thankful to Prof. Per Agrell for useful comments and suggestions that helped to improve this paper.

This work was supported by the ARC (Action de Recherche Concertée – Concerted Research Action) project on Managing Shared Resources in Supply Chains of the Communauté Française de Belgique. It also presents research results of the P7/36 PAI project COMEX, part of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1 Introduction

In this paper we study how two firms belonging to two different supply chains can collaborate. In particular, we are going to concentrate on collaborations involving operations pooling, so that firms may benefit through economies of scale, risk sharing, know-how transfer, etc. Obviously, each firm expects the collaboration to lead improvements of its situation, however, the firms have other expectations that may depend on other factors, which in general are omitted in the literature on collaboration in operations. We propose more realistic models of the behaviour of firms aiming to start a collaboration.

One element that characterizes the behaviour of the firms arises from the fact that setting up the collaboration agreement requires a significant effort by the future partners. Consequently, the firms have to be convinced that their gains will justify this effort. As a result, a necessary condition for implementing the collaboration is that the overall benefits are sufficiently large. Therefore, the challenge for firms interested to collaborate is to answer the question: ‘How do we expand the pie of benefits between collaborative parties?’(Jap, 1999). We denominate efficiency as the measure of the overall benefit achieved through collaboration.

Another aspect of the behaviour of the firms is that they must be sufficiently confident in advance in the success of a collaborative agreement. Such confidence depends strongly on the agreement on how to share collective benefits. Jap (2001) summarizes the cost allocation challenge by the question: ‘How do we divide the pie of benefits between the firms?’. The notion of fairness is crucial to answer this question, widely discussed in the literature, resulting in multiple definitions, see e.g. Nash (1950); Rawls (1971); Kalai and Smorodinsky (1975); Camerer and Thaler (2010); Bertsimas et al. (2011). Experiments showed that players might prefer a self destructing behavior to unfair collaboration, i.e. the firms can desist from collaborating, even though each firm improves its performance compared to the stand-alone situation. An example is the experiment of the ultimatum game, in which a proposer offers some allocation of money between himself and a responder, who can decide to accept it or not. Contrary to the expected rational behaviour, if the offered proportion is strictly positive, but smaller than a certain value, the responder tends to reject the offer (Camerer and Thaler, 2010; Jap, 2001). Motivated by this behaviour, recent literature shows that the expectations of participants to economic relations are complex, and they also involve the relative results between firms. In this sense, our work is based on the Rawlsian notion of fairness, such that the joint decisions of the firms prioritize the improvement the partner that is least well off (Rawls, 1971).

In order to build a detailed model of the behaviour of the firms, the agreement should take the dimensions of efficiency and fairness into account. If utility can be transferred between firms, the feasibility of the maximum efficiency is independent of the allocation of gains between firms. Indeed, the firms can always implement the decision leading the maximum efficiency, because they can set adequate payments that ensure meeting the fairness criteria. But the firms are generally reluctant to transfer payments, because the collaboration takes into account several intangible costs (for example, holding and back-ordering costs) which are difficult to evaluate even when the production plan is carried out in isolation. Moreover, the requirements of contracts between firms and/or the presence of legal restrictions are additional impediments for the implementation of transfer payments. This has been widely reported in the literature, see e.g. Granot and Sosic (2005); Makadok (2010). In the absence of such payments, implementing the optimal decision may be incompatible with the fairness expectations of the firms, thus a trade-off exists between the fairness and the efficiency dimensions of the problem. Hence the challenge for the firms that start a collaboration is to answer the question: ‘How do we achieve an equilibrium between the fair division of the pie and its expansion?’. Given that the operations will directly determine the shares of the

benefits when payments are not possible, the management of operations is crucial for answering such question.

Obviously, the reduction of costs is the essential incentive for each company to collaborate, however, partners can also expect other benefits from the collaboration. When the firms agree to collaborate in the long term, they must decide on the joint planning of operations without perfect information about their future demands. In such case, each firm is interested in minimizing its costs, but also, expects to control the variability of its cost. Thus, the success of the agreement depends on meeting both expectations.

Given the expectation of firms for a successful implementation of collaborative agreements: fairness, risk reduction and absence of transfer payments, we propose a methodology for jointly planning operations in order to overcome such hurdles. Our methodology gives priority to meet the fairness criteria, while minimizing the potential inefficiencies of the firms. Moreover, we measure the effect of this methodology on the risk of each firm in comparison to other mechanisms of collaboration.

As our interest is to study the collaboration in operations, we model the production line of each firm as a Capacitated Lot Sizing problem (*CLSP*), a well known production-inventory problem that establishes when and how much to produce in order to minimize set-up, production, holding and back-ordering costs (see Wolsey (1995); Karimi et al. (2003); Brahimi et al. (2006) for an extended literature review of this problem). The formulation of the *CLSP* is consistent with the practical cases that motivate our work, where two independent firms interchangeably use their production lines to process their requests. An example of a practical case corresponding to such setting would be a group of printing firms whose requests correspond to books or newspapers. As the quality of a book or a newspaper may not differ significantly from line to line, the companies have the opportunity to pool their operations. In this way, if one of the lines has slack capacity in a production period, the products of other firms can be allocated to this line, in such way the firms benefit from the reduction of the set-up and/or inventory costs. Some other examples are electronic firms producing components by using either their own production line or the production line of some partner (for example LCD panels for LCD TVs, Choi et al. (2010)), and small- and medium-sized furniture manufacturers pooling their resources in order to compete with larger-manufacturers (Bjornfot and Torjussen, 2012). In the previous examples, pooling operations brings gains from the economies of scale of the set-up costs, and also by reducing the stocks and back-orders as a result of a better utilization of the production capacities of the firms.

Our work contributes to the scientific discussion by providing a methodology for joint planning decisions by combining games or group decisions, multi-criteria approaches and production-inventory problems. Generally, the literature on collaboration in operations assumes that the incentive of the partners to collaborate is determined by the individual rationality criterion, i.e. the collaboration does not hurt the performance of a firm. We go beyond such criterion by utilizing a more complex notion of fairness, that better corresponds to the interest of firms in real applications. In this work, we also provide a novel analysis for operations pooling in which we evaluate the quality of an agreement not only in terms of the costs of the partners, but in terms of the risk associated to the firms when decisions are made under uncertainty.

From the managerial point of view, this paper can inspire the implementation of collaborations for real problems, since the proposed methodology helps the firms to jump hurdles for collaboration (e.g. fairness, risk or absence of transfer payments). So, we contribute to shorten the gap between the theory of collaboration and the implementation of such practice.

The paper is structured as follows. In Section 2 we highlight the literature related to our research. Section 3 describes the problem of firms when they jointly plan their operations. Also, we present and analyse different criteria of fairness, and we define the indicator of risks to measure

the quality of a collaborative agreement. In Section 4 we introduce schemes leading to decisions that make a balance between the efficiency and the fairness of the collaboration. Based on such schemes, we extend the formulation of the *CLSP* to a collaborative context, which is presented in Section 5. In Section 6 we provide a numerical analysis of the collaboration under different schemes. Finally, in Section 7 we summarize the main contributions and discuss the future research related to this work.

2 Literature Review

We will study horizontal collaborations, defined as joint activities between different supply chains, in such way that similar parties collaborate on a particular business function (Stadtler, 2009; Erhum and Keskinocak, 2007). There is another type of collaboration in supply chains, the vertical collaboration, which focuses on a single supply chain and it is out of the scope of our work. In practice, horizontal collaborations may be advantageous to the partners, however there are difficulties in implementing it.

We start by discussing the advantages of the horizontal collaboration. Corbett and de Groote (2000) and Simatupang and Sridharan (2002) suggest that the essential advantages lie in the improvements in comparison to an initial solution (stand-alone situation). According to Crujissen et al. (2007), such improvements arise from reducing the costs of operations and increasing the productivity of the firms. Note that, there may be other advantages such as improving service level (lead times, geographical coverage, reliability, etc.) and better competitive position of the firms in their markets, however, these are in general secondary objectives to the cost reduction. For such reason, our study focuses on the effect of collaboration on the costs of the firms.

We continue by describing the impediments for implementing horizontal collaborations. The main difficulty in implementing such practice is the division of the gains (Crujissen et al., 2007). Brandenburgger and Nalebuff (1996) emphasize that collaboration should prioritize the maximization of the overall benefit of the companies, but it is essential to take into account that the different parties will compete for such gains. To handle the stress arising from such competition, the partners must perceived that the collaborative agreement is fair. Indeed, Bertsimas et al. (2011) suggest that in many environments fairness is more important than optimality. Further, the authors propose that when different parties collaborate, a decision may not be practically implementable, because some of the parties might consider it ‘unfair’.

Although the relevance of fairness is clear in collaboration, the meaning of such concept is ambiguous and vastly discussed in the literature. Nevertheless, most of the existing notions of fairness compare the relative results achieved by the partners. In this line, Camerer and Thaler (2010) suggest that participants do not care about the other’s welfare per se, but desire some type of equity. Thus, participants may even prefer self destructing behaviour to unfair collaborations. In order to implement such notion of fairness, Fishburn and Sarin (1994); Boiney (1995); Brams and Taylor (1995) utilize the concept of equity, which establishes that each partner must receive the same proportion of the gain resulting from collaboration. Alternatively, many authors follow the Rawlsian notion of fairness (Rawls, 1971), in which the collaboration focuses efforts to the party that is least well off. With the aim of establishing common criteria for collaborative agreements, Nash (1950) proposes an axiomatic characterization of fairness, which is extended to more general problems by Kalai and Smorodinsky (1975); Kalai (1977). Each of these authors proposes a notion of fairness in order to satisfy the proposed axioms, however, no notion in the literature satisfies simultaneously the whole set of fairness axioms. Based on that fact, the literature related to collaboration aims to implement notions of fairness covering the maximum number of axioms. In

this work, we consider that firms expect that their decisions pursue a Rawlsian notion of fairness when pooling operations, so that the collaboration results in a reduction of costs that are balanced between firms.

The existing formulations for addressing fairness in collaboration can be divided into two categories depending on the existence, or not, of transfer payments between the firms. When such payments are possible, the problem of collaboration can be divided in two stages: first, the firms make decisions in order to achieve the globally optimality; based on such decisions, the resulting benefits are divided between firms. Note that, the globally optimal decision is always feasible, since transfer payments lead to an equilibrium between the interests of the different parties. The problem of collaboration with transfer payments is largely studied in the literature of cooperative games with transferable utility (TU-game), see e.g. von Neumann and Morgenstern (1943); Shapley (1959); Gillies (1959); Myerson (1991). In the same line, but more related to our work, Fiestras-Janeiro et al. (2011) review the most relevant approaches dealing with collaboration in the context of production-inventory problems. A first group of such approaches focuses on the case where demand is continuous and constant, so that firms collaborate in an economic order quantity (EOQ) environment. Meca et al. (2004); Federgruen and Zheng (1992); Anily and Haviv (2007) deal with the EOQ problem with multiple firms. The authors propose a formulation, in which once the firms minimize their joint inventory cost, the costs are allocated between firms by utilizing a game theory frame. Another group of approaches focuses on dynamic demand problems, whose game theory formulations are known as production-inventory games. Guardiola et al. (2009) deal with this type of problem, but in contrast to our case, their problem ignores the fixed ordering costs, so the analysis omits the economies of scale resulting from collaboration. van den Heuvel et al. (2007) study what they call Economic-lot sizing game, in which the operations of the firms involve fixed ordering costs. The authors model the operations as an Uncapcitated Lot-sizing problem, but the game associated to the cost allocation between firms has a non-linear cost structure caused by the presence of such costs. Sambasivan and Yahya (2005); Drechsel (2010); Drechsel and Kimms (2011) extend the problem to its capacitated version. Sambasivan and Yahya (2005) proposes a Lagrangean relaxation based heuristic to solve the problem, but the authors do not address the allocation of costs between firms. Drechsel (2010) addresses the allocation problem through a max-min formulation such that the difference between the maximum relative reduction of costs of the firms is minimized. This work is in many aspects close to this current investigation, but there are two essential differences: (i) their formulation considers transfer payments between firms, and (ii) their performance measurement is different.

There are practical difficulties linked to collaborations that arise in both calculating the exact benefits of each firm (consequently difficulties on determining the amount of side payments) and possible legal restrictions (Granot and Sosic, 2005). Moreover, when two competitors decide to collaborate, their transfer payments are likely to be reviewed by the antitrust authority. Thus the absence of such payments facilitates such agreements. The problem of collaboration in the absence of payments is studied in the literature as non-transferable utility games (NTU-game), see e.g. Aumann (1961); Myerson (1991); Borm et al. (1992). The main difference in comparison to the TU-games lies in the fact that the operational decisions of the firms directly determine the allocation of gains. Then, if the firms make agreements taking the fairness dimension into account, the efficiency of the collaboration in the NTU-games is reduced in comparison to the TU-games (Jain and Mahdian, 2007). Bertsimas et al. (2011) study the inefficiency in NTU-games when fairness considerations are introduced. The authors propose upper bounds for such inefficiency under different notions of fairness. However, the bounds are only valid for convex utility sets, we try in this article to determine decision rules that implement different notion of fairness in the framework of production planning where the utility set is typically not convex. In the same vein,

Drechsel (2010), Frisk et al. (2010) propose a cost allocation methodology for the operations of firms based on the Rawlsian notion of fairness. In their case the absence of transfer payments has only a limited impact on the efficiency of the decision, because they use a linear model for operations.

As we discuss in Section 1, the collaboration may have an effect on the risk of the operations. Similar to fairness, the risk is a concept widely discussed in the literature, but it also has multiple definitions. One of the most utilized measures of risk is the value at risk, however, Artzner et al. (1999) show the weakness of such measure. Alternatively, the authors introduce an axiomatic characterization for risk measures. The risk measures satisfying such axioms are classified as coherent. Rockafellar and Uryasev (2000) introduce a coherent risk measure known as conditional value at risk. Such measure has become common in the literature of risk and in real life applications, therefore, we utilize it for measuring the risk of the collaboration between firms.

Our work contributes to the scarce literature on horizontal collaboration in operations by proposing a methodology that prioritizes the fairness of the agreement, whose implementability is independent of the existence of transfer payments between firms. Furthermore, although there are operations management approaches for controlling the conditional value at risk (Ahmed et al., 2007; Choi and Ruszczyński, 2008; Chen et al., 2009), to the best of our knowledge, our work represents the first study including risk in operations when multiple firms pool their production resources.

3 General Framework

In this section we give a formal description of the problem introduced in Section 1. Also, we describe the conditions that determine the fairness of an agreement. Further, we focus on collaboration between risk averse firms, and we define indicators for measuring the risk of the resulting operations of the firms.

3.1 Problem Description

Consider two self-interested and risk-averse firms, Firm 1 and Firm 2, that operate interchangeable production lines. If the firms agree to collaborate, they make a joint planning of production activities such that the production lines of both firms are planned together.

We represent the planning decisions of the firms by the pair $X = (X_1, X_2)$. Moreover, we define $\Omega^G(d)$ as the set of all the planning decisions that are operationally feasible under the agreement when the firms face a demand d , and the function $C : \Omega^G \rightarrow \mathbf{R}^2$, such that $C_i(X)$ represents the cost of firm i resulting from X . If the firms do not collaborate, they operate independently. In such scenario, the objective of each firm is to minimize the cost of its own operations. Let $X^o(C, d) = (X_1^o(C, d), X_2^o(C, d))$ be the pair of decisions that minimize the cost of each firm when they work separately.

In order to represent a wide range of problems of collaboration, we assume that the sum of the optimal costs of the firms is subadditive when the firms collaborate. In other words, the inequality $e^T \cdot C(X^o(C, d)) \geq e^T \cdot C(X^*(C, d))$ is always satisfied, where $X^*(C, d) = \arg \min_{X \in \Omega^G(d)} \{e^T \cdot C(X)\}$ and

e is the vector of all ones. This condition is essential for the firms, it indicates that they can reduce costs by pooling operations.

When setting up the agreement, firms spend time and resources in the bargaining process, building trust, adapting operations, etc. Therefore, if the advantages related to the agreement do not remain valid in the long run, such effort may not be worthwhile. Consequently, we focus

on long-term agreements, where the firms learn gradually their demands for a certain interval of time, what we call a planning block. Thereby, firms plan operations repeatedly as reliable information of demand becomes available for a planning block. This assumption makes sense in long term collaborative agreements, since on the one hand, it may be difficult for firms to get reliable information about their future demand when they decide the agreement, but on the other hand, they should plan operations in the short term. Note that, in the problems that we study, the decision in each planning block depends on the realization of demand, i.e. $X(d)$, where d is a realization of the random pair $D = (D_1, D_2)$ that represents the demands faced by the firms in a planning block.

We will call a collaboration scheme as the sets of rules upon which the firms will jointly plan operations for a block of periods, when the demand is known for this block. Once the firms carry out the planned operations for a production block, they make a new joint plan for the next block. Note that, determining a collaboration scheme is the the main decision involved in the agreement.

Given that the reduction of costs is the essential incentive for firms to pool their production lines, the main goal of a collaboration scheme is the minimization of the total joint cost. Nevertheless, the firms expect more from the collaboration. If the scheme focuses only on such objective, two key issues for implementing a collaborative agreement are omitted:

- On the one hand, the firms may have apprehensions regarding how the cost is allocated among them. For instance, the implementation of the scheme may be hindered if the cost of a firm may turn out to be higher than when operating independently, or if the gains of the joint operations may be very unbalanced. This dimension corresponds to what we call fairness.
- On the other hand, it is natural that firms expect to shrink the variability of their operations as a result of pooling. So, the firms would like that the chosen collaboration scheme supports such reduction.

In the remainder of this section, we discuss the measures and criteria for determining the efficiency in reducing costs, the fairness of the agreement, and the resulting risk from the collaboration.

3.2 Efficiency Measures

In order to evaluate the performance of a collaborative scheme, the firms can utilize the results associated with $X^*(C, d)$ and $X^o(C, d)$. Obviously, $e^T \cdot C(X^*(C, d))$ is a lower bound for the sum of the costs of the firms when the agreement is carried out during a planning block in which the realized demand is d . Moreover, $C(X^o(C, d))$ constitutes a benchmark for such agreement, because each element of the pair represents the alternative to pooling operations. Hence, $E_D [(1 - e^T \cdot C(X^*(C, D))/e^T \cdot C(X^o(C, D))) \cdot 100\%]$ is a natural benchmark for measuring the efficiency of reducing costs resulting from the agreement. We will say that a scheme is efficient if its associated expected cost reduction is similar to the maximum achievable reduction.

3.3 Criteria for Fair Collaboration

When firms expect fairness in the agreement, the negotiation for a collaboration can be model as a bargaining problem in each production block. Such problem is characterized by $(C, X^o(C, d), \Omega^G(d))$, i.e. the firms make decisions considering their individual cost functions, the operations of the firms in the stand-alone situation given the realized demand for the planning block, and the feasible solutions for such demand. We call $U(d)$ the set of all triples $(C, X^o(C, d), \Omega^G(d))$ defining bargaining problems for the realization of demand d . Based on this bargaining problem, we give the following definition for a collaboration scheme:

Definition 1. *The collaboration scheme F is characterized by the function $X^F : U(d) \rightarrow \Omega^G(d)$, i.e. $X^F(C, X^o(C, d), \Omega^G(d))$ is the planning decision obtained by implementing F for the bargaining problem $(C, X^o(C, d), \Omega^G(d))$. The pairs of cost under the collaboration scheme F is $C^F(C, X^o(C, d), \Omega^G(d)) = C(X^F(C, X^o(C, d), \Omega^G(d)))$.*

A set of axioms has become common in the literature of bargaining problems, where each of them represents some dimension of fairness. Based on the descriptions on Kalai and Smorodinsky (1975), Kalai (1977) and Bertsimas et al. (2011), we give a formal definition of such axioms for the bargaining problems in $U(d)$. For a better understanding of these definitions, we utilize the notation $a \geq b$ for $a, b \in \mathbf{R}^n$ to denote $a_i \geq b_i$, $i = 1, \dots, n$, and the notation of the bargaining problem is reduced to $u(d) = (C, X^o(C, d), \Omega^G(d))$, where $u(d) \in U(d)$.

Axiom 1. *Individual Rationality.* *The collaboration scheme F is individual rational if each firm reduces its cost compared with its situation in the absence of the agreement, i.e.*

$$C^F(u(d)) \leq C(X^o(C, d)). \quad (1)$$

This axiom establishes that no firm is hurt by the implementation of the collaborative agreement. Thereby, the firms ensure that the collaboration is worthwhile for each of them.

Axiom 2. *Pareto-optimality.* *The collaboration scheme F is pareto-optimal if no firm can be made better off without making at least one firm worse off, i.e. $\nexists X \in \Omega^G(d)$ such that $C(X) \leq C^F(u(d))$ and $C(X) \neq C^F(u(d))$.*

The pareto-optimality imposes that any planning decision must avoid unnecessary losses for the firms. Clearly, $X^*(C, d)$ meets this criterion, however, such decision may be incompatible with other axioms of fairness (for example Axiom 1) when transfer payments are not allowed. When the agreement between firms ensures other axioms of fairness, the pareto-optimality discriminates favourably towards most efficient solutions.

Axiom 3. *Symmetry.* *Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the permutation operator defined by $T((a_1, a_2)) = (a_2, a_1)$. Then, $T(C^F(u(d))) = C^F(T(u(d)))$.*

The symmetry implies that the fair planning decisions cannot be affected by how the firms are named. Consequently, the axiom imposes that firms with similar characteristics should get similar outcomes.

Axiom 4. *Invariance with respect to affine transformations of costs.* *Let $A : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be an affine operator defined by $A(a_1, a_2) = (A_1(a_1), A_2(a_2))$, with $A_i(a) = \alpha_i \cdot a + \beta_i$ and $\alpha_i \geq 0$. If $u'(d) = (A(C), X^o(C, d), \Omega^G(d))$, then $A(C^F(u(d))) = C^F(u'(d))$.*

This axiom establishes that the relative results of the firms must be invariant to the way that each firm measures the gains of the collaboration. If a collaboration scheme satisfies this requirement, its implementation allows firms to obtain fair operations even when the individual gains are measured in different units. Nevertheless, there are critics to this axiom, because the comparison between the results between firms becomes more difficult (Nydegger and Owen, 1974). Thereby, Kalai (1977) proposes a weak version of Axiom 4, which is known as homogeneity.

Axiom 5. *Homogeneity.* *Let $H : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the homogeneity operator defined by $H(h_1, h_2) = (\alpha * h_1, \alpha * h_2)$ and $\alpha \geq 0$. If $u'(d) = (H(C), X^o(C, d), \Omega^G(d))$, then $H(C^F(u(d))) = C^F(u'(d))$.*

The homogeneity requires that the planning decisions remain identical if the gains of the firms are linearly modified by the same factor. Thereby, the planning decisions of the firms are independent of the unit used to measure the costs.

Axiom 6. *Independence of irrelevant alternatives.* Let $u'(d) = (C, X^o(C, d), \Omega^{G'}(d))$ be a bargaining problem such that $\Omega^G(d) \subset \Omega^{G'}(d)$. If $X^F(u'(d)) \in \Omega^G(d)$, then $C^F(u(d)) = C^F(u'(d))$.

This axiom states that preferring a planning decision over another one is independent of other available options. But there are objections to this axiom in the literature (see Kalai and Smorodinsky (1975)), which leads to the following alternative axiom.

Axiom 7. *Monotonicity.* Let $u'(d) = (C, X^o(C, d), \Omega^{G'}(d))$ be a bargaining problem, such that the minimum achievable cost of Firm 1 is identical for $u(d)$ and $u'(d)$. If for every cost that Firm 1 may demand, the minimum cost that Firm 2 can derive simultaneously is smaller or equal in $u'(d)$, then $C^F(u(d)) \geq C^F(u'(d))$.

The monotonicity establishes that if, for every cost level that Firm 1 may demand, the minimum feasible cost level that Firm 2 can simultaneously reach is decreased, then the fair planning decision should imply a reduction of the cost of Firm 2.

It should be noted that the satisfaction of an axiom in a planning block can be extrapolated to the whole horizon involved in the agreement. This feature is a consequence of the independence between the decisions of the different planning blocks. Thus, if a collaboration scheme satisfies an axiom in a planning block, we say that the agreement satisfies such axiom.

Once the axioms of fairness are established, the challenge is to determine a scheme that ensures the fulfillment of such axioms. However, the literature states that it is not possible to guarantee the simultaneous satisfaction of all axioms. Thus, the literature proposes notions of fairness aiming to cover as many axioms as possible. Nash (1950) proposes the proportional fairness approach in which the percentage decrease in the cost of one firm is larger than the percentage increase in cost of the other firm. This approach satisfies Axioms 1 - 6. Given the objections of Kalai (1977) to the Axiom 6, the collaboration should be based on an alternative notion of fairness that satisfies Axiom 7 instead of Axiom 6. A notion satisfying Axioms 1 - 3, 5 and 7 is derived from the Rawls' theory of justice, in which the agreement prioritizes the firm that is least well off. The mathematical formulation of this notion corresponds to a max-min approach for the case of utility maximization problems (Kalai and Smorodinsky, 1975). Obviously, when the objective of the partners is to minimize the costs, the approach is of the min-max type. Note that, the Rawlsian notion fairness satisfies Axiom 5, that is a weak version of Axiom 4. The schemes of collaboration that we propose in this paper are based on the Rawlsian notion of fairness.

3.4 Risk Measures

We study collaborations in which the demand faced by each firm is uncertain when agreeing on a scheme of collaboration. Consequently, an agreement reached by the firms is subject to risk. Given the risk-averse nature of the firms, one of the challenges of the collaboration is to agree on a scheme that enables the firms to contain their risk.

If the firms would like to include the risk dimension in their decisions, the first step for them is to determine the way of measuring such risks. Artzner et al. (1999) propose a set of axioms that a risk measure might have. If a risk measure satisfies such axioms, then it is called a coherent risk measure. The concept of coherent risk measure has become common for problems of portfolio optimization. Our objective is to present a way to adapt those axioms to operations involving the collaboration of two firms. To this end we say that the set \mathcal{F} contains all the existing collaboration schemes.

Also, we say that a function $\rho : (\mathcal{F}, U(D)) \rightarrow \mathbf{R}^2$ measures the risk of a collaboration scheme, i.e. $\rho_i(F, u(D))$ represents the risk of firm i when a collaboration scheme $F \in \mathcal{F}$ is implemented for the bargaining problem $(u(D))$.

Note that, our measure of risk is based on the per unit production cost of the firms. To this end, we define the random variable resulting from the function $c : \Omega^G(D) \rightarrow \mathbf{R}^2$ that provides the unit cost of the firms, i.e. $c_i(X) = C_i(X)/D$. Accordingly to the notation for the cost of collaboration schemes, we will denote $c^F(u(D)) = C^F(u(D))/D$.

Axiom 8. *Monotonicity of risk.* If F and $F' \in \mathcal{F}$, and $c_i^F(u(d)) \leq c_i^{F'}(u(d))$, for any realization of the demand d , then $\rho_i(F, u(D)) \leq \rho_i(F', u(D))$.

This axiom establishes that if the unit cost of firm i under the collaboration scheme F is smaller than under the collaboration scheme F' , for all the possible scenarios, then the risk of firm i under F is smaller than under F' .

Axiom 9. *Positive homogeneity.* If F and $F' \in \mathcal{F}$, and $c_i^{F'}(u(d)) = k \cdot c_i^F(u(d))$ for any realization of demand d , then $\rho_i(F', u(D)) = k \cdot \rho_i(F, u(D))$, $\forall k > 0$.

The positive homogeneity states that if the unit cost of firm i resulting of the collaboration scheme F is doubled by the scheme F' , then the risk of firm i associated to F' is the double of the risk associated to F .

Axiom 10. *Translation invariance.* If F and $F' \in \mathcal{F}$ and $c_i^{F'}(u(d)) = k + c_i^F(u(d))$, for any realization d , then $\rho_i(F', u(D)) = k + \rho_i(F, u(D))$.

In general, the risk measure represents the amount of money that a firm has to add to its position in order to make it acceptable.

Axiom 11. *Risk subadditivity.* Let $J : \mathcal{F}^2 \rightarrow \mathcal{F}$. If $J(F, F')$ implies that if the planning decision is obtained through F with a probability p , and through F' with a probability $1 - p$, then $\rho_i(J(F, F'), u(D)) \leq p \cdot \rho_i(F, u(D)) + (1 - p) \cdot \rho_i(F', u(D))$, $\forall i$.

This axiom establishes that the risk of combining two collaboration schemes cannot get any worse than the weighted sum of the two risks separately. This axiom pursuits the diversification principle commonly used in portfolio problems, however, such principle has a limited compatibility with collaboration problems.

A widely used measure of risk in portfolio problems is the value at risk. This measure computes a threshold value for a given probability level α , such that the unit cost of a firm exceeds this value with a probability α . In the context of the bargaining problem $u(D)$, we define the value at risk of the collaboration scheme F as $VaR_\alpha(F, u(D))$, whose formulation is as follows:

$$VaR_{\alpha,i}(F, u(D)) = \inf \{c \in \mathbf{R} \mid P [c_i^F(u(D)) \leq c] \geq \alpha\} \quad (2)$$

The measure of risk $VaR_\alpha(F, u(D))$ satisfies Axioms 8 - 10, but it may not respect Axiom 11. Therefore, $VaR_\alpha(F, u(D))$ is not a coherent measure of risk. Further, $VaR_\alpha(F, u(D))$ presents a real disadvantage in measuring risk, because it omits the computation of the costs that exceed the threshold value, since the tail end of the distribution of costs is not assessed.

Alternatively, Rockafellar and Uryasev (2000) introduce the conditional value at risk, which avoids the shortcomings of $VaR_\alpha(F, u(D))$. This new measure is an extension of the $VaR_\alpha(F, u(D))$, but it measures the expected costs of the realizations in the tail of the percentile, and it satisfies the Axiom 11. Consequently, the conditional value at risk is a coherent risk measure. For the

bargaining problem $u(D)$, we say that $CVaR_\alpha(F, u(D))$ is the conditional value at risk of scheme F , whose formulation is as follows.

$$CVaR_{\alpha,i}(F, u(D)) = E [c_i^F(u(D)) | c_i^F(u(D)) \geq VaR_{i,\alpha}(F, u(D))] \quad (3)$$

Both, $VaR_\alpha(F, u(D))$ and $CVaR_\alpha(F, u(D))$, are useful for measuring the risk of collaboration in operations. Indeed, the firms can utilize such measures for determining the level of coordination between the operational and financial objectives of the firms.

4 Collaboration Schemes

As we discuss in section 2, it is in general easier for firms to agree on collaborations in which transfer payments are not necessary. In the absence of such payments, if the firms include the principles of fairness in their agreement, implementing the globally optimal decision may be infeasible. This relation between fairness and efficiency is the essential challenge of the collaboration, and therefore, the objective of the firms is to agree on a collaboration scheme that meets fairness criteria, while it avoids potential inefficiencies. In this section we discuss three collaboration schemes based on the different notions of fairness: first we describe the individual rationality scheme, which is common for problems of operations in collaborative environments; second, we extend the max-min normalized costs reduction proposed by Drechsel (2010); finally, we propose a new scheme that maximizes the minimum unit cost of the firms.

4.1 Individual Rationality Scheme

This scheme minimizes the sum of the costs of the firms, subject to the satisfaction of the axiom of individual rationality. The new feasible set of planning decisions is Ω^R , which contains the elements in Ω^G satisfying the condition established in expression (1). Therefore, given a demand realization d and the corresponding bargaining problem $u(d)$, the planning decision of the firm in a planning block is $X^R = \arg \min_{X \in \Omega^R(d)} \{e^T \cdot C(X)\}$.

It must be pointed out that the individual rationality scheme satisfies Axioms 1 and 3, which are directly related to the fairness dimension of the agreement. Quite apart from that, the firms can get advantages in terms of their risk when the agreement satisfies these axioms, this is summarized in Proposition 1. Let us henceforth denote $VaR_\alpha^o(u(D))$ and $CVaR_\alpha^o(u(D))$ to be the indicators of risk of the firms in the stand alone situation, and we introduce the following definitions.

Definition 2. $\mathcal{F}_S \subseteq \mathcal{F}$ is the set of all the collaboration schemes satisfying the Axiom of Symmetry. $\mathcal{F}_R \subseteq \mathcal{F}$ is the set of all the collaboration schemes satisfying the Axiom of Individual Rationality.

Proposition 1. Let Firm 1 and Firm 2 be two identical firms. If the collaboration scheme $F \in (\mathcal{F}_S \cap \mathcal{F}_R)$, then $VaR_\alpha(F, u(D)) \leq VaR_\alpha^o(u(D))$, $CVaR_\alpha(F, u(D)) \leq CVaR_\alpha^o(u(D))$, $VaR_{\alpha,1}(F, u(D)) = VaR_{\alpha,2}(F, u(D))$, and $CVaR_{\alpha,1}(F, u(D)) = CVaR_{\alpha,2}(F, u(D))$.

Proof. Since the firms are identical, if F satisfies the Axiom of Symmetry, then the probability distribution of the random variable $c_i^F(u(D))$ is identical for both firms. Therefore, $VaR_{\alpha,1}(F, u(D)) = VaR_{\alpha,2}(F, u(D))$, and $CVaR_{\alpha,1}(F, u(D)) = CVaR_{\alpha,2}(F, u(D))$.

If F satisfies the Axiom of Individual Rationality, then $c^F(u(d)) \leq c(X^o(C, d))$, $\forall d \in \mathcal{D}$. Consequently, $VaR_\alpha(F, u(D)) \leq VaR_\alpha^o(u(D))$ and $CVaR_\alpha(F, u(D)) \leq CVaR_\alpha^o(u(D))$. \square

From Proposition 1, the individual rational scheme (or any scheme satisfying Axioms 1 and 3) reduces the risk of identical firms in comparison to the stand-alone situation. Indeed, we can extrapolate Proposition 1 to problems involving more firms. So, the risk of the collaboration decreases with the number of participants if any sub-coalition of firms is not hurt by the agreement. Further, this Proposition states that the risk of the collaboration is identical for both firms. Nevertheless, the individual rational scheme is a limited way of dealing with the bargaining problem, and its implementation may not be fair in terms of the risk of the collaboration in comparison to other schemes. We present evidence of such weakness in Section 6.

4.2 Max-min Normalized Cost Reduction Scheme

We derive this scheme from the max-min notion of fairness related to the Rawls' 'A Theory of Justice'. In order to describe this scheme, we utilize $C_i(X^o(C, d))$ as benchmark for the costs of the firms, where $X^i(C, d) = \arg \min_{X \in \Omega^R(d)} \{C_i(X)\}$ is the minimum cost that firm i can get from the collaboration when the individual rationality constitutes a necessary condition of the agreement. $C_i(X^o(C, d)) - C_i(X^i(C, d))$ is thereby the maximum achievable savings of firm i . In this scheme, we focus on the proportion between the percentage of savings achieved by each firm and its maximum achievable reduction, what we measure by $\Phi(C, X, d) \in \mathbf{R}^2$, where $\Phi_i(C, X, d) = \frac{C_i(X^o(C, d)) - C_i(X)}{C_i(X^o(C, d)) - C_i(X^i(C, d))}$. The scheme aims to achieve equiproportional improvements between the firms. Given a realization of demand d , the formulation for deciding operations in each planning block is as follows.

$$\max_{\phi \in \mathbf{R}, X \in \Omega^G(d)} \{\phi \mid \phi \cdot e \leq \Phi(C, X, d)\}. \quad (4)$$

Note that expression (4) implies the minimization of ϕ , which is the minimum fraction obtained by any of the firms. Nevertheless, the solution obtained may not be pareto-optimal. With the aim of avoiding potential inefficiencies, the implementation of the scheme consists of two steps: first, we solve the problem (4), without loss of generality, we assume that $\phi = \Phi_1(C, X, d)$; second, we minimize the costs of Firm 2, but fixing the planning decisions for Firm 1.

This scheme satisfies Axioms 1 - 3, 5 and 7, and its objective is of the max-min type. Therefore, the scheme fits the Rawlsian notion of fairness. Nevertheless, the firms may go further by considering collaborative agreements based on other schemes. Particularly, the firms should pay particular attention to the measurements of their cost. Indeed, if the agreement is based on the Rawlsian notion of fairness and the firms measure their performance in terms of the unit cost, then the firms may reduce significantly their risk. We propose collaboration schemes based on this measure.

4.3 Min-max Unit Cost Scheme

Based on the Rawls' 'A Theory of Justice', a propose the min-max unit cost scheme. By contrast with the previous scheme, here the agreement implies the minimization of the maximum unit cost among firms, such that planning decisions satisfy the individual rationality. In other words, this scheme concentrates the efforts to help the firm facing the more critical situation in terms of its unit cost. The formulation for deciding operations in each planning block is as follows.

$$\min_{\psi \in \mathbf{R}, X \in \Omega^R(d)} \{\psi \mid \psi \cdot e \geq c(X)\}. \quad (5)$$

Expression (5) implies the minimization of ψ , which is the maximum unit cost of the firms resulting of collaboration. Here again, the optimal solution of the formulation may not be pareto-

optimal. Thus, the joint production plan is determined by a two step procedure similar to the Max-min normalized cost reduction scheme.

This scheme fits the Rawlsian notion of fairness, then it satisfies Axioms 1 - 3, 5 and 7. Note that, satisfying Axiom 5 helps to counter the absence of Axiom 4. Consequently, the outputs of the scheme are invariant to the unit used to measure the cost of both firms.

Since the scheme pursuits the minimization of the maximum unit cost of the firms in each production block, we could expect some side effects on the variability of the unit costs of the firms. Such reduction will imply that the value at risk and the conditional value at risk resulting from this scheme should be smaller than for the other schemes. In Section 6 we show through numerical experiments that this scheme overcomes other schemes in terms of the resulting risk of each company, while the efficiency of the agreement is not significantly damaged.

Note that, when firms can determine in advance natural differences in their costs per unit, the proposed formulation can be easily modified, without changing the notion of fairness. For instance, if the firms expect that the unit cost of the Firm 1 is k_c times higher than for the Firm 2, we replace expression 5 by $\min_{\psi \in \mathbf{R}, X \in \Omega^R(d)} \{\psi \mid \psi \geq c_1(X), \psi \geq k_c \cdot c_2(X)\}$.

5 The Collaborative Lot Sizing Problem

Even though our work can be applied to any collaboration with the previously described characteristics, we will now demonstrate how the collaboration schemes presented above can be implemented for the case of production lines that are modelled as a *CLSP*. Thus, there is a holding cost for units produced in advance of demand and a back-order cost for units produced late. Also, there are fixed production costs each period a production line is used and an unit production cost. Finally, we suppose that demand is known with enough accuracy to build a reliable production plan for a planning block consisting of T periods in advance.

As mentioned earlier, we are seeking to set up an agreement that does not involve transfer payments. As a consequence, the imputation of the costs plays an important role in the agreement. Also, the aim of the agreement we are studying is to pool the production activities while keeping the firms as independent as possible. We reflect this in our model as follows:

- each firm supports the holding and back-ordering costs for its products,
- the fixed production costs are supported by the owner of the production line, independently of the firm for whom the products are made,
- for the unit production costs we consider two cases, either these costs are supported by the owner of the line, independently of the product, or this cost is supported by the firm for whom the product is made. The first case would correspond to a situation where the production costs are difficult to isolate (workforce drawn from the common pool of the plant, utilities where it might not be easy to determine the real cost of steam, energy, etc.). The second case will be more adequate if the direct production costs for the jointly operated line are easily determined. The general idea is that our methodology is flexible in terms of the firms' imputation of unit production costs.

We propose the following notations for our analysis:

\mathcal{H}_n : set of periods of the n^{th} planning block

$d_{i,t}$: demand faced by firm i for period t

$f_{i,t}$: fixed production cost for the production line of firm i in period t

$p_{i,t}$: unit production cost of firms i 's line in period t

$h_{i,t}$: unit storage cost of firm i in period t

$b_{i,t}$: unit back-ordering cost of firm i in period t

$CAP_{i,t}$: production capacity of firm i 's line in period t

The decision variables of the problem are as follows:

$x_{i,j,t}$: amount produced on firm j 's line in period t , which is destined to firm i

$y_{i,t}$: 1 if firm i 's production line is in use during period t ; 0 otherwise

$s_{i,t}$: stock level of firm i at the end of period t .

$u_{i,t}$: back-ordered units of firm i at the end of period t .

We define $CLSP^G$ as the problem of minimizing the sum of the costs of Firm 1 and Firm 2 when the production lines are pooled. The formulation of this problem is the following:

$$\min \sum_{i=1,2} \sum_{t \in \mathcal{H}_n} [f_{i,t} \cdot y_{i,t} + p_{i,t} \cdot (x_{i,1,t} + x_{i,2,t}) + h_{i,t} \cdot s_{i,t} + b_{i,t} \cdot u_{i,t}] \quad (6)$$

$$s_{i,t-1} + x_{1,1,t} + x_{1,2,t} + u_{i,t} = d_{i,t} + s_{i,t} + u_{i,t-1} \quad \forall t \in \mathcal{H}_n; i = 1, 2 \quad (7)$$

$$x_{1,i,t} + x_{2,i,t} \leq CAP_{i,t} \cdot y_{i,t} \quad \forall t \in \mathcal{H}_n; i = 1, 2 \quad (8)$$

$$x_{i,j,t} \geq 0, y_{i,t} \in \{0, 1\} \quad \forall t \in \mathcal{H}_n; i, j = 1, 2 \quad (9)$$

The objective function (6) is the sum of all cost incurred by both firms. Constraint (7) establishes the product flow conservation and constraint (8) is the capacity restriction in each period.

We extend the notation X , X^o , X^* , Ω^G and Ω^I to the $CLSP^G$ faced by the firms during the n^{th} planning block. For simplicity we omit the suffix n . Thus, $X \in \mathbf{R}^2 \times \mathbf{R}^T \times \mathbf{R}^T$ represents the planning decisions in a planning block, where $X_{i,j,t} = x_{i,j,t}$ (note that, the variables $y_{i,t}$, $s_{i,t}$ and $u_{i,t}$ can be deduced from X). We calculate the cost of each firm by the operator $C : \mathbf{R}^2 \times \mathbf{R}^T \times \mathbf{R}^T \rightarrow \mathbf{R}^2$, whose functional form depends on the assumptions about the imputations of the unit production cost of the firms: $C_i(X) = \sum_{t \in \mathcal{H}_n} [f_{i,t} \cdot y_{i,t} + p_{i,t} \cdot (x_{1,i,t} + x_{2,i,t}) + h_{i,t} \cdot s_{i,t} + b_{i,t} \cdot u_{i,t}]$, if the unit production cost is linked to the production line; $C_i(X) = \sum_{t \in \mathcal{H}_n} [f_{i,t} \cdot y_{i,t} + p_{i,t} \cdot (x_{i,1,t} + x_{i,2,t}) + h_{i,t} \cdot s_{i,t} + b_{i,t} \cdot u_{i,t}]$, if the unit production cost is supported by the firm to whom the product is destined. Despite the cost imputation, constraints (7)-(9) characterize the set Ω^G , and X^* corresponds to the optimal solution of the $CLSP^G$. The set Ω^I consists of the elements of Ω^G satisfying the constraint $x_{i,j,t} = 0, \forall t \in \mathcal{H}_n, i = 1, 2, i \neq j$. Therefore, we obtain X^o by including such constraint in the formulation of the $CLSP^G$.

6 Numerical Experiments

The main goal of our numerical experiments is to investigate the schemes introduced in Section 4 in terms of their efficiency, fairness and risk. Also, we aim to determine the characteristics of the firms which are potentially more advantageous when pooling production lines, and we study the effect

of the unit production cost imputations on the results of the collaboration. Finally, we analyse the sensitivity of the efficiency of the collaboration with the aim to understand the source of the gains of the collaboration and to identify the characteristics of the partners which are potentially more advantageous for collaboration.

We analyse the results for a series of instances of the collaboration problem, where an instance represents a specific set of parameters of the $CLSP^G$. We use the following assumptions in order to build each instance of the problem:

- The firms are identical, i.e., the cost parameters and the distribution of their demands are identical.
- The fixed cost, direct production cost, holding cost and back-ordering cost of firm i do not vary with periods, i.e. $f_{i,t} = f^i, p_{i,t} = p^i, h_{i,t} = h^i, b_{i,t} = b^i, \forall t$.
- Each firm has a fixed production capacity per period, i.e. $CAP_{i,t} = CAP^i, \forall i, t$.
- The capacity tightness of firm i (CT_i) is the ratio between the expected demand per period of this firm (\bar{d}_i) and the production capacity per period, i.e. $\bar{d}_i = CT_i \cdot CAP^i$.
- $D_{i,t}$ is the random variable representing the demand faced by firm i in a production period. $D_{i,t} \sim \Gamma(\frac{\bar{d}_i}{CV_i^2}, \cdot CV_i^2)$, where CV_i is the coefficient of variation of $D_{i,t}$.
- The length T of the planning block is identical for both firms.

Each instance is constructed with different demand values that are randomly generated using the parameters previously described. Our analysis distinguishes between different demand loads of the system, thus three scenarios of capacity tightness are considered: low, medium and high demand load ($CT = 0.2, 0.5, 0.8$, respectively). We take the second column of the Table 1 as the base scenario of our experiments, and we perform a full factorial experiment by testing all combinations of parameters of the third column of Table 1. For each parameter set 2,000 instances are generated with different demands in order to obtain reliable values for the studied measures of efficiency, fairness and risk. Consequently, 120 parameter combination are studied involving the resolution of $120 \cdot 2,000 = 240,000$ instances of the problem.

Parameter	Base Set	Range of Values
T	10	$\{1, 2, \dots, 10\}$
f	100	100
p	$p = f \cdot k_p, k_p = 0.1$	$k_p = \{0.1, 0.2, \dots, 1.0\}$
h	$h = p \cdot k_h, k_h = 1.5$	$k_h = \{0.5, 1.0, \dots, 5.0\}$
b	$b = 2 \cdot h$	$b = 2 \cdot h$
CT	$\{0.2, 0.5, 0.8\}$	$\{0.2, 0.5, 0.8\}$
CV	0.4	$\{0.2, 0.4, \dots, 2.0\}$

Table 1: Parameters of the Problem.

We denote by R , N and A , the collaboration schemes individual rationality, max-min normalized cost reduction and min-max unit cost, respectively. Also, G and I denote the problem of minimizing the sum of the costs of the firms and the stand-alone situations, respectively. In terms of the cost structure of the firms, we denominate $CS1$ the cost structure in which the unit production cost is supported by the firm for whom the product is made and $CS2$ the cost structure in which the unit production cost is linked to the production lines.

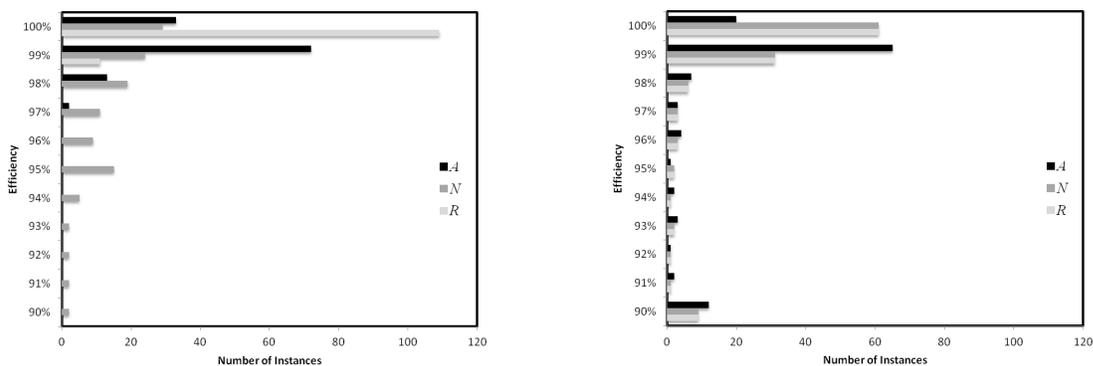
6.1 Efficiency.

The formulations of the collaboration schemes allow us to make some conclusions about their efficiencies without the need of simulations. For instance, the collaboration scheme R leads to operations which are at least as efficient as the ones obtained by implementing the schemes N or A , however, we cannot conclude about the relative efficiency of these two schemes beforehand. With this in mind, our simulations aim to provide evidence about both the relative efficiency of schemes N or A , and also, the relative efficiencies of the different schemes for different cost structures.

Figure 1 summarizes the observed efficiencies for the instances considered. From these results, the high efficiency of schemes R , N and A is evident. In fact, the efficiencies of such schemes are always greater than 90.0% when the cost structure $CS1$ is considered, however, the efficiency decreases under the cost structure $CS2$. The different level of efficiency between $CS1$ and $CS2$ is explained by the link between the unit production cost and the production lines in both cost structures. For instance when $CS1$ is considered, the unit production costs of the firms are independent of the production lines. Therefore, if the two firms produce in the same period in the stand-alone situation, then pooling the lines helps to reduce one the fixed costs of the firms by assigning units from one production line to the other. Even though the units are produced by only one of the lines, the unit production cost is still allocated to the firm for whom the product is made, hence no firm increases its cost while the efficiency may be not affected. When collaboration considers $CS2$ the unit production costs are associated to the production lines. Therefore, if the units to be produced by Firm 1 in the stand-alone situation are reallocated to another line, then Firm 1 reduces its cost, but this cost is now allocated to the Firm 2. Thus, under $CS2$ the global optimality can lead difficulties in reaching an equilibrium between the costs of the firms.

Another important observation in terms of the efficiency of the agreements is the impact of the length of the planning blocks. The instances with low efficiency are characterized by short planning blocks (e.g. $T = 1$), then the collaboration is done on a extremely myopic basis, and the collaboration schemes provide operations similar to the stand-alone situation.

In order to measure the relative difference between the efficiencies of schemes N and A , we track the number of instances in which one scheme outperforms the other one. When the firms carry out the collaboration under the cost structure $CS1$, the scheme A outperforms scheme N in 115 instances. We observe an opposite tendency when the cost structure $CS2$ is considered. In such case, the scheme A outperforms the scheme N in only 29 instances.



(a) Cost Structure $CS1$.

(b) Cost Structure $CS2$.

Figure 1: Observed Cost Efficiency of Different Collaboration Schemes.

6.2 Fairness

As expected, the collaboration between identical firms under any of the schemes considered leads to unit costs identically distributed between firms. Nevertheless, through simulation we can study the disparities of the results between firms in each planning block. In Figure 2 and Figure 3 the dark line represents the minimum between the observed unit cost of the firms in a planning block, and the grey dots represent the cost associated to the other firm for the corresponding planning block. As we can observe in both figures, the collaboration scheme G and R are characterized by a high difference in the relative unit costs during a planning block. The collaboration scheme M , and particularly the collaboration scheme A , reduce drastically this difference. This effect is explained by the lower variability of the unit costs when the collaboration scheme A is implemented. Note that, if the firms collaborate under the cost structure $CS2$, the advantages of the collaboration scheme A are more significant. Such effect occurs because of the low divisibility of the costs when firms collaborate under the cost structure $CS2$. Thus, the globally optimal solution may bring a larger disparity to the result of the firms, however, such disparity can be reduced by the collaboration scheme A .

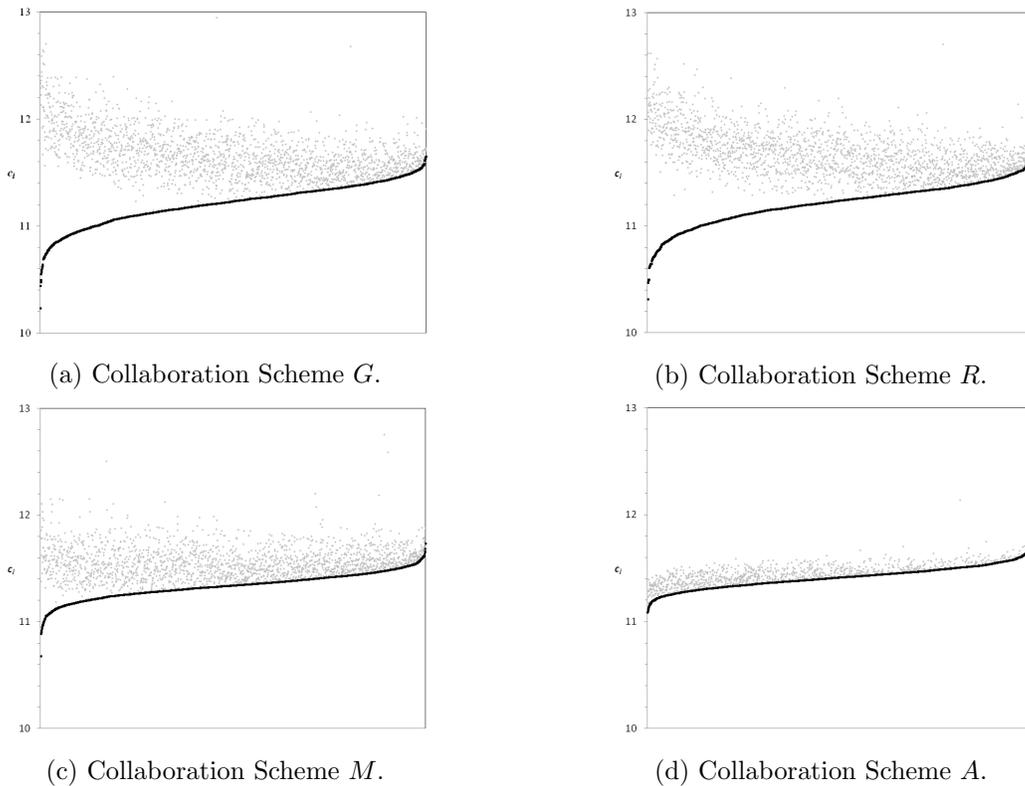


Figure 2: Observed Unit Cost per Planning Block. Cost Structure $CS1$ and $CT = 0.5$.

6.3 Risk of the collaboration.

Figure 2 and Figure 3 also provide evidence about the risk of each firm associated to the different collaboration schemes. We can observe that the collaboration scheme A has associated a lower risk of having higher costs in comparison to other schemes. Nevertheless, as we discuss in Section 3, firms

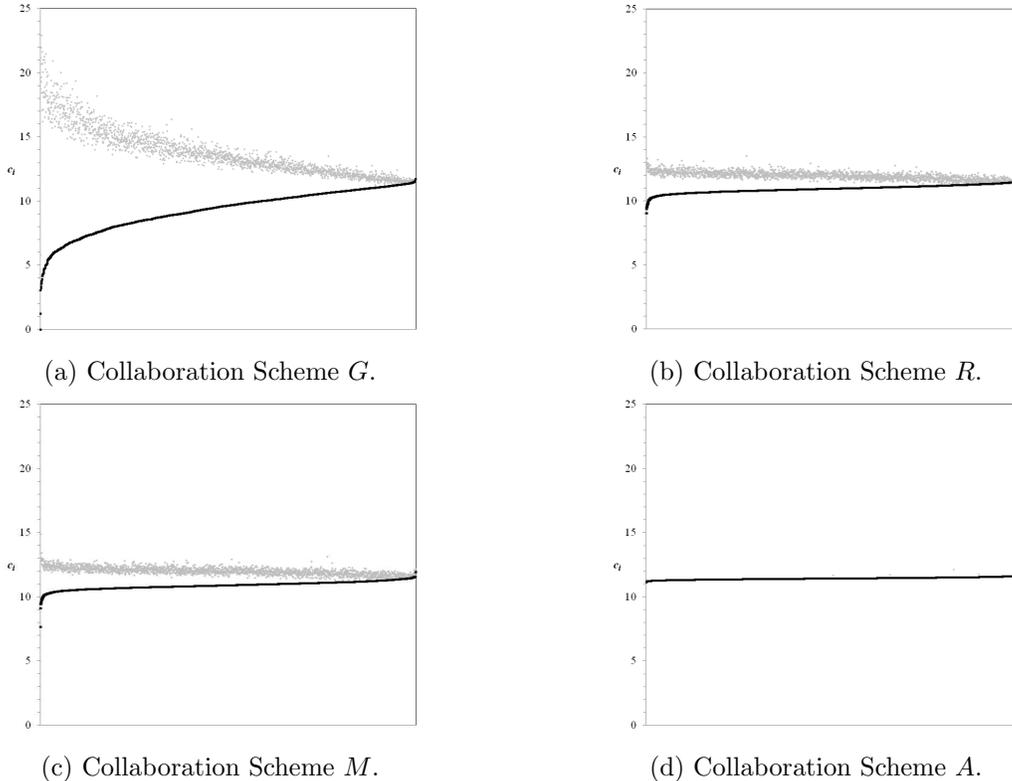
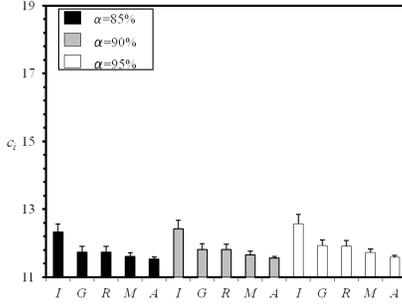


Figure 3: Observed Unit Cost per Planning Block. Cost Structure $CS2$ and $CT = 0.5$.

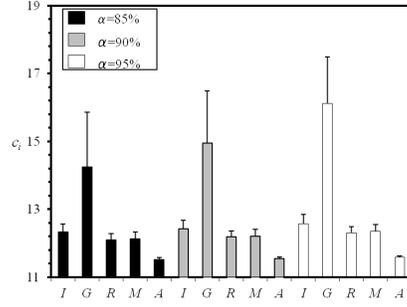
must consider an adequate indicator for measuring their individual risk, e.g. the value at risk or the conditional value at risk. We calculate the values of such indicators for different collaboration schemes and different levels of risk (parameter α) for the instances considered of Figure 2 and Figure 3. These results are summarized in Figure 4, in which the bars represent the observed value at risk for a certain value of α and the whiskers show the corresponding observed conditional value at risk. Figure 4 illustrates the advantages of the collaboration scheme A , since both risk indicators are lower under this scheme. Note that, we have considered values of $\alpha > 0.8$, which are commonly used values for measuring risk. Moreover, here again the cost structure $CS2$ brings more difficulties for implementing the collaboration, since the globally optimal operation of the firms imply higher level of risk in comparison to the cost structure $CS1$. Nevertheless, collaboration scheme A helps to reduce significantly such risk.

Furthermore, we are interested to study how the risk associated to different collaboration schemes changes with the parameters of the problem. We present two examples about such relation in Figure 5. In these figures the bars represent the observed value at risk for $\alpha = 95\%$, the whiskers show the corresponding observed conditional value at risk, and the white dots are the expected unit costs.

In general, the values of both risk indicators are much lower when the collaboration scheme A is implemented. The exception occurs for the instances in which the demand has a high variability. In such case, the operations obtained through G are less risky, but as we discussed earlier, the implementation of the collaborations disregarding the fairness dimension of the problem may bring additional stress in terms of the relative results of the firms. If we compare schemes addressing the fairness dimension of the problem, we can observe the dominance of scheme A over R and N .



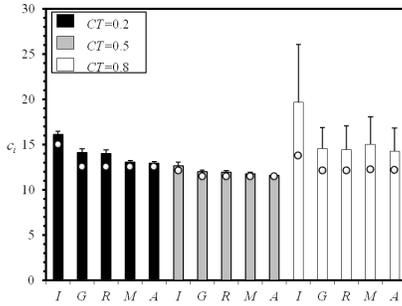
(a) Value at risk.



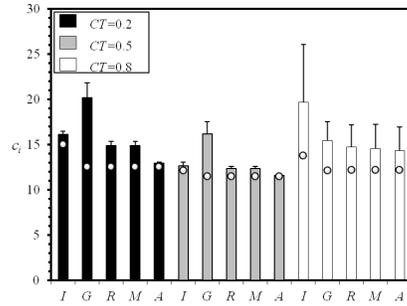
(b) Conditional value at risk.

Figure 4: Observed Risk Measures for Different Collaboration Schemes and $CT = 0.5$.

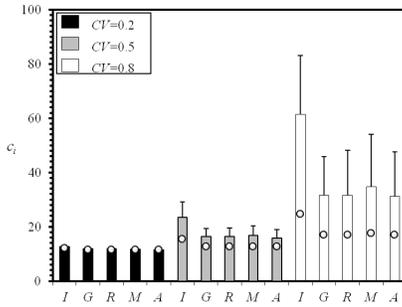
From Figure 5 we additionally get some evidence about the relation between the magnitude of the savings and the risk of the collaboration. For instance, when the variability of the demand is higher, the collaboration implies larger cost reductions, but at the same time, the risk of the collaboration is much higher. Therefore, considering a scheme that controls the risk of the agreement is essential for implementing the collaboration between firms.



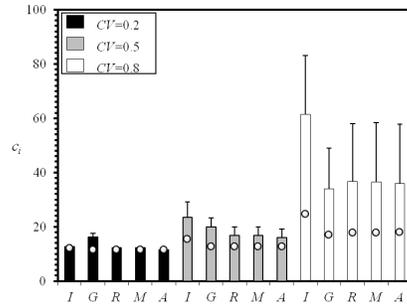
(a) Different Values of CT . Cost structure $CS1$.



(b) Different Values of CT . Cost structure $CS2$.



(c) Different Values of CV . Cost structure $CS1$.



(d) Different Values of CV . Cost structure $CS2$.

Figure 5: Observed Unit Cost for Different Collaboration Schemes.

6.4 Source of the savings of the collaboration

We continue our analysis by studying the impact of each parameter on the maximum savings that the collaboration can provide in comparison to the stand alone situation. Such analysis has two objectives. On the one hand, we study the characteristics of the firms that yield more advantages

when pooling production lines. On the other hand, we aim to understand the source of the gains when operations are pooled. In particular, we simulate the proportion of savings resulting from the economies of scale of the fixed production costs. In order to measure such proportion we utilize the indicator $FC = \frac{FC^o - FC^*}{C^o}$, where FC^o and FC^* are the total global fixed cost associated to G and I , respectively. On the one hand, if $FC \approx S$ the savings of the collaboration come mainly from the economies of scale of the fixed costs. On the other hand, if $FC \ll S$ such savings come from the reduction of stocks and back-orders. The observed values for different parameters of the problem are summarized in Tables 2. In such tables each cell contains two values: the left side value corresponds to S , and the right side value is FC .

Impact of the capacity tightness: Tables 2 provide evidence about the inverse relation between the capacity tightness of the firms and FC . Further, such tables also show that the magnitude of the savings of the collaboration depends on FC . The reasons of such phenomena are as follows:

- If the capacity tightness is medium or low, the capacity available for production is large in comparison to the demand of each period. Therefore, in the stand alone situation, each firm carries out its operations by producing in large batches (when the fixed production costs are large) or by a low utilization of the capacity in each period (holding and back-ordering costs are large). When the operations are pooled, the firms use the slack of capacity to reduce their fixed production costs.
- If the capacity tightness is high, the stand-alone situation of each firm is characterized by operations leading to high utilization of the production lines and large amounts of holding and back-ordering units. When the operations are pooled, there are difficulties in consolidating the batches of both firms in a single production line, because the high load of demand. Thus, the firms use the slack of capacity to reduce their holding and back-ordering costs.

Impact of the planning block length: In general, the savings resulting from collaboration are independent of the length of each planning block, see Table 2a. Note though that when the utilization rate is high the horizon needs to be sufficiently long for pooling opportunities to exist.

Impact of fixed and unit production costs: Based on the results of Table 2b, the savings of the collaboration have an inverse relation with k_p . In other words, the proportion between the unit operational costs and the fixed production costs has a negative effect on the savings of the firms, e.g., if the fixed costs are relatively larger than the unit production costs, then there are more opportunities for reducing costs by pooling lines. We can clearly observe such phenomenon when the capacity tightness is low or medium, since the main savings lie in the economies of scale of the fixed production costs. In contrast, when k_p is larger, the fixed production costs are less dominant for the firms.

Impact of holding and back-ordering costs: In general, the savings that firms can get from the collaboration tend to grow with the value of the holding and back-ordering costs, see Table 2c. We analyse such results for different levels of capacity tightness:

- If the capacity tightness is high, there is a direct relation between the savings of the collaboration and k_h . Such relation results from the fact that higher values of k_h imply larger holding and back-ordering costs for the firms in the stand-alone situation. This phenomenon arises because the operations of the firms involve a level of stocks and back-orders larger than if the lines are pooled.
- If the capacity tightness is medium, the savings resulting from the collaboration are independent of the value of k_h . For this level of demand the magnitude of k_h does not influence the

T	$CT = 0.2$		$CT = 0.5$		$CT = 0.8$	
1	17.3	16.1	6.1	5.8	5.0	1.2
2	16.9	18.2	5.9	5.5	6.7	0.7
3	16.9	17.2	5.7	5.5	7.7	0.6
4	16.8	17.2	5.6	5.5	8.6	0.6
5	16.7	16.6	5.6	5.4	9.1	0.5
6	16.7	16.8	5.4	5.2	9.6	0.5
7	16.7	16.9	5.4	5.2	9.4	0.5
8	16.7	16.7	5.5	5.2	9.4	0.5
9	16.7	16.5	5.5	5.3	9.2	0.5
10	16.7	16.5	5.5	5.3	10.0	0.5

(a) Different Values of T .

k_p	$CT = 0.2$		$CT = 0.5$		$CT = 0.8$	
0.1	16.7	16.5	5.5	5.3	9.7	0.4
0.2	10.1	10.1	3.2	2.7	10.0	0.2
0.3	7.2	7.2	2.4	1.8	10.3	0.1
0.4	5.6	5.6	1.9	1.4	10.1	0.1
0.5	4.6	4.6	1.7	1.1	10.5	0.1
0.6	3.9	3.9	1.5	0.9	10.1	0.1
0.7	3.4	3.4	1.4	0.8	10.5	0.1
0.8	3.0	3.0	1.3	0.7	10.4	0.1
0.9	2.6	2.7	1.3	0.6	10.6	0.0
1.0	2.4	2.4	1.2	0.6	10.2	0.0

(b) Different Values of k_p .

k_h	$CT = 0.2$		$CT = 0.5$		$CT = 0.8$	
0.1	12.7	5.8	5.1	5.2	4.1	0.7
0.2	16.3	14.5	5.4	5.5	7.4	0.6
0.3	16.7	16.5	5.5	5.3	10.0	0.5
0.4	16.8	16.8	5.5	5.1	12.2	0.4
0.5	16.7	16.9	5.7	5.0	14.2	0.4
0.6	16.8	16.9	5.9	4.9	16.6	0.4
0.7	16.7	16.8	5.9	4.9	18.1	0.3
0.8	16.7	16.8	6.2	4.8	19.1	0.3
0.9	16.7	16.9	6.2	4.8	20.5	0.3
1.0	16.7	16.9	6.4	4.8	22.2	0.3

(c) Different Values of k_h .

CV	$CT = 0.2$		$CT = 0.5$		$CT = 0.8$	
0.2	16.7	16.9	5.0	5.6	1.9	0.0
0.4	16.7	16.6	5.4	5.2	9.8	0.5
0.6	16.0	14.4	8.4	5.0	17.5	0.8
0.8	14.7	11.9	12.7	4.2	21.2	0.9
1.0	13.5	9.9	16.3	3.4	23.8	0.8
1.2	12.6	8.0	19.9	2.6	26.6	0.7
1.4	12.5	6.6	22.5	2.1	28.5	0.6
1.6	13.1	5.6	25.0	1.6	28.2	0.5
1.8	14.4	4.8	25.8	1.4	29.1	0.4
2.0	15.4	4.1	28.0	1.1	29.2	0.4

(d) Different Values of CV .

Table 2: Observed Savings of the Collaboration

optimal batch sizes of the operations in the stand-alone situations. Then, the reduction of costs resulting from pooling the production lines do not vary significantly with h .

- If the capacity tightness is low, there is a significant effect of k_h on the savings of the collaboration. In the the stand-alone situation, the operations of each firm prioritize higher utilization of capacity per period instead of additional stocks or back-orders. This phenomenon is more prominent when the value of k_h is increased. Therefore, pooling the production lines brings significant savings, which arise from the economies of scale of the fixed production costs, and consequently, the impact of the value of the holding and backordering costs is less significant.

Impact of the coefficient of variation of the demand: Table 2d shows how the savings resulting from the collaboration depend on the variability of the demand of the firms. Since there is a direct relation between the value of CV and the amount of stocks and back-orders of the firms, we explain the savings of the collaboration based on the capacity tightness of the system.

- If the capacity tightness is medium or high, planning the operations in an isolated way involves larger amounts of stock and back-orders as the value of CV grows. When operations are pooled, the firms use the available capacity to reduce these stocks and back-orders, and consequently, the magnitude of the reduction of costs associated to such units increases with CV .
- When the capacity tightness is low, larger values of CV imply less frequent demands, so the plan of the operations of the firms considers that the production lines are used during

few periods. Given that the economies of scale of the fixed production costs are the main source of savings when the demand is low, there is an inverse relation between FC and CV . Nevertheless, this relation is weak, because of the presence of large amounts of holding and backorder units when CV is high.

7 Summary and Conclusions

In this paper we studied the collaboration resulting from pooling the operations of independent firms. The challenge is to determine a collaboration scheme such that the gains of each firm surpass the implementation effort of the agreement. Even though such gains are attractive for the firms, these firms may also expect that the costs are divided fairly among them.

Our main contribution is to propose collaboration schemes which are flexible in terms of the existence, or not, of transfer payments between firms. This flexibility is in tune with the infeasibility of such payments in many real problems. The absence of payments generates a trade-off between the efficiency and the fairness of an agreement, because pursuing balanced costs between partners restricts the space of operations that can be implemented by the firms. Hence, there is a direct link between the operations management and the collaboration scheme agreed by the firms.

We propose a collaboration scheme which determines the joint operations of the firms by implementing a Rawlsian notion of fairness, such that the maximum unit production costs of the partners is minimized. Our numerical results provide evidence that this approach leads to joint operations in which the production costs of the firms are more balanced than with other notions of fairness. Although fairness may reduce the efficiency of the collaboration, implementing our scheme is still worthy for the firms, because the efficiency of the agreement is not significantly hurt compared to the global optimization.

The numerical tests we carried out show also link between fairness and variability of the costs achieved by the firms. Although, the firms may expect that the collaboration will reduce the variability in comparison to their stand alone situation, such reduction will be more significant when the scheme of collaboration involves more advanced notions of fairness. Hence, our scheme outperforms other schemes in terms of the variability in costs. This result may be of interest for firms that make decisions under uncertainty, or when they make decisions repeatedly. In such cases, implementing our model represents not just an efficient way of ensuring fairness, but the firms can decrease the risk of their operations.

Modelling the collaboration in the absence of transfer payments, the inclusion of fairness in the agreement and the effects on variability of the costs of the firms, are three elements that ease the implementation of theoretical bargaining models in real applications. Indeed, when firms discuss a potential agreement, our work can guide the decision making process in order to keep a successful partnership.

A research avenue to extend our work is the implementation of the proposed scheme to horizontal collaboration problems involving more than two firms. In such case, we expect that the gains obtained would not be very different, but other elements related to the stability of the coalition should be studied. Further, our work focuses on inventory-production problems, it would be interesting to analyse the proposed scheme in other settings where the firms can benefit from the economies of scale resulting of a collaboration. Finally, it is interesting to explore the collaboration in fully-decentralized systems under asymmetric information.

References

- Ahmed, S., Cakmak, U., Shapiro, A., 2007. Coherent risk measures in inventory problems. *European Journal of Operational Research* 182, 226–238.
- Anily, S., Haviv, M., 2007. The cost allocation problem for the first order interaction joint replenishment model. *Operations Research* 55, 292–302.
- Artzner, P., Delbaen, F., Eber, J., Heath, D., 1999. Coherent measures of risk. *Mathematical Finance* 9, 203–228.
- Aumann, R., 1961. The core of a cooperative game without side payments. *Transactions of the American Mathematical Society* 98, 539–552.
- Bertsimas, D., Farias, V., Trichakis, N., 2011. The price of fairness. *Operations Research* 59, 17–31.
- Bjornfot, A., Torjussen, L., 2012. Extent and effect of horizontal supply chain collaboration among construction sme. *Journal of Engineering, Project, and Production Management* 2, 47–55.
- Boiney, L., 1995. When efficient is insufficient: fairness in decision affecting a group. *Management Science* 41, 1523–1537.
- Borm, P., Keiding, H., McLean, P., Oortwijn, S., Tijs, S., 1992. The compromise value for ntu-games. *International Journal of Game Theory* 21, 175–181.
- Brahimi, N., Dauzere-Peres, S., Najid, N. M., Nordli, A., 2006. Single item lot sizing problems. *European Journal of Operational Research* 168, 1–16.
- Brams, S., Taylor, A., 1995. On envy-free cake division. *Journal of Combinatorial Theory* 70, 170–173.
- Brandenburgger, A., Nalebuff, B., 1996. *Co-opetition*. Doubleday. New York.
- Camerer, C., Thaler, R., 2010. Anomalies - ultimatums, dictators and manner. *Journal of Economic Perspectives* 9, 209–2192.
- Chen, Y., Minghui, X., Zhang, Z., 2009. A risk-averse newsvendor model under the cvar criterion. *Operations Research* 57, 1040–1044.
- Choi, P., Garcia, R., Friedrich, C., 2010. The drivers for collective horizontal cooperation: a case study of screwcap initiatives in the international wine industry. *International Journal of Strategic Business Alliances* 1, 271–290.
- Choi, S., Ruszczyński, A., 2008. A risk-averse newsvendor with law invariant coherent measures of risk. *Operations Research Letters* 36, 77–82.
- Corbett, C., de Groote, X., 2000. A supplier's optimal quantity discount policy under asymmetric information. *Management Science* 46, 444–450.
- Crujissen, F., Cools, M., Dullaert, W., 2007. Horizontal cooperation in logistics: Opportunities and impediments. *Transportation Research Part E* 43, 129–142.
- Drechsel, J., 2010. *Cooperative lot sizing problems in supply chain*. Springer-Verlag Berlin Heidelberg.

- Drechsel, J., Kimms, A., 2011. Cooperative lot sizing with transshipments and scarce capacities: solutions and fair cost allocations. *International Journal of Production Research* 49, 2643–2668.
- Erhum, F., Keskinocak, P., 2007. In: Kempf, K., Keskinocak, P., Uzsoy, R. (Eds.), *Handbook of production planning*. Kluwer International Series in Operations Research and Management Science. New York: Kluwer, Ch. Collaborative supply chain management.
- Federgruen, A., Zheng, Y., 1992. The joint replenishment problem with general joint cost structures. *Operations Research* 40, 384–403.
- Fiestras-Janeiro, M., García-Jurado, I., Meca, A., Mosquera, M., 2011. Cooperative game theory and inventory management. *European Journal of Operational Research* 210, 459–466.
- Fishburn, P., Sarin, R., 1994. Fairness and social risk i: unaggregated analyses. *Management Science* 40, 1174–1188.
- Frisk, M., Gothe-Lundgren, M., Jornsten, K., Ronnqvist, M., 2010. Cost allocation in collaborative forest transportation. *European Journal of Operational Research* 205, 448–458.
- Gillies, D., 1959. Solutions to general non-zero sum games. Princeton University Press, Princeton, in A. W. Tucker, R. D. Luce (Eds.) *Contribution to the theory of games IV*.
- Granot, D., Sobic, G., 2005. Formation of alliances in internet-based supply exchanges. *Management Science* 51, 92–105.
- Guardiola, L., Meca, A., Puerto, J., 2009. Production-inventory games: A new class of totally balanced combinatorial optimization games. *Games and Economic Behavior* 65, 205–219.
- Jain, K., Mahdian, M., 2007. In: Nisan, N., Roughgarden, T., Tardos, V., Vazirani, V. (Eds.), *Algorithmic game theory*. Cambridge University Press, Cambridge, Ch. Cost Sharing.
- Jap, D. S., 1999. Pie expansions efforts: Collaboration processes in buyer-supplier relationships. *Journal of Marketing Research* 36, 461–475.
- Jap, D. S., 2001. ‘Pie sharing ’in complex collaboration contexts. *Journal of Marketing Research* 37, 86–99.
- Kalai, E., 1977. Proportional solutions to bargaining situations: interpersonal utility comparisons. *Econometrica* 45, 1623–1630.
- Kalai, E., Smorodinsky, M., 1975. Other solutions to nash’s bargaining problem. *Econometrica* 43, 513–518.
- Karimi, B., Fatemi Ghomia, S., Wilson, J., 2003. The capacitated lot sizing problem: a review of models and algorithms. *Operations Research* 43, 264–281.
- Makadok, R., 2010. The interaction effect of rivalry restraint and competitive advantage on profit : Why the whole is less than the sum of the parts. *Management Science* 56, 356–372.
- Meca, A., Timmer, J., García-Jurado, I., Borm, P., 2004. Inventory games. *European Journal of Operational Research* 156, 127–139.
- Myerson, R., 1991. *Game theory: Analysis of conflict*. Harvard University Press, Cambridge, Massachusetts.

- Nash, J., 1950. The bargaining problem. *Econometrica* 18, 155–162.
- Nydegger, R., Owen, G., 1974. Two-person bargaining: an experimental test of the nash axioms. *Int. Journal of Game Theory* 3, 239–249.
- Rawls, J., 1971. *A theory of justice*. Harvard University Press, Cambridge, MA.
- Rockafellar, R., Uryasev, S., 2000. Optimization of conditional value-at-risk. *Journal of Risk* 2, 21–41.
- Sambasivan, M., Yahya, S., 2005. A lagrangean-based heuristic for multi-plant, multi-item, multi-period capacitated lot-sizing problems with inter-plant transfers. *Computers and Operations Research* 55, 537–555.
- Shapley, L., 1959. A value for n-person games. Princeton University Press, Princeton, in A. W. Tucker, H. W. Kuhn (Eds.) *Contribution to the theory of games II*.
- Simatupang, T., Sridharan, R., 2002. The collaborative supply chain. *International Journal of Logistics Management* 13, 289–308.
- Stadtler, H., 2009. A framework for collaborative planning and state-of-the-art. *OR Spectrum* 31, 5–30.
- van den Heuvel, W., Borm, P., Hamers, H., 2007. Economic lot-sizing games. *European Journal of Operational Research* 176, 1117–1130.
- von Neumann, J., Morgenstern, O., 1943. *Theory of games and economic behavior*, 2nd Edition. Princeton University Press.
- Wolsey, L. A., 1995. Progress with single-item lot-sizing. *European Journal of Operational research* 86, 395–401.

Recent titles

CORE Discussion Papers

- 2013/35 Kristof BOSMANS, Koen DECANCO and Erwin OOGHE. What do normative indices of multidimensional inequality really measure?
- 2013/36 Alain PHOLO BALA, Dominique PEETERS and Isabelle THOMAS. Spatial issues on a hedonic estimation of rents in Brussels.
- 2013/37 Lionel ARTIGE, Antoine DEDRY and Pierre PESTIEAU. Social security and economic integration.
- 2013/38 Nicolas BOUCKAERT and Erik SCHOKKAERT. Differing types of medical prevention appeal to different individuals.
- 2013/39 Pierre M. PICARD. Trade, economic geography and the choice of product quality.
- 2013/40 Tanja B. MLINAR and Philippe CHEVALIER. Pooling in manufacturing: do opposites attract?
- 2013/41 Chiara CANTA and Marie-Louise LEROUX. Public and private hospitals, congestion, and redistribution.
- 2013/42 Mathieu LEFEBVRE, Pierre PESTIEAU and Gregory PONTIERE. FGT poverty measures and the mortality paradox: Theory and evidence.
- 2013/43 Nada BELHADJ, Jean J. GABSZEWICZ and Ornella TAROLA. Social awareness and duopoly competition.
- 2013/44 Volker BRITZ, P. Jean-Jacques HERINGS and Arkadi PREDTETCHINSKI. On the convergence to the Nash bargaining solution for action-dependent bargaining protocols.
- 2013/45 Pasquale AVELLA, Maurizio BOCCIA and Laurence WOLSEY. Single item reformulations for a vendor managed inventory routing problem: computational experience with benchmark instances.
- 2013/46 Alejandro LAMAS, Tanja MLINAR, Liang LU and Philippe CHEVALIER. Revenue management for operations with urgent orders.
- 2013/47 Helmuth CREMER, Firouz GAHVARI and Pierre PESTIEAU. Uncertain altruism and the provision of long term care.
- 2013/48 Claire DUJARDIN, Vincent LORANT and Isabelle THOMAS. Self-assessed health of elderly people in Brussels: does the built environment matter?
- 2013/49 Marc FLEURBAEY, Marie-Louise LEROUX, Pierre PESTIEAU and Grégory PONTIERE. Fair retirement under risky lifetime.
- 2013/50 Manuel FÖRSTER, Ana MAULEON and Vincent VANNETELBOSCH. Trust and manipulation in social networks.
- 2013/51 Anthony PAPAVALIOU, Yi HE and Alva SVOBODA. Self-commitment of combined cycle units under electricity price uncertainty.
- 2013/52 Ana MAULEON, Elena MOLIS, Vincent VANNETELBOSCH and Wouter VERGOTE. Dominance invariant one-to-one matching problems.
- 2013/53 Jean GABSZEWICZ and Skerdilajda ZANAJ. (Un)stable vertical collusive agreements.
- 2013/54 François MANIQUET and Massimo MORELLI. Approval quorums dominate participation quorums.
- 2013/55 Mélanie LEFÈVRE and Joe THARAKAN. Intermediaries, transport costs and interlinked transactions.
- 2013/56 Gautier M. KRINGS, Jean-François CARPANTIER and Jean-Charles DELVENNE. Trade integration and the trade imbalances in the European Union: a network perspective.
- 2013/57 Philip USHCHEV, Igor SLOEV and Jacques-François THISSE. Do we go shopping downtown or in the 'burbs'? Why not both?
- 2013/58 Mathieu PARENTI. Large and small firms in a global market: David vs. Goliath.
- 2013/59 Paul BELLEFLAMME and Francis BLOCH. Dynamic protection of innovations through patents and trade secrets.
- 2013/60 Christian HAEDO and Michel MOUCHART. Specialized agglomerations with areal data: model and detection.
- 2013/61 Julien MARTIN and Florian MAYNERIS. High-end variety exporters defying distance: micro facts and macroeconomic implications.

Recent titles

CORE Discussion Papers - continued

- 2013/62 Luca G. DEIDDA and Dimitri PAOLINI. Wage premia, education race, and supply of educated workers.
- 2013/63 Laurence A. WOLSEY and Hande YAMAN. Continuous knapsack sets with divisible capacities.
- 2013/64 Francesco DI COMITE, Jacques-François THISSE and Hylke VANDENBUSSCHE. Vertical differentiation in export markets.
- 2013/65 Carl GAGNÉ, Stéphane RIOU and Jacques-François THISSE. How to make the metropolitan area work? Neither big government, nor laissez-faire.
- 2013/66 Yu. NESTEROV and Vladimir SHIKHMAN. Algorithmic models of market equilibrium.
- 2013/67 Cristina PARDO-GARCIA and Jose J. SEMPERE-MONERRIS. Equilibrium mergers in a composite good industry with efficiencies.
- 2013/68 Federica RUSSO, Michel MOUCHART and Guillaume WUNSCH. Confounding and control in a multivariate system. An issue in causal attribution.
- 2013/69 Marco DI SUMMA. The convex hull of the all-different system with the inclusion property: a simple proof.
- 2013/70 Philippe DE DONDER and Pierre PESTIEAU. Lobbying, family concerns and the lack of political support for estate taxation.
- 2013/71 Alexander OSHARIN, Jacques-François THISSE, Philip USHCHEV and Valery VERBUS. Monopolistic competition and income dispersion.
- 2013/72 N. Baris VARDAR. Imperfect resource substitution and optimal transition to clean technologies.
- 2013/73 Alejandro LAMAS and Philippe CHEVALIER. Jumping the hurdles for collaboration: fairness in operations pooling in the absence of transfer payments.

Books

- V. GINSBURGH and S. WEBER (2011), *How many languages make sense? The economics of linguistic diversity*. Princeton University Press.
- I. THOMAS, D. VANNESTE and X. QUERRIAU (2011), *Atlas de Belgique – Tome 4 Habitat*. Academia Press.
- W. GAERTNER and E. SCHOKKAERT (2012), *Empirical social choice*. Cambridge University Press.
- L. BAUWENS, Ch. HAFNER and S. LAURENT (2012), *Handbook of volatility models and their applications*. Wiley.
- J-C. PRAGER and J. THISSE (2012), *Economic geography and the unequal development of regions*. Routledge.
- M. FLEURBAEY and F. MANIQUET (2012), *Equality of opportunity: the economics of responsibility*. World Scientific.
- J. HINDRIKS (2012), *Gestion publique*. De Boeck.
- M. FUJITA and J.F. THISSE (2013), *Economics of agglomeration: cities, industrial location, and globalization*. (2nd edition). Cambridge University Press.
- J. HINDRIKS and G.D. MYLES (2013). *Intermediate public economics*. (2nd edition). MIT Press.
- J. HINDRIKS, G.D. MYLES and N. HASHIMZADE (2013). *Solutions manual to accompany intermediate public economics*. (2nd edition). MIT Press.

CORE Lecture Series

- R. AMIR (2002), Supermodularity and complementarity in economics.
- R. WEISMANTEL (2006), Lectures on mixed nonlinear programming.
- A. SHAPIRO (2010), Stochastic programming: modeling and theory.