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When Borch's Theorem does not apply: some key implications of market incompleteness, with policy relevance today

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When Borch's Theorem does not apply: some key implications of market incompleteness, with policy relevance today

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Abstract

Markets are incomplete when the assets available to the agents do not span the space of future contingencies. Efficiency is then assessed by the weak criterion of "constrained efficiency" (efficiency relative to the set of allocations compatible with the asset structure). That criterion requires firms to optimise relative to shadow-prices reflecting shareholders' preferences. Yet, even when firms do so, competitive equilibria on the markets for assets and commodities fail (generically) to be constrained efficient (section 3). Pareto-superior allocations can be implemented through price/wage rigidities and quantity constraints (section 4). But nominal rigidities are conducive to multiple equilibria, implying endogenous macroeconomic uncertainties that compound the primitive (exogenous) uncertainties (section 5). Various policy implications can be drawn, which are of some relevance to the current crisis.

Keywords: general equilibrium, incomplete markets, temporary equilibrium, constrained efficiency, price rigidities, multiple equilibria, coordination failures, Phillips curve

JEL Classification: D50, D52, D82

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CONTENTS.

1. From Borch's theorem to incomplete market economies.

Borch's theorem is the beautiful, transparent statement of a basic property of efficient risk-sharing among a set of agents. Let N agents each be endowed with a random wealth prospect x_i of known probability distribution, and with preferences representable by the expectation of a concave function of wealth; then, every efficient risk-sharing arrangement calls for **pooling all risks** and **sharing the aggregate wealth** $X = \Sigma_i x_i$ among all the agents. The share of the aggregate risk borne by each agent is allowed to vary with the level of that risk, reflecting individual risk-tolerances (a property of the utility functions). (Borch 1960.)

In Borch's 1960 paper, reference is made to the equally general result in Arrow (1953), where random prospects are defined with reference to an underlying set of S exogenous **states of the world.** A wealth prospect is then defined by an S-vector of state-dependent wealth levels. The preferences of an agent are assumed representable by a concave function of such vectors. Arrow's theorem states that every efficient risk-sharing arrangement corresponds to a **competitive equilibrium** on the S markets for claims contingent on the states (for "Arrow securities"). It is readily verified that Arrow's efficient arrangements satisfy Borch's theorem.

Borch's research was motivated by reinsurance problems. He noted modestly that these provide a unique application of Arrow's model, which otherwise would miss an empirical counterpart: markets for contingent claims are not common, and opportunities for risk sharing remain limited in today's world.

Within advanced economies, substantial programs of mutual insurance of individual risks are organised through social security (unemployment insurance, health insurance, pensions,…) and through progressive income taxation. In large federal nations (US, Canada, Germany..), there is partial mutualisation of regional (state) risks – like 40% in the US and Canada, more in Germany. But across nations, there is almost no risk-sharing[3], in spite of the fact that aggregate national

[3] An immediate corollary of Borch's theorem asserts that efficient global risk-sharing could be organised on a two-tier basis, with individual risks pooled efficiently within nations or regions, and aggregate national or regional risks pooled at an upper tier; see e.g. Drèze (2000) or Drèze (2009) for an application to Belgium's constitutional debate.

risks (say, uncertainty about the evolution of average real disposable incomes per capita) remain substantial in the long run and are *imperfectly correlated*.[4] This shortcoming is related to the absence of assets indexed on national incomes. Existing assets traded on stock exchanges account only for a few percentage points of national incomes.[5] Clearly, the extent to which Borch's theorem applies remains limited – in spite of the remark in Borch (1968) that incomplete markets entail incentives for the agents to create additional assets.

The reference to uncertainty about the evolution of national incomes is important, because it points to the obvious fact that many relevant risks are of a macroeconomic nature, hence largely endogenous. Today (spring 2009), there is much *uncertainty about the likely depth and duration of the recession* currently under way. This is quite different from, say, exogenous uncertainty about the size of Norway's oil reserves or about the speed of global warming…

When assessing the extent of organised risk-sharing, it is important to understand the nature of risky activities and to draw some distinctions. In the Borch model, and in the exchange version of Arrow's model, *risks are given:* each agent is endowed with a given state-distribution of wealth, and risk-sharing calls for redistribution of these endowments, neither more nor less.

A different problem arises in production economies, where the state distribution of wealth tomorrow can be, to some extent, *chosen through production decisions* (investments). Thus, a simple model of a production economy endows firms with production sets linking outputs tomorrow to inputs today. The firm then chooses a state-dependent production plan from a convex feasible set. In that case, the state-distribution of returns is effectively *chosen* by the firm. This raises a new set of problems, discussed at length below.

A third level of complexity arises when agents exert *some control over the occurrence of the states*. Models with that feature are commonplace in the theory of individual decision making. For instance, they enter the

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[4] Thus, Forni and Reichlin (1999) estimate that *prospects* for international risk-sharing across member states of the European Union are comparable to those *implemented* across states in the US or Canada.

[5] In his introdution to a general appraisal of market incompleteness, Shiller (1993, p. ix) notes: "..corporate dividends amount to .. about 3% of national income in the United States".

insurance literature under the name of *moral hazard*. But they are not standard in general equilibrium theory.

2. A basic model.

2.1 Primitives.

Throughout this lecture, I rely on the simplest model suitable for discussing incomplete markets (see also Table **1**). It concerns a production economy extending over two periods under uncertainty with no markets for contingent claims and a limited ("incomplete") set of assets. Incomplete markets imply that *households face multiple budget constraints* and *firms' profits do not have well-defined market values*.

There are two explicit **periods**: the present (today), or period 0, and the future (tomorrow), or period 1. There is a single "state of the environment" or "**state of the world**" in period 0 and there are S possible states s = 1,..,S in the future. Which state will obtain tomorrow is unknown today. Let $\mathcal{S} = \{0, 1,..,S\}$ denote the set of **nodes**.

The economy consists of H + J + 1 **agents**: H **households** indexed h ε $\mathcal{H} = \{1,..,H\}$; J **firms** indexed j ε $\mathcal{J} = \{1,..,J\}$ and a **bank** indexed 0. There are L **physical goods** or commodities both in period 0 and in period 1, state s; they are indexed sl ε {s1,..,sL}, s ε $\mathcal{S}$. There is one **money**, used for transaction purposes by households and firms, according to a general transactions technology introduced below. Money holdings are denoted m_s, s ε $\mathcal{S}$. Money balances are supplied by the bank at the (here exogenous) rates of interest r_s, s ε $\mathcal{S}$. The bank is owned by the households with given ownership fractions θ^{h0} . Its profits are denoted r_sM_s, s ε $\mathcal{S}$.[6]

The **assets** consist of a safe **nominal bond** and the **shares of stock** of the J firms. The assets are traded by households alone. Holdings of bonds by household h are labelled b^h ε R, with initial holdings equal to 0. Shares of firm j held by household h are labelled θ^{hj} ε R_+, with initial holdings δ^{hj}, $\Sigma_h \delta^{hj} = 1$ for each j.

Spot **markets** for commodities operate in period 0 with **prices** p_0 ε R^L_+ and in period 1 with prices p_s ε R^L_+, s = 1,..,S. Shares of the firms are traded in period 0 on a stock market with prices π ε R^J_+. Bonds, which

[6] The modelling of money here follows Drèze and Polemarchakis (2001), with a slight generalisation suggested by Jean-Jacques Herings and an extension to distinct interest rates for bank loans and safe bonds.

pay one unit of money in every state s tomorrow, are traded today at a price $\varphi \varepsilon [0, 1]$.
Household h is defined by a vector of initial **endowments** w^h and a **consumption-and-transactions correspondence** $\Xi^h(p) \subset R^{(L+1)(S+1)}$ that defines the feasible set of consumptions and money holdings at given prices p[7]. (Cash-in-advance is a transparent illustration, but the specification here is more general.) The preferences of household h are given by a **preference relation** $\geq^h$ on Ξ^h.
Firm j is similarly defined by a **production-and-transactions correspondence** $\Psi^j(p) \subset R^{(L+1)(S+1)}$ that defines the feasible productions and money holdings at given prices[8].

2.2 Constrained feasibility.

The **budget constraints** of household h are defined by:

$$p_0(x^h_0 - w^h_0) + r_0 m^h_0 + \pi(\theta^h - \delta^h) + \varphi b^h \leq \Sigma j \theta^{hj}(p_0 y^j_0 + r_0 m^j_0) + \theta^{h0} r_0 M \tag{2.1}$$

$$p_s(x^h_s - w^h_s) + r_s m^h_s \leq b^h + \Sigma_j \theta^{hj}(p_s y^j_s + r_s m^j_s) + \theta^{h0} r_s M_s,\ s = 1,..,S. \tag{2.2}$$

There are thus altogether S+1 *distinct budget constraints*, one per node. A household facing these constraints can reallocate (monetary) resources over time and across states *exclusively through its portfolio of assets* (θ^h, b^h). Markets are said to be "complete" when portfolio choices permit unlimited reallocations, so that the S+1 constraints (2.1)-(2.2) boil down to a single one. Define the values of "dividends" paid by firm j in state s as $V^j_s := p_s y^j_s + r_s m^j_s$, an element of the J x S matrix V. Taking the existence of nominal bonds into account, add to the matrix V a row of 1's, and label V^+ the resulting (J+1) x S matrix. Markets are complete if and only if V^+ has full rank S. This can only happen if $J+1 \geq S$ *and* S rows of V^+ are linearly independent. These are clearly strong requirements, hence the relevance of the "incomplete markets" case that arises when $\text{rank}V^+ < S$.
Thus, the asset structure defined by V^+ entails *constraints* on (defines) the set of consumption plans attainable by the households: there should exist *prices and portfolios* for which conditions (2.2) are satisfied, all h.

[7] Prices come in because the money balances needed for the trades x^h-w^h depend upon p.
[8] Because inputs are negative quantities, money balances used by firms are also negative.

Definition D.2.1. A **constrained feasible allocation** is a tuple $((x^h, m^h) \varepsilon\ \Xi^h(p), h\ \varepsilon\ H, (y^j, m^j)\ \varepsilon\ \Psi^j, j\ \varepsilon\ J)$ such that *there exist* (p, θ, b) for which (2.2) *is satisfied* with

$$\Sigma_h(x^h - w^h) - \Sigma_j y^j \leq 0 \qquad (2.3.1)$$

$$M = \Sigma_h m^h - \Sigma_j m^j \qquad (2.3.2)$$

$$\Sigma_h \theta^{hj} = 1, j\ \varepsilon\ J, \text{ and } \Sigma_h b^h = 0. \qquad (2.3.3)$$

Condition (2.3.1) states that the allocation must be *physically feasible*. Condition (2.3.2) imposes that the sum of money balances held by all agents corresponds to the amount issued by the bank; it is thus akin to a physical feasibility condition for "bank notes". Condition (2.3.3) amounts to clearance of the asset markets. Together with (2.2), it implies that *the allocation is decentralised through the portfolios* (θ^h, b^h) and prices (p, r): the consumption of each household is *financed* in every state *tomorrow* by the endowment and portfolio returns of the household. That is, the definition of *constrained* feasibility embodies the limitations on transfers of resources across states that result from market incompleteness (from the structure of the assets). Note that, although the set of assets is a primitive, their relevant properties (state distributions of dividends) are endogenous (production decisions).

D.2.1 is a definition of *pure feasibility*: it *allows for transfers* of wealth across households at node 0, hence the asset prices (π, φ) play no role (do not appear). Further, **D.2.1** incorporates *no requirement of optimisation* by the agents – households or firms. The reference to the prices (p, r) is a complication – unfortunate perhaps, but unavoidable to capture the crucial feature that not all reallocations of resources across states are *financially feasible*. **D.2.1** imposes this, and *no further requirement*.[9]

2.3 Constrained efficiency.

Using **D.2.1**, a concept of **constrained efficiency** is at hand.

[9] Definition 2.1 does not place any restrictions on admissible prices (beyond non-negativity). Thus, by setting all prices equal to 0, the constraints (2.2) become irrelevant, and full Pareto efficiency is at hand. Under the constraints on prices introduced in Section 5, that possibility no longer exists.

Definition D.2.3 The tuple $((x^h, m^h), h \varepsilon H, (y^j, m^j), j \varepsilon J,)$ is a **constrained efficient allocation** if it is constrained feasible and there does not exist another constrained feasible allocation strictly preferred to it by every household.[10]

The concept of constrained efficiency is of course *weaker than first-best* (Pareto) efficiency. In the model under discussion, the two coincide when markets are complete – or else when restrictive assumptions hold, assumptions that are so extreme as not to deserve attention. Conversely, **constrained inefficiency is a much stronger property than first-best inefficiency.** It is accordingly of great interest to investigate *whether market equililbria are constrained efficient*.

3. Constrained inefficiency of market equilibria.

3.1 .Preliminaries.

In a seminal paper, Diamond (1967)[11] studied an economy where L = 1 and where the production sets entail *no choice other than scale*: the state-distribution of outputs is a given *function* of the input level[12]. He could then prove that competitive equilibria on the asset markets are constrained efficient. This nice property follows from the fact that production decisions are implicitly taken by households: their demands for shares determine the input levels, hence the full production plans.

In general, production decisions are more complex, and a definition of the **decision criterion of firms** is called for. This is not a trivial task: as noted above, market incompleteness entails that *profit maximisation is not well defined*. Indeed, when rank$V^+ < S$, the asset prices do not imply unique prices for state-specific dividends (for "Arrow securities" promising one unit of money contingent on a given state s = 1,..,S).

For the model under discussion, two approaches to equilibrium analysis have been developed, namely *General Equilibrium with Incomplete markets* (GEI) and *Temporary General Equilibrium* (TGE)[13].

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[10] Because transfers of wealth in period 0 are allowed, this is equivalent to the more standard "preferred or equivalent for every household, with strict preference for at least one of them".

[11] That seminal paper was the first to introduce the concept of constrained efficiency.

[12] Diamond's model has been extended some by Ekern-Wilson (1974), Radner (1974).

[13] For a comparison of the two approaches, see e.g. Magill and Shafer (1991) or Drèze (1999b). GEI started with Radner (1968), whereas modern TGE was first introduced by

Under GEI, it is assumed that (i) all spot markets clear at competitive prices, and (ii) all agents hold *common and correct point expectations* about spot prices in every state tomorrow.[14] One can then try to characterise the production decisions that are compatible with constrained efficiency, under that specification. The virtue of GEI *for normative analysis* is that the approach permits focusing on the potential implications (for constrained efficiency) of market incompleteness *per se*, when other potential sources of inefficiency (like imperfect information about future prices) are assumed away.
In contrast, TGE develops the more realistic approach based on *idiosyncratic price expectations*. That is clearly the relevant starting point for *positive analysis*. It is adopted in section **5** below. Prior to that however, it is useful to state the main lessons of the normative analysis permitted by GEI.

3.2 Constrained efficient production decisions.

A characterisation of constrained-efficient production decisions in the model under discussion was first stated in Drèze (1974), for the special case L = 1.[15] The extension to L > 1 appears in Geanakoplos et al. (1990) and in Bonnisseau and Lachiri (2004). In either case, the characterisation is reasonably transparent.
Under competitive spot markets, the production decisions of a firm affect households in two ways: (i) through a state-distribution of dividends; and (ii) when L > 1, through a possible impact on commodity prices. If markets were complete, the state distribution of dividends would be valued in the market, and the second theorem of Welfare Economics would apply, calling for profit maximisation at given prices. Drèze and his followers show that constrained efficiency calls for profit maximisation at *shadow prices reflecting the valuation of dividends by shareholders*. Denote by λ^h_s the marginal rate of substitution of household h for dividends in state s versus dividends today. The shadow price β^j_s to be attached by firm j to profits (dividends) in state s is defined by $\beta^j_s = \Sigma_h \theta^{hj} \lambda^h_s$. *Profit maximisation by*

Grandmont (1974) at a conference held in Bergen in 1971. (The TGE idea goes back to Hicks 1939)

[14] Clearly, the assumption of "perfect foresight" goes way beyond "rational" expectations à la Muth (1961).

[15] Drèze's paper was presented first at the 1971 conference in Bergen.

all firms at shadow prices averaging final shareholders' shadow prices is a ***necessary condition*** *for constrained efficiency.*[16]

Remark 3.2.1 When markets are complete, the individual marginal rates of substitution are all equal to the market prices for dividends, and we are back to straight profit maximisation – as was to be expected. When markets are incomplete, the stock and bond prices still carry some implications *that are reflected in the marginal rates of substitution of shareholders* through the first order conditions for optimal portfolios. There is no need for firms to take them into account separately.

Remark 3.2.2 The individual rates λ^h_s are multiplied by the shareholdings θ^{hj}, reflecting the fact that dividends accrue to shareholders in proportion to their holdings. The formula recognises this feature. Note that the formula relies on *terminal shareholdings* θ^{hj}, *not initial shareholdings* δ^{hj}. This feature emerges as a *necessary condition* for constrained efficiency, reflecting the fact that dividends will accrue to the final holders. But who is deciding about the production plan, the initial or the final shareholders? This depends upon the sequence in time of the production decisions and the clearing of the stock market. In the equilibrium analysis, there is an implicit assumption of simultaneity, calling for consistency between stock prices and production decisions – for instance through an adjustment process of the kind studied in section 5 of Drèze (1974).

3.3 Generic constrained inefficiency.

The formula for shadow prices given above characterises neatly a necessary condition for constrained efficiency. It invites a *conjecture*: with perfect foresight, competitive clearing of spot markets today and tomorrow, *coupled with profit maximisation at the shadow prices given above* - "**stockholders' equilibria**" in Drèze's terminology - could be conducive to constrained efficiency in the economy under discussion. *Unfortunately, such is not the case*. Drèze (1974) gives robust examples to the contrary. The main theorem in Geanakoplos et al. (1990) gives non-pathological assumptions under which stockholders' equilibria are *generically constrained-inefficient*! (Generically, i.e. "almost always" - with reference to initial endowments in that paper.) So **there should**

[16] It is also a sufficient condition for Pareto efficiency of the production plan from the viewpoint of the firm's shareholders; in that sense, it is amenable to decentralisation.

exist Pareto superior constrained feasible allocations that are not competitive equilibria – i.e. failure of the two welfare theorems.[17]
This property will not surprise readers familiar with the **general theorem of the second best,** stated as follows by Lipsey and Lancaster (1956): "The general theorem for the second best optimum states that if there is introduced into a general equilibrium system a constraint which prevents the attainment of one of the Paretian conditions, the other Paretian conditions, although still attainable, are, in general, no longer desirable." At stake here is a natural extension of this general theorem from Pareto efficiency to constrained efficiency: when first-best allocations are not attainable, we may expect some of their properties, like competitive market clearing, to be no longer desirable…
A general result to the effect that stock market equilibria are gener(ic)ally inefficient, not only with reference to a first-best Pareto criterion, but also with reference to what can be accomplished when insurance possibilities are limited (when Borch's theorem does not apply) is **a striking warning about the limitations of market capitalism.** It is a timely warning, both given the liberal creed that has been prevalent over the pas twenty years (since the fall of the iron curtain), and given the current crisis that originated in the proliferation of "toxic assets" (about which more in section **3.5**) The result should thus be understood properly.
In the model of Drèze (1974) with L = 1 (so that commodity prices are trivially given), constrained-inefficient stockholders' equilibria are potentially dominated by alternatives that differ with respect to *both production plans and portfolios*. Indeed, portfolios are optimal given the production plans, and these plans are optimal given the portfolios; but *simultaneous changes* in portfolios and plans can be Pareto improving. The fact that a *necessary condition* (the decision criterion of the firms) is satisfied does not imply that the condition is *sufficient*. Indeed, it is not, because *the set of constrained-feasible allocations is* ***not convex***: the dividend incomes entering tomorrow's budget constraints are defined as *products of endogenous variables*, namely $\theta^{hj}(p_s y^j_s + r_s m^j_s)$. This bilinearity results in a non-convex feasible set,

[17] An early example appears in Plolemarchakis (1979). A general result appears in Geanakoplos and Polemarchakis (1986). A recent further contribution is Herings and Polemarchakis (2005).

over which necessary conditions for optimality are not sufficient. When a stockholders' equilibrium is not constrained-efficient, simultaneous changes in θ's and y's can be Pareto improving. But bringing these about is not a trivial task, because the simultaneous changes must be *identified* as Pareto improving. This may call for communication between a firm and households that are *not currently shareholders*. No specific suggestions to that effect have appeared in the literature, to the best of my knowledge.[18] Hence my interest in a positive theory of the firm, needed for section **5.**

Remark 3.3.1 The generic result in Geanakoplos et al. assumes that spot markets for commodities clear at competitive prices, both today and tomorrow; Pareto improvements come from the assets only (production plans and portfolios). A complementary result, not available in the literature, would take as given the decision criterion of the firms and the competitive clearing of the asset markets, but allow for non-competitive allocation of the commodities. I conjecture that a generic inefficiency result similar to that in Geanakoplos et al. holds for that alternative specification, as illustrated in section **4**.

Remark 3.3.2 A further comment about *generic* inefficiency is in order. The assumptions leading to the theorem in Geanakoplos et al., though "non-pathological", are still restrictive.[19] *Does "genericity" matter?* Yes and no… If some of these assumptions fail, it does not follow that any stockholders' equilibrium is efficient! There is no *guarantee* that it is inefficient, only that it *could be*. The *concern* about potential inefficiency is there, and it remains advisable to investigate measures apt to generate Pareto-improvements.

3.4 A positive theory of firms' decisions.

It was noted above that "constrained efficiency calls for profit maximisation at *shadow prices reflecting the valuation of dividends by shareholders*".

In the case of a firm run by a *single owner*, that decision criterion is natural and realistic: the owner simply values state-distributions of dividends through his or her expected utility for wealth – full stop.[20]

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[18] Bisin et al. (2009) assume that firms know the values attached by every household to every element of their production sets; that is a rather extreme and gratuitous assumption…

[19] In particular, there is an unnatural assumption about the relative numbers of states and agents.

[20] The distinction between alternative ownership structures is developed more fully in Magill and Quinzii (1996).

In the case of a firm run by a few *partners,* agreement among them about production decisions is in sight. If they entertain divergent degrees of risk aversion, or different state-dependent preferences, some compromises are called for. Possibly they may introduce transfers or other compensations to reach unanimity. The underlying principle of "Pareto efficiency for the shareholders" is a natural agenda for the partnership.

Of course, firm ownership by a single or a few partners is unlikely to prove efficient from the viewpoint of risk sharing.

When it comes to *corporations*, with shares widely disseminated among anonymous shareholders, the situation is quite different. Today, these corporations are typically controlled by a *board of directors*, elected by and accountable to the *general assembly of shareholders*. Outside of exceptional circumstances, that assembly would typically be attended by a few shareholders only, with some proxies.[21]

The situation just described is captured quite faithfully in a decision criterion that I have labelled the **control principle**. It was introduced first in my 1983 Yrjö Jahnsson Lectures, with a non-technical presentation in my Harry Johnson Lecture (1984); see Drèze (1989, 1985). It goes as follows:

(i) each firm is endowed with by-laws (a primitive) that stipulate how directors are selected from among shareholders;

(ii) a production plan y is preferred to the alternative plan y' if and only if it is preferred by all members of a set of shareholders holding a *majority* of shares and including *all* directors.

This definition leads to a *partial ordering* of production alternatives, thereby opening the door to *multiple equilibria* - a theme addressed in section **5**.

The definition of **equilibrium of the firm** requires that the chosen production plan y is such that there does not exist a feasible y' preferred to y. Existence of equilibrium follows from standard assumptions, plus

[21] Exceptional circumstances: after the Government rescued Belgium's largest bank (FORTIS) in 2008, several assemblies witnessed lively proxy fights with large attendance…

a weak condition of continuity on the definition of the board of directors.[22]

The decision criterion reflecting this "control principle" has received very little attention in the literature. Still, I claim that it remains to date the *more general and more realistic* formulation of a decision criterion for business firms under incomplete markets.[23] The realism should be obvious. The generality results from encompassing the "shareholders efficiency" condition of section **3.2** by simply defining the board of directors to consist of all shareholders (perhaps with transfers or vote trading); and from encompassing such standard formulations as "the utility function of a manager" – as in Radner (1968) - by simply defining the board as consisting of that manager alone. Also, the criterion is readily extended to allow for partial delegation of responsibilities, for absentee ownership, a.s.o..

Remark 3.4.1 There remains one delicate issue about reliance on the control principle, an issue related to that raised in Remark **3.2.2**. On which basis are the board of directors and the general assembly formed: initial shareholders (δ^h's), or final shareholders (θ^h's)? If initial shareholders, then the composition of the board and of the assembly are part of the primitives, which is a simplification; but then one must take into account the fact that initial shareholders choosing production plans will realise that they may *trade shares* on the period 0 stock exchange. In that case, they are concerned not only about future dividends, but also about trading prices. Thus, an initial shareholder intending to sell her shares will favour a production plan apt to bring about a high selling price, even if the underlying dividend profile does not appeal to her; and conversely for a prospective buyer. This calls for forming expectations about the *impact of future state-dependent dividends on today's stock prices*, a further difficulty.

In the spirit of developing a theory susceptible of *multiperiod extensions*, the "initial shareholders" approach is the right one. It was introduced for the first time in the important paper by Grossman and Hart (1979), together with the assumption that each shareholder h uses *her own* marginal rates of substitution λ^h_s

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[22] The composition of the board is endogenous, because it reflects stock ownership as resulting from clearing of the stock market. The continuity condition states that the mapping defining the board of directors as a function of the vector of shareholdings by the H households must be upper hemi-continuous in the discrete topology. Transparent examples include the n largest shareholders, the smallest set of shareholders holding together α percent of the shares, every shareholder holding at least β percent of the shares, and more.

[23] The criterion is developed in Drèze (1989) for L = 1; an extension to L > 1 is described in section **5**.

to evaluate the *impact* of dividends in state s on asset prices today. The authors label that assumption “competitive price perceptions”. Unfortunately, that label is deceptive. As explained in detail by Drèze et al. (2007), the Grossman-Hart assumption goes beyond competitive perceptions: it imposes “egocentric price perceptions” that do not, in general, correspond to rational expectations. Drèze et al. use a more general definition. There is no room here to treat that technical issue more fully; hence, it is also eschewed in section **5**.

3.5 A digression: “toxic” assets.

It is intriguing to wonder whether the proliferation of so-called “toxic assets” over the last decade relates in any way to the generic inefficiency of stockholders’ equilibria. Although the term “toxic assets” does not appear in the economic literature, a close analogue is standard, namely “lemons”. An asset may be characterised as a “lemon” if there exists another asset (or portfolio) that *dominates* it: for a comparable investment at time 0, that other asset yields (weakly) higher dividends in all states tomorrow. Under *absence of arbitrage*, lemons should not exist. Conversely, existence of lemons reflects *information failures*.

In Drèze et al. (2008), *information asymmetries* are introduced through incomplete information on the part of firms about shareholders’ preferences, and on the part of shareholders about production plans[24]. It is then shown (through robust examples) that such asymmetries may lead to proliferation of lemons. And information asymmetries (as under securitisation of mortgages…) provide a natural explanation for the proliferation of toxic assets.

4. Non-competitive spot-market clearing.

In the general model (L > 1), relative prices enter the picture, and open new avenues towards Pareto improvements (as noted in Remark **3.3.1**). The principle is elementary: suitably chosen commodity prices offer additional prospects towards risk-sharing, when markets are incomplete (when Borch’s theorem does not apply).

Take a country like Ivory Coast whose GDP consists substantially of coffee production. World coffee prices are volatile, and there are no

[24] Technically, this is done through investment funds that do not detect lemons: they hold the same fraction of all firms in a given sector; the analogy to funds pooling toxic mortgages is close…

markets for trading claims on the country's GDP. Futures markets for coffee exist for short terms only, and renewing options continuously is impractical and expensive. Also, these markets do not offer insurance on uncertainties about production levels. Stabilising the world price of coffee (possibly with some inventory management) would reduce substantially the uncertainty affecting the country's prospective GDP. That is, price rigidities offer an alternative for insurance or asset trading.[25]

In a previous lecture devoted to incomplete markets (Drèze 2001), I devoted a section to the theme: *"Incomplete markets breed wage and price rigidities"*. That theme has both a positive dimension and a normative dimension.

On the **positive side**, I noted that downwards price rigidity in the presence of excess capacity is often due to fixed costs, costs that must be covered *state-by-state under incomplete markets*;[26] this calls for average rather than marginal cost pricing, a form of price rigidity.[27] I also note that under monopolistic competition and risk aversion, *uncertainty about demand elasticity* introduces a kink in perceived demand, which entails price stickiness.[28]

On the **normative side**, privileged here, one application has retained much attention, namely wage rigidities. For instance, Drèze and Gollier (1993)[29] show how, in a stylized version of the model of section **2**, *wage rigidities permit Pareto improvements* upon competitive stockholders' equilibria.[30]

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[25] My own interest in equilibria with price rigidities (Drèze 1975) arose from considerations of Marcel Boiteux about the merits of price rigidities over time in the face of demand fluctuations for electricity (with emphasis on investment decisions by users).

[26] General equilibrium with fixed costs is studied in Dehez and Drèze (1988) and Dehez et al. (2003).

[27] The illuminating analogy is with peak-load pricing, under which fixed costs are fully covered through the excess of marginal over average cost at peak periods; see Drèze (1964). Under uncertainty, this would call for covering fixed costs under favourable states only, and transferring surpluses to other states through asset trading – if markets permitted!

[28] On this point, see Drèze (1979b). See also Drèze and Herings (2008) for a general equilibrium model with kinky perceived demands.

[29] That paper builds upon preliminary considerations in Drèze (1979a, 1989) and Gollier (1991).

[30] The Pareto improvements emerge in Drèze-Gollier from allowance for upward wage rigidity in states exceptionally favourable to labour.

It is not claimed that wage rigidities *alone* can restore full constrained efficiency, in a general model. But it is important to realise that they *permit Pareto improvements, if properly defined*. The importance derives from the *prevalence of downward nominal wage* rigidities in contemporary advanced economies. A prima facie case for Pareto improvement is linked to the current (spring 2009) crisis. Without downward rigidities, market wages would drop to reservation levels, and labour incomes would fall appreciably – with severe consequences for workers' welfare, for further mortgage failures, and then for aggregate demand and employment.

The argument in Drèze and Gollier is reasonably transparent. Insurance of labour income risks is at the same time highly desirable and severely limited. It is desirable, because labour incomes are seldom open to diversification. In the words of James Meade (1972): "While property owners can spread their risks by putting small bits of their property into a large number of concerns, a worker cannot put small bits of his effort into a large number of different jobs". Of course, labour contracts[31] offer scope for risk sharing between *a specific firm and its employees*. But the main point, and the starting point of the Drèze-Gollier paper, is that *prospective entrants or re-entrants*[32] to the labour market are not covered by such contracts. This is a severe limitation to the extent of risk sharing between workers and property owners. Downwards wage rigidities coupled with unemployment benefits provide an avenue towards limiting the risks affecting labour incomes.[33] It is conducive to Pareto improvements, *if* the loss in productive efficiency is more than compensated by the gain in ex ante risk sharing efficiency.

Relying upon a model derived from the well-known Capital Asset Pricing Model (CAPM), Drèze and Gollier derive (i) a sufficient condition for constrained inefficiency of a stockholders' equilibrium (Proposition 2); then (ii) a characterisation of constrained efficient allocations (Proposition 3). I shall not enter here into details of that work. I simply wish to record the conclusion that, **under quite general**

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[31] For an introduction to the theory of labour contracts, see the seminal papers by Azariadis (1975), Baily (1974) and Gordon (1974), or the account in Drèze (1990).

[32] Entrants: today's graduating students; re-entrants: inactive workers (including young mothers), prospective victims of lay-offs, a.s.o..

[33] As explained in Drèze (1979a), labour income insurance without minimum wages would be impractical.

conditions, downward wage rigidities can help improve upon constrained inefficient equilibria – especially when coupled with unemployment benefits.
The Drèze-Gollier model is real, so the rigidities are also real. In practice, however, indexation of wages on the nominal price level is rare (with Belgium a notable exception). Thus, downward *nominal* wage rigidities provide the natural starting point, also for my section **5.** But assessing the significance of nominal rigidities calls for unveiling their implications for the underlying real wages that permit improved risk sharing and thus address the real concerns.

Remark 4.1 Inspired by the above considerations, I have explored various avenues of potential improvements on prevailing modes of wage formation. These include (partial) indexation of wages on national income per capita – Drèze (1990) -, indexation of social security contributions on business conditions – Drèze (1993) – or even time sharing – Drèze (1986). I have also extended the model of section **3** to labour contracts – Drèze (1989, chapter 3). There is no space here to develop these themes.

5. Multiple equilibria.

5.1 Some general results.

Economies subject to price/wage rigidities share an important property: they are prone to exhibit **a continuum of quantity-constrained equilibria reflecting coordination failures.**
This property was first brought out by Roberts (1987), who considered the intriguing case where the rigid prices or wages are compatible with a competitive equilibrium.[34] Although the (real) economy studied by Roberts is quite special[35], an extension of his multiplicity result for a standard Arrow-Debreu economy is given in Drèze (1997), following Herings (1996). The equilibrium concept used by Roberts (and Drèze) is known as a "*supply-constrained equilibrium*".[36] Under that concept, (some) prices are subject to lower bounds, and quantity constraints on

[34] The possibility of quantity-constrained equilibria at competitive prices was first brought out by Hahn (1978).
[35] All firms operate under constant returns to scale; all households are initially endowed with productive inputs supplied to firms and not consumed; consumption preferences are homothetic.
[36] That specific concept was introduced by van der Laan (1982), building upon the work of Drèze (1975). Although Drèze and van der Laan consider a pure exchange economy, the extension to production appears in Dehez and Drèze (1984).

individual net supplies come in when (and only when) the downward rigidities are binding. The main theorem in Drèze (1997) reads as follows: "Given an arbitrary strictly positive vector $\bar{s}$ of individual supply constraints for the commodities subject to downward price rigidities, there exists a supply-constrained equilibrium with constraints $s^\circ \leq \bar{s}$." That result is further strengthened in Citanna et al. (2001) where it is shown that there exists a *continuum of supply-constrained equilibria with arbitrary severe rationing of supplies*. Although availability of inputs sets an upper limit to output, there is no lower limit (as we might experience tomorrow?); hence multiplicity manifests itself as underutilisation of resources. In special cases, the alternative equilibria are Pareto-ranked.

An intuitive explanation of these results is stated as follows by Citanna et al. (p. 170): "We fix the prices of a subset of commodities, L^{II} in number. This freezes $L^{II} - 1$ relative prices. But we allow rationing of the supply of these L^{II} commodities. This leaves one degree of freedom, corresponding to the overall level of rationing for these L^{II} commodities and to the level of flexible prices relative to the L^{II} fixed prices… If firms expect that the total demand for their output is low, then they will hire only a limited amount of labour. Workers, expecting to be (partially) unemployed … express low demands for commodities, thereby confirming the firms' expectations." (It is only too easy to interpret the last two sentences in terms of today's situation… with privileged reference to the automobile industry!) In other words, underemployment equilibria may quite generally be interpreted as **coordination failures**.[37]

The work under review concerns real economies, for which multiplicity of allocations is a "real" phenomenon. The monetary model of section 2, expanded to complete markets, has been studied by Drèze and Polemarchakis (2001). These authors show that, under homogenous transactions technologies and in absence of initial monetary commitments, there exists a continuum of equilibria with *identical allocations* but arbitrary price levels at each node - hence arbitrary inflation rates between today and state s tomorrow. That is, under

[37] Other forms of coordination failures are studied, e.g. by Cooper and John (1988) or Jones and Manuelli (1992). Note that multiple equilibria reflecting coordination failures are *not* "sunspot equilibria"; on this concept, see e.g. Azariadis and Guesnerie (1986).

complete markets, both inflation rates *and inflation variability across states* have no real consequences, under the stated assumptions.[38] The multiplicity of real allocations in the previous paragraphs is of a different nature. The two specifications are combined in section **5.**

These results show that the static general equilibrium model must imperatively be complemented by a specification of a **selection process** through which a specific equilibrium emerges. The more natural complement is a *dynamic process of short-run adjustment* with initial conditions inherited from yesterday and with expectations about future realisations evolving along the process. In the absence of price rigidities, this calls for *a positive theory of inflation.*[39] There is not much general equilibrium literature on dynamic adjustment with price rigidities; see however Drèze (1991b, 1999a). There is of course a rich macroeconomic literature; the link remains to be formalised.

Remark 5.1.1 These results stand in apparent conflict with earlier work by Laroque and Polemarchakis (1978) or Laroque (1981) who display sufficient conditions for local uniqueness of equilibria with fixed prices. The explanation rests with model formulation and assumptions. The more recent papers use a different (and hopefully less restrictive) framework. In the same way that I explained in Remark **3.3.2** why genericity is not of paramount significance, I note here that existence of a *continuum* of supply-constrained equilibria (which requires suitable assumptions) is not paramount either. Again, the *concern* about coordination failures is the main point, which may stand under weaker conditions.

5.2 A TGE model with price/wage rigidities.

In order to connect these general results to market incompleteness, I have studied properties of the economy defined in section **2** above under downward nominal rigidities of prices or wages.[40] For the *positive analysis* of that economy, I rely on the Temporary General Equilibrium (TGE) approach, and I adopt the more relevant decision criterion for firms defined by the "control principle" (section **3.4**). That work is of necessity quite technical, but basic features can be described as follows (with summary in Table **2**).

[38] Of course, under incomplete markets, inflation variability has real consequences.
[39] On this topic, see also Drèze (1993a).
[40] My research covers real as well as nominal rigidities, but I concentrate here on nominal rigidities to alleviate an already arduous exposition. This section is based upon the more general formulation in Drèze (2013).

The L commodities are partitioned into two sets, L^{I} and L^{II}. The prices of commodities in L^{I} are flexible; the prices of commodities in L^{II} are subject to *downward nominal rigidities*. At time 0, for each l in L^{II}, I impose $p_{l0} \geq \overline{p}_{l0}$. When a price $p_{l0} = \overline{p}_{l0}$, the supply of that commodity by households and firms is subject to quantity constraints, defined for each agent by a continuous function of a global rationing index σ_{l0} (an endogenous, market variable). That is:

$$x^{h}_{l0} - w^{h}_{l0} \geq l^{h}(\sigma_{l0})\ \forall\ h\ \varepsilon\ H, \quad y^{j}_{l0} \geq l^{j}(\sigma_{l0})\ \forall\ j\ \varepsilon\ J. \qquad (5.1)$$

The levels of the variables $l^{h}(.)$, $l^{j}(.)$ are part of the signals observed by the agents.[41] Tomorrow in state s the lower bounds $\overline{p}_{ls}$ are allowed to depend (continuously) upon today's market realisations. Analogues of conditions (5.1) prevail, with rationing indices σ_{ls}. Henceforth, (5.1) stands for the whole set of such constraints, with 0 replaced by s ε *S*.

The **information** of the agents consists of the signals $\xi^{0} = (p_{0}, r_{0}, \sigma_{0}, \pi, \varphi)$, $\zeta^{0} = V_{0} := (V^{0}_{0}, V^{1}_{0}, ..,V^{J}_{0})$ at node 0, then $\xi^{s} = (p_{s}, r_{s}, \sigma_{s})$, and $\zeta^{s} = V_{s}$ in state s, where $V^{j}_{s} := p_{s}y^{j}_{s} + r_{s}m^{j}_{s}$ denotes the dividends from firm j, and $V^{0}_{s} = r_{s}M_{s}$ denotes the dividend from the bank.

The **household decision**, denoted $d^{h} = (d^{h}_{0}(\xi_{0}), d^{h}_{s}(\xi^{s}, \zeta^{s}))$. consists of demands today $d^{h}_{0} = (x^{h}_{0}, m^{h}_{0}, \theta^{h}, b^{h})$ and of a **plan** for demands tomorrow $d^{h}_{s} = (x^{h}_{s}, m^{h}_{s})$ in all states as functions of the signals (ξ^{s}, ζ^{s}) observed then. It must satisfy $d^{h}\ \varepsilon\ \Xi^{h}(.)$, the budget constraints (2.1)-(2.2) and the quantity constraints (5.1). Tomorrow's decision d^{h}_{s} maximises a conditional utility $\overline{u}^{h}(d^{h}_{0}, d^{h}_{s})$ subject to (2.2).

Having observed (ξ^{0}, ζ^{0}), household h forms **expectations about the signals** (ξ^{s}, ζ^{s}). I assume that, *given* (ξ^{0}, ζ^{0}), the relevant expectations on the *occurrence of state s and associated signal* (ξ^{s}, ζ^{s}) are defined by a (subjective) **probability measure** $\psi^{h}(\xi^{s}, \zeta^{s} | \xi^{0}, \zeta^{0})$.

Then, given a decision d^{h}_{0}, household h has a well-defined **expected utility**

$$U^{h}(d^{h}_{0} | \xi^{0}, \zeta^{0}) = \Sigma_{s}\psi^{h}(\xi^{s}, \zeta^{s} | \xi^{0}, \zeta^{0})\ u^{h}(d^{h}_{0}, d^{h}_{s}(\xi^{s}, \zeta^{s})). \qquad (5.2)$$

The household problem is to maximise $U^{h}(d^{h}_{0} | \xi^{0}, \zeta^{0})$ w.r.t. d^{h}_{0} subject to (2.1). Under strong continuity assumptions on $\psi^{h}(.)$, that problem is well-behaved, and the optimal decisions $(d^{h}_{0} | \xi^{0}, \zeta^{0})$ constitute an upper

[41] To illustrate: σ_{l0} may denote the overall rate of unemployment for unskilled labour; the quantity of labour that household h can sell is a function of that rate (possibly defined by a random drawing over all unemployed unskilled workers).

hemi-continuous correspondence from the space of (ξ^0, ζ^0) to the space of d^h_0. I write $P^h(d^h)$ for *the set of decisions* $d^{h'}$ *preferred* (strictly) by h to d^h.

Turning to firm j, I first add to the primitives a set $I^j \subseteq \{h \in H \mid \delta^{hj} > 0\}$ that defines the *control group* of firm j. Firm j is assumed to select a feasible decision $d^j = (d^j_0, d^j_s(\xi^s))$ such that there does not exist an alternative feasible decision $d^{j'}$ which is preferred to d^j by a set of shareholders containing I^j *and* holding together at least 50% of the final shares.

This invites me to *define shareholders' preferences over the decisions of firms*. These decisions affect final shareholders solely through the dividends issued both in period 0 and in period 1, state s = 1,..,S. So I need to define shareholders' preferences over streams of dividends $V^j := (V^j_0, V^j_1,..,V^j_S)$. I assume that such preferences are defined *taking as given* the signals (ξ^0, ζ^0), h's portfolio (θ^h, b^h) and h's expectations $\psi^h(\xi^s|\xi^0, \zeta^0)$. These preferences may then be represented by an *indirect utility function for income streams* v^h *and signals* $\xi^{\cdot} := (\xi^1,..\xi^S)$, where $v^h = (v^h_0, v^h_1,..,v^h_S)$, $v^h_0 = p_0w^h_0 + \Sigma_j\pi^j\delta^{hj} + \Sigma_j\theta^{hj}V^j_0 + \theta^{h0}r_0M_0$ and $v^h_s = p_sw^h_s + \Sigma_j\theta^{hj}V^j_s + b^h + \theta^{h0}r_sM_s$, s = 1,..,S. Write Λ^h_s for that indirect utility given state s and Λ^h for its expectation, expressed as

$$\Lambda^h(v^h|\theta^h, b^h, \xi^0, \zeta^0) := \Sigma_s\, \psi^h(\xi^s|\xi^0, \zeta^0)\, \Lambda^h_s(v^h_0, \xi^0, v^h_s, \xi^s/\theta^h, b^h). \qquad (5.3)$$

In comparing the dividend streams V^j and $V^{j'}$, household h will prefer $V^{j'}$ if and only if

$$\Lambda^h(v^h + \theta^{hj}(V^{j'} - V^j), \xi) > \Lambda^h(v^h, \xi). \qquad (5.4)$$

Applying the control principle, a feasible decision $d^{j'}$ is *preferred by firm* j to the feasible decision d^j if and only if there exists a set of shareholders, holding at least 50% of the shares and containing I^j, such that $d^{j'}$ is preferred to d^j *according to* (5.3) by every member of that set. And I write $P^j(d^j)$ for the set of decisions preferred by firm j to d^j.

After tediously listing these definitions, I can now define:

Definition 5.1 A **temporary equilibrium** is a tuple $((d^h)_{h \in H}, (d^j)_{j \in J}, \xi^0)$ such that:

(a) $\forall$ h, $d^h \in \Xi^h(.)$ and $\Xi^h \cap P^h(d^h)$ is empty;

(b) $\forall$ j, $d^j \in \Psi^j(.)$ and $\Psi^j \cap P^j(d^j)$ is empty;

(c) $\Sigma_h(x^h_0 - w^h_0) \leq \Sigma_j y^j_0$ and $M_0 = \Sigma_h m^h_0 - \Sigma_j m^j_0$;

(d) $\forall\ l\ \varepsilon\ L^{II}$, $p_{l0} \geq \bar{p}_{l0}$ implies $x^h_{l0} - w^h_{l0} > l^h(\sigma_{l0})\ \forall\ h$ and $y^j_{l0} > l^j(\sigma_{l0})\ \forall\ j$;

(e) $\Sigma_h\ b^h = 0$ and $\forall\ j\ \Sigma_h\theta^{hj} = 1$.

5.3 An inflation-unemployment locus.

The following theorem (here stated informally) is a first transposition to temporary equilibria of results mentioned in section **5.1.** Let $\bar{\mathbf{u}}$ denote an **average rate of underutilisation of all resources** (all: an "output gap", combining underutilisation of capacities as well as unemployment of labour), and let **i** denote **the overall inflation rate** at period 0**.**

Theorem 5.1. Under standard assumptions on the primitives of the economy, and strong assumptions of continuity on the households' probabilistic expectations, **for every A in $\mathbf{R}_{++}$, there exists a temporary equilibrium with**

$$(1 - \bar{\mathbf{u}}) . (1 + \mathbf{i}) = \mathbf{A}. \tag{5.5}$$

A suggestive special case occurs when price rigidities are not binding, as could for instance be the case for high enough overall price levels, say A > A*. In that special case, formula (5.5) allows for $\bar{\mathbf{u}} = 0$, with **i** and A co-determined (**i** = A – 1) with a level given by whatever process of nominal price formation is at work. A then simply stands for the ratio of today's price level to yesterday's price level – thus reminding us of the need to complement the static equilibrium theory with a positive theory of inflation (as also noted at the end of section **5.1**).

Conversely, if **i** were determined by some process of nominal price formation, then $\bar{\mathbf{u}}$ and A would be codetermined with $\bar{\mathbf{u}} = 1 + A(1 + \mathbf{i})^{-1}$ inversely related to today's price level – as could be expected under nominal price rigidities. The multiplicity of equilibrium values for $\bar{\mathbf{u}}$ would then result from the arbitrariness of A (of the extent of coordination failures).

In general, theorem **5.1** asserts existence of *a set of temporary equilibria* with a combination of underutilisation of resources and of inflation *indexed by a parameter* A. A would then reflect both initial conditions (including yesterday's price level) and the expectations about future realisations that underlie the decisions and preferences of the households at time 0 given their information (ξ^0, ζ^0). The level of A should thus reflect the working of a short-run adjustment process selecting a specific equilibrium. That selection may yield a more or less attractive allocation, possibly linked to a coordination failure, as mentioned in section **5.1**. The selection process should also entail a process of expectation formation. But the multiplicity under (5.5) complicates that task. In particular, **rational expectations must take into account the potential multiplicity of equilibria.** That is, rational expectations are bound to be multi-valued.

It is not asserted that the set of equilibria characterised by (5.5) forms a continuum (is connected); but it is implied by the formula that they are generally **distinct**, hence genuinely multiple.[42] Again, it is not asserted that they can be Pareto-ranked. All this reflects the "temporary" nature of my research in progress.

Formula (5.5) has an intriguing implication: it allows for **two distinct tradeoffs between inflation and unemployment**! Indeed:

- **keeping A fixed,** $\bar{\mathbf{u}}$ and **i** can move in **parallel**, either increasing (stagflation) or decreasing (recovery) together;
- **under variations of A,** $\bar{\mathbf{u}}$ and **i** can (yet need not..) move in **opposite** directions (Phillips curve).

The existence of both possibilities is confirmed empirically, as vividly illustrated by figure 2. (In contrast, only the second case is allowed under the fashionable *New Keynesian Phillips curve*.) This feature illustrates my claim – in Drèze (1987c) and passim – that *it may be more fruitful to look for the macroeconomic implications of microeconomics than for the microeconomic foundations of macroeconomics…* But the link to macroeconomics remains to be spelled out fully.

A comment is in order on the role of incomplete markets in understanding **demand volatility**. Drèze (2001) contains a section entitled *"Incomplete markets breed demand volatility",* where I refer to *changes in the degree of uncertainty* embodied in the agents' expectations (as when the prospects of bankruptcy for automobile manufactures rise…). I note that *uncertainty* about future resources typically induces households to save more, as shown by Drèze and

[42] In the special case $\bar{\mathbf{u}} = 0$, the (competitive) real equilibrium can be unique, but sustained by alternative price levels. With $\bar{\mathbf{u}} > 0$, identical real equilibria at different price levels would require special conditions on primitives.

Modigliani (1972) or Sandmo (1970). At the same time, according to the theory of *Investment under Uncertainty* due to Dixit and Pindyck (1994), investments are undertaken when the net present value of the associated profit stream is *greater than the value of an option to carry out the same investment at a later date*. Because the option value increases with the degree of uncertainty, an increase in that degree (as experienced in 2008!) encourages *postponement of investment*: the associated loss is of second order, the gain of first order![43] These arguments would not apply under complete markets. Thus, **incomplete markets breed demand volatility, reflected in multiple equilibria[44], hence in multivalued expectations, and calling for policies** apt to steer the short-run adjustment mechanism in the desired direction while reducing the uncertainty about future realisations; that is, while *anchoring* expectations.[45]

At the same time, the multiplicity has an undeniable flavour of **realism**: as we wonder today about the growth rates of western economies over the coming months, the existence of a full range of possibilities is flagrant and inescapable! Which econom(etr)ic model embodies all the contingencies apt to influence these growth rates (including, for instance, "changes in the degree of uncertainty")?

Remark 5.3 The multiplicity result in Theorem 5.1 should be invoked with caution. The result is obtained under a flexible price (interest rate) for bonds. If the bank (or the "treasury") controlled the bond rate through open market operations, the result would no longer follow from the maintained assumptions – though it would probably remain valid under natural assumptions about the elasticity of price expectations (w.r.t. today's prices). As already hinted in remarks **3.3.2** and **5.1.1**, I do not regard it as essential to prove a result like Theorem 5.1 with full generality. What matters most is to recognise that *coordination failures are a genuine possibility, of which we should be conscious at all times*, even if they do not occur all the time.

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[43] The reasoning is analogous to that underlying the "menu cost" theory for price adjustments; see e.g. Mankiw (1985).

[44] The close association between volatility of investment and volatility of GDP is illustrated in figures 1.a and 1.b.

[45] There is an obvious analogy with monetary policy aimed at controlling today's inflation (as resulting from short-run adjustment) while anchoring inflation expectations.

6. Conclusion.

6.a The material collected in this paper is easy to summarise. I have imported (or extended) and integrated **three key theoretical implications of market incompleteness**:[46]

(i) "competitive" market allocations display **generic inefficiency**, not only relative to unattainable Pareto optima, but also relative to constrained optima attainable within the existing asset structure;

(ii) price/wage **rigidities** offer scope for constrained-feasible Pareto improvements over competitive allocations,

(iii) but are conducive to **multiplicity** of equilibria, linked to coordination failures.

These three properties, and especially (iii), further imply:

(iv) a realistic model cannot rest with a description of uncertainty through exogenous "states of the world"; it must imperatively recognise that a major source of uncertainty is associated with **macroeconomic developments**, which are largely **endogenous**, and over which **rational expectations** remain to be defined;[47]

(v) there is a role for macroeconomic **policy** in anchoring expectations and overcoming coordination failures.

6.b These are **theoretical** implications: I have not claimed any empirical, econometric support for my assertions – in spite of being a strong advocate of integrating theory and econometrics on the way to policy.[48] As always, theoretical results are **model dependent** – there is no other way! The model underlying this presentation claims **some generality**: it belongs to the illustrious *general equilibrium family* dating back to Walras, contemporaneously developed by Arrow, Debreu, McKenzie and their followers, and more recently extended to

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[46] Implication (i) is at the heart of market incompleteness; (ii) and (iii) follow immediately, but also have independent status in economic theory.

[47] It has been claimed, in particular by Kurz (1993), that prices tomorrow belong in the definition of the states; see also Drèze (1999c) for a discussion of that claim. The issue raised here bears some analogy: do macroeconomic variables (GDP, inflation, unemployment..) belong in the definition of the states? The question deserves more attention than I could devote to it so far.

[48] To wit: Drèze (1972, 1987b, 1991a).

incomplete markets by the GEI and TGE theorists quoted above. At first sight, doing *general equilibrium theory* –as opposed to *partial equilibrium* or *aggregate modelling* - would justify a claim of *full* generality, not just "*some*" generality. Why the caution? For a simple, obvious reason: the theory remains *incomplete* in that it deals with *static, equilibrium* theory, leaving out altogether *short-run dynamics*![49]
Interestingly, my theoretical journey has led me to derive, from implications of the initial model, **two distinct calls for extensions**. The first, as just recalled, is the need to complement the equilibrium analysis with a specification and analysis of *short-run dynamics*. The second is the need to *model uncertainty*, not just through exogenous states of the world, but also *through endogenous macroeconomic uncertainties*. As recognised above, this has momentous methodological as well as substantive implications, starting with the need of a suitable treatment of *expectations*, and hopefully leading to a better understanding of the role assets indexed on macroeconomic realisations (so-called "*macrosecurities*").[50]
This theoretical conclusion is vividly illustrated by current concerns: our major uncertainty today concerns the depth and duration of the recession, *not* exogenous developments of a physical or purely political nature.

6.c My conclusions suggest emphatically that the **relevant model** for eliciting the **macroeconomic implications of general equilibrium theory** is not the standard Walras-Arrow-Debreu model, but the **incomplete markets model**, in its twin versions: **GEI**, mostly useful for normative analysis; and **TGE**, a must for positive analysis.
It is unfortunate that incomplete-markets analysis starts with the challenge of defining a *decision criterion for business firms*. The more relevant work on that topic, starting with Diamond (1967) and leading to the developments listed in section **3** then **5,** is not part of today's mainstream thinking, nor even of much current work on financial economics.[51] It is equally unfortunate that both GEI and TGE are

[49] The model here remains incomplete in several other respects, the most glaring of which include the absence of a public sector and of imperfect competition (due to fixed costs?).
[50] On this last concept, see Shiller (1993).
[51] To wit: such recent books as Tirole (2006) or Demange and Laroque (2006) fail to incorporate these developments.

technically demanding subjects, a feature that hinders their adoption as standard (mainstream) tools.

And yet, it is difficult to imagine how a proper discussion of policy alternatives towards *coping with the current crisis* could be organised in a less demanding framework. Item (i) above is part of the starting point to study asset monitoring aimed at avoiding the proliferation of lemons (of toxic assets) and at improving the insurance prospects offered by asset markets. Item (ii) is part of the starting point to guide wage policies in the face of growing unemployment. Items (iii) and (iv) are part of the starting point to orient fiscal (and other) policies towards promoting recovery (v).

Of course, much remains to be done, and the key implications collected here are just a starter towards a full policy analysis. To illustrate, consider the timely issue of choosing the *desirable mix of consumption and investment under today's fiscal policy*. The current crisis entails an unexpected loss of wealth that could meaningfully be spread over several generations. That argument invites debt-financed fiscal expenditures aimed at sustaining the consumption of those households suffering from a loss of income and devoid of the savings needed to spread that loss over time.[52].That policy transfers to future generations the part of today's loss of wealth corresponding to the correlative reduction in private investment ("crowding out" effect). But debt financing raises the issue of (partial) Ricardian equivalence – especially in those countries, like the EU, where today's fiscal deficits are presented as short-lived, thereby raising concerns about forthcoming tax hikes. An answer to that concern might be to concentrate the fiscal stimulation on productive public investments whose returns offset the debt service.[53] But the stimulation of investment does not entail sharing today's losses with future generations… So, what is the desirable mix?[54] My model in section **5** does not (yet!) include a public sector, and does not lend itself naturally to balance the interests of successive generations: the GEI approach is better suited to that normative end…Clearly, the work presented here remains very much "incomplete"…another kind of transfer to the next generations!

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[52] They are also the households whose propensity to consume is highest, a natural advantage for effectiveness of fiscal stimulation.

[53] Public housing for low-income dwellers is a front example; see Drèze et al. (1999).

[54] My own inclination would privilege the maximal impact on aggregate demand…

Epilogue.

My conclusion suggests that there exist chapters of economic theory whose relative neglect by our profession has co-existed with relative neglect of issues relevant to the current crisis; resource allocation under market incompleteness, with its corollaries (i)-(iii) in my conclusion, falls in that category.

Of course, in writing that, I am exposing myself to the stricture of being self-centred! Indeed, this lecture abounds in self-citations, citations of works that have been in the public domain for many years and whose relative neglect is perhaps best interpreted as the profession's verdict on their merits…

My excuse is that I have quoted works that embody relatively demanding technical requirements and that cover separate issues (investment under uncertainty, price/wage rigidities, multiple equilibria) that were not integrated before, to my knowledge – both for lack of obvious motivation, and because facing several technical issues simultaneously adds another dimension to our theoretical challenges.[55]

The responsibility for making technical developments accessible to general economists lies with the technical authors. A public lecture offers a privileged occasion to bring esoteric material to the attention of a broad audience. Many thanks to Karl Borch and NHH for creating the current occasion.

[55] My own prior familiarity with these three separate issues is a happy coincidence, partly explained in Dehez and Licandro (2005), perhaps also enhanced by career overextension?

Table 1: notation for section 2.1.

Time: $t = 0, 1$.
States at time 1: $s = 1,..,S$; $\mathcal{S} = \{ 0, 1,..,S \}$.

Agents: H households $h \varepsilon \mathcal{H} = \{ 1,..,H \}$
J firms $j \varepsilon \mathcal{J} = \{ 1,..,J \}$
1 bank 0.

Commodities: L commodities indexed ls, $l \varepsilon \mathcal{L} = \{ 1,..,L \}$, $s \varepsilon \mathcal{S}$, are traded on spot markets at **prices** $p = (p_0,..,p_S) \varepsilon \Pi_s R^L_+$.
Initial endowments of commodities are $w^h = (w^h_0,..,w^h_S) \varepsilon R^{(S+1)L}$.

Money holdings: $m^h = (m^h_0, m^h_1,.., m^h_S) \varepsilon R^{(S+1)}_+$
$m^j = (m^j_0, m^j_1,.., m^j_S) \varepsilon R^{(S+1)}_-$ (input sign convention)
$M = \Sigma_h m^h - \Sigma_j m^j$.
Money balances are supplied by the bank at a cost $r = (r_0,..,r_S) \varepsilon R^{(S+1)}_+$.

Assets: all assets are held by households, not by firms;
they are traded on the stock exchange at $t = 0$.
Bond holdings are $b^h \varepsilon R$, with price ϕ, no initial holdings.
Shares of stock of the J firms, with prices $\pi \varepsilon R^J_+$, are held in amounts $\theta^{hj} \varepsilon [0, 1]$ with initial holdings δ^{hj}, $\Sigma_h \delta^{hj} = 1 \ \forall\ j$.

Firm j chooses a production plan $(y^j, m^j) \varepsilon \Psi^j(p) \subset R^{(S+1)L} \times R^{(S+1)}_-$.

Houshold h chooses, on basis of its preferences $\geq^h$, a consumption plan $(x^h, m^h) \varepsilon \Xi^h(p) \subset R^{(S+1)L} \times R_-^{(S+1)}$ and a **portfolio** $(\theta^h, b^h) \varepsilon [0, 1]^J \times R$ subject to the **budget constraints**

$$0 \geq p_0(x^h_0 - w^h_0) + r_0 m^h_0 + \pi(\theta^h - \delta^h) + \varphi b^h - \Sigma j \theta^{hj}(p_0 y^j_0 + r_0 m^j_0) - \theta^{h0} r_0 M_0 \quad (2.1)$$

$$0 \geq p_s(x^h_s - w^h_s) + r_s m^h_s - b^h - \Sigma_j \theta^{hj}(p_s y^j_s + r_s m^j_s) - \theta^{h0} r_s M_s,\ s = 1,..,S. \quad (2.2)$$

Table 2: notation for section 5.2.

There are **two sets of commodities**: L^I with flexible prices
L^{II} with *downward nominal* rigidity.
At time 0, $p_{l0} \geq \bar{p}_{l0}$ $\forall$ l ε L^{II}. When $p_{l0} = \bar{p}_{l0}$, supply rationing sets in, through a *market parameter* σ_{l0}.
The agents observe **supply constraints** $l^h(\sigma_{l0})$, $l^j(\sigma_{l0})$:

$$x^h_{l0} - w^h_{l0} \geq l^h(\sigma_{l0}) \ \forall \ h \ \varepsilon \ H, \ \ y^j_{l0} \geq l^j(\sigma_{l0}) \ \forall \ j \ \varepsilon \ J. \tag{5.1}$$

At node s ε S, the **dividends** of firm j are $V^j_s := p_s y^j_s + r_s m^j_s$, and the dividends of the bank are $V^0_s = r_s M_s$. In vector form: $V^+_s = (V^0_s, V^1_s,..,V^J_s) = (V^0_s, V_s)$.

The **signals** observed by the households at node s are (ξ^s, ζ^s), where $\xi^0 = (p_0, r_0, \sigma_0, \pi, \varphi)$, $\xi^s = (p_s, r_s, \sigma_s)$, and $\zeta^s = V^+_s$, s ε S. Given the signals at time 0, household h has **expectations** about the *occurrence of state s together with signals* (ξ^s, ζ^s); these expectations are given by the **probability measure** $\psi^h(\xi^s, \zeta^s \mid \xi^0, \zeta^0)$.

A **decision** for household h given (ξ^0, ζ^0), denoted $d^h(\xi^0, \zeta^0)$, consists of a time 0 component $d^h_0 = (x^h_0, m^h_0, \theta^h, b^h)$ and **contingent plans** $d^h_s = (x^h_s, m^h_s \mid \xi^s, \zeta^s)$.

Preferences over decisions are represented by their **expected utilities**

$$U^h(d^h_0 \mid \xi^0, \zeta^0) = \Sigma_s \psi^h(\xi^s, \zeta^s \mid \xi^0, \zeta^0) \ \ \bar{u}^h(d^h_0, d^h_s(\xi^s, \zeta^s)). \tag{5.2}$$

I denote by $P^h(d^h)$ the set of decisions **strictly preferred** by h to d^h.

Given (θ^h, b^h), these preferences also admit representation by an *indirect utility* Λ^h, a function of the parameters of the budget constraints (2.2). For ease of notation, denote the **income** of h at node s by v^h_s, where
$v^h_0 = p_0 w^h_0 + \Sigma_j \pi^j \delta^{hj} + \Sigma_j \theta^{hj} V^j_0 + \theta^{h0} r_0 M_0$ and
$v^h_s = p_s w^h_s + \Sigma_j \theta^{hj} V^j_s + b^h + \theta^{h0} r_s M_s$, s = 1,..,S.
Write $v^h := (v^h_0, v^h_1,..v^h_S)$, $\xi := (\xi^1,..\xi^S)$.
Then the indirect utility of (v^h, ξ) given (ξ^0, ζ^0) and (θ^h, b^h) may be written as

$$\Lambda^h(v^h, \xi / \xi^0, \zeta^0, \theta^h, b^h) := \Sigma_s \ \psi^h(\xi^s, \zeta^s / \xi^0, \zeta^0) \ \Lambda^h_s(v^h_0, v^h_s, \xi^s / \ \xi^0, \zeta^0, \theta^h, b^h).$$

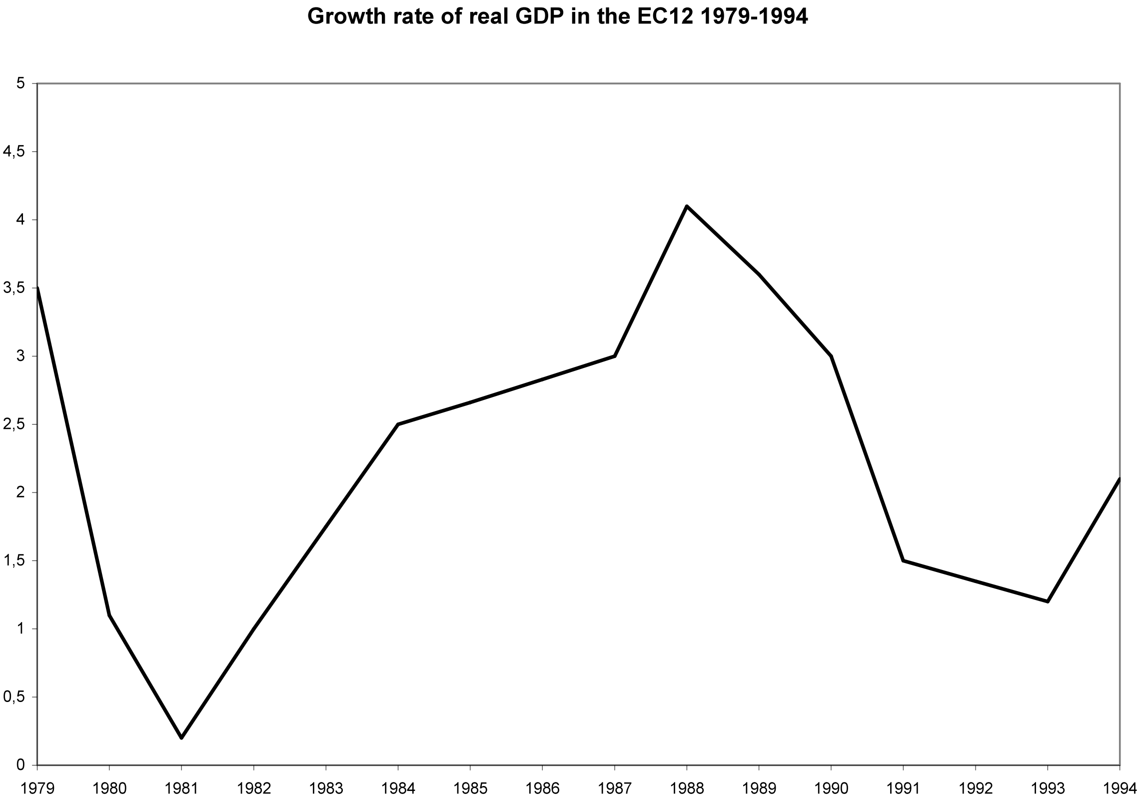
Turning to **firms**, their decisions are similarly defined as $d^j := (d^j_0, d^j_s(\xi^s))$. Their decision criterion is based on the *control principle*, with a priori given control groups $I^j \subseteq \{h \in H \mid \delta^{hj} > 0\}$.

A household h with $\theta^{hj} > 0$ will favour a decision $d^{j\prime}$ entailing the dividend stream $V^{j\prime}$ over a decision d^j entailing the stream V^j if and only if

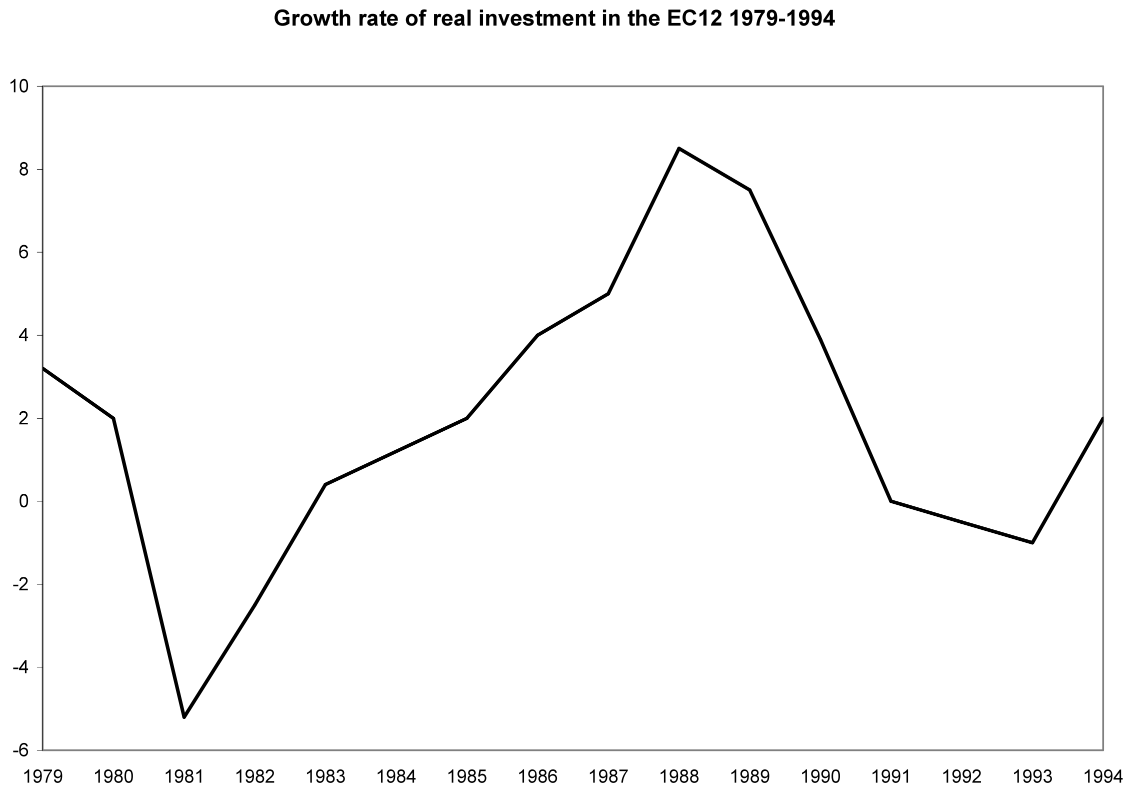
$$\Lambda(v^h + \theta^{hj}(V^{j\prime} - V^j), \xi) > \Lambda^h(v^h, \xi). \tag{5.4}$$

I write $P^{hj}(d^j)$ for the set of decisions by firm j strictly preferred by h to d^j; and $P^j(d^j/\theta)$ for the set of decisions by firm j strictly preferred to d^j under the "control principle" given the share ownership θ.

Figure 1.a

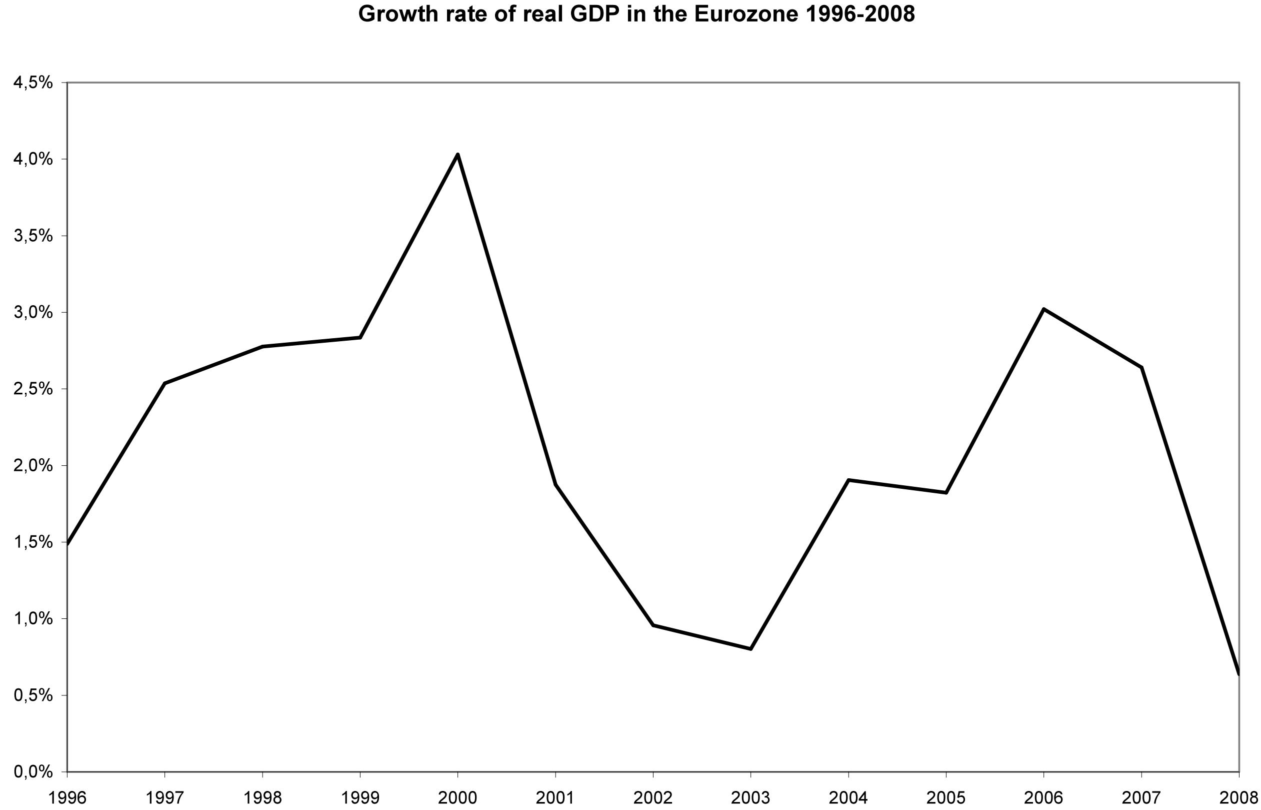


Growth rate of real GDP in the EC12 1979-1994
5
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994

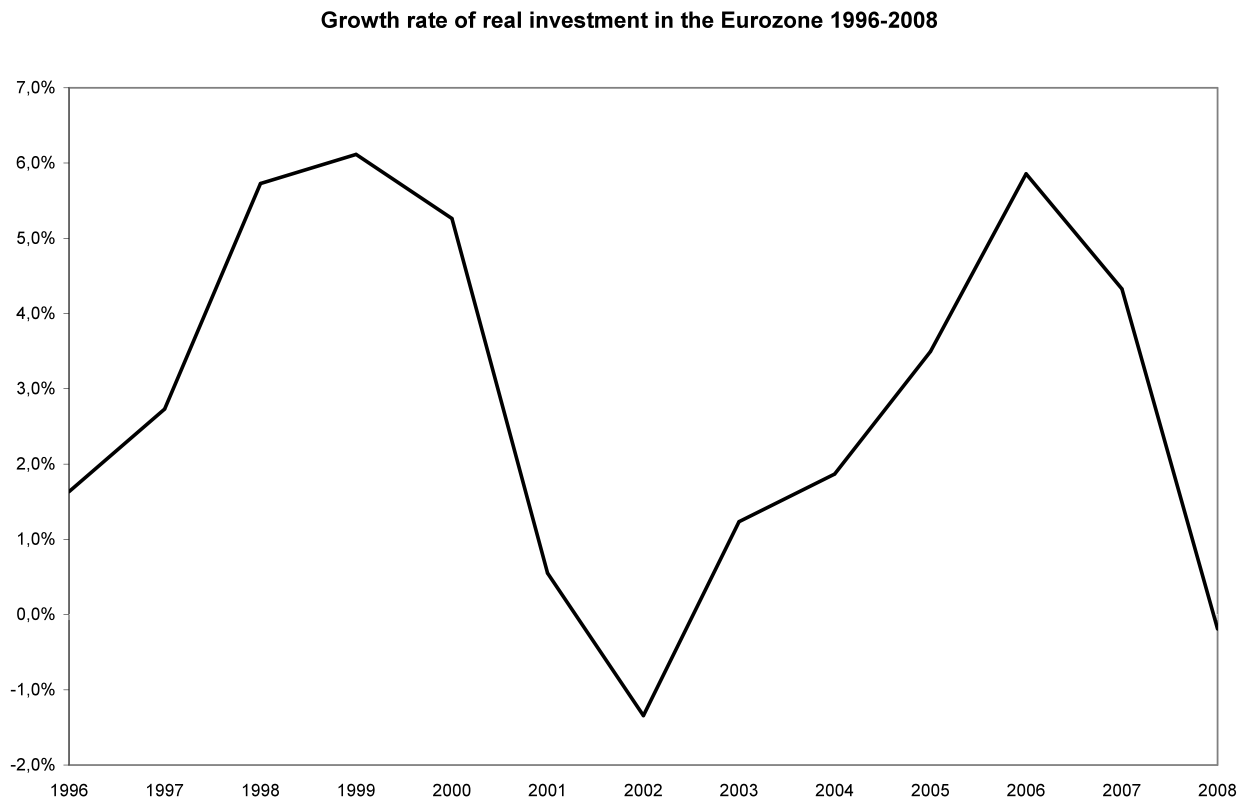


Growth rate of real investment in the EC12 1979-1994
10
8
6
4
2
0
-2
-4
-6
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994

Figure 1.b



Growth rate of real GDP in the Eurozone 1996-2008
4,5%
4,0%
3,5%
3,0%
2,5%
2,0%
1,5%
1,0%
0,5%
0,0%
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008



Growth rate of real investment in the Eurozone 1996-2008
7,0%
6,0%
5,0%
4,0%
3,0%
2,0%
1,0%
0,0%
-1,0%
-2,0%
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008

Figure 2

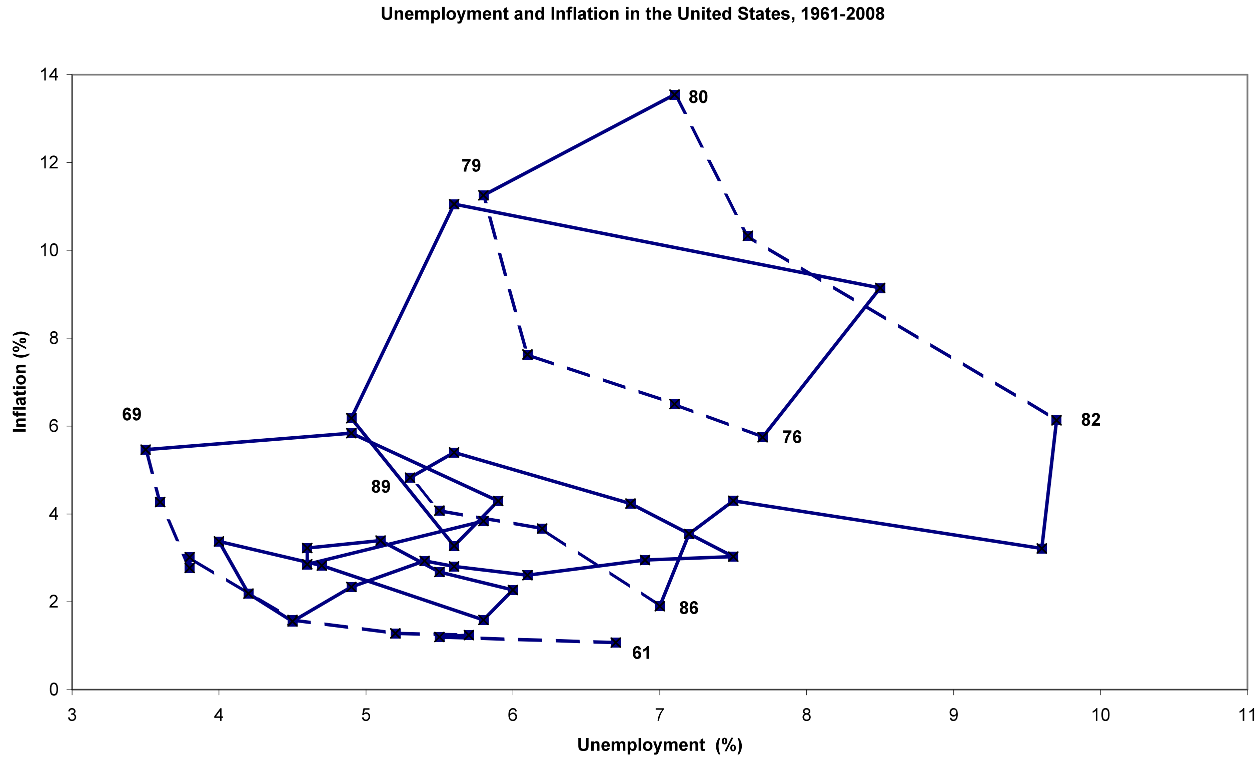
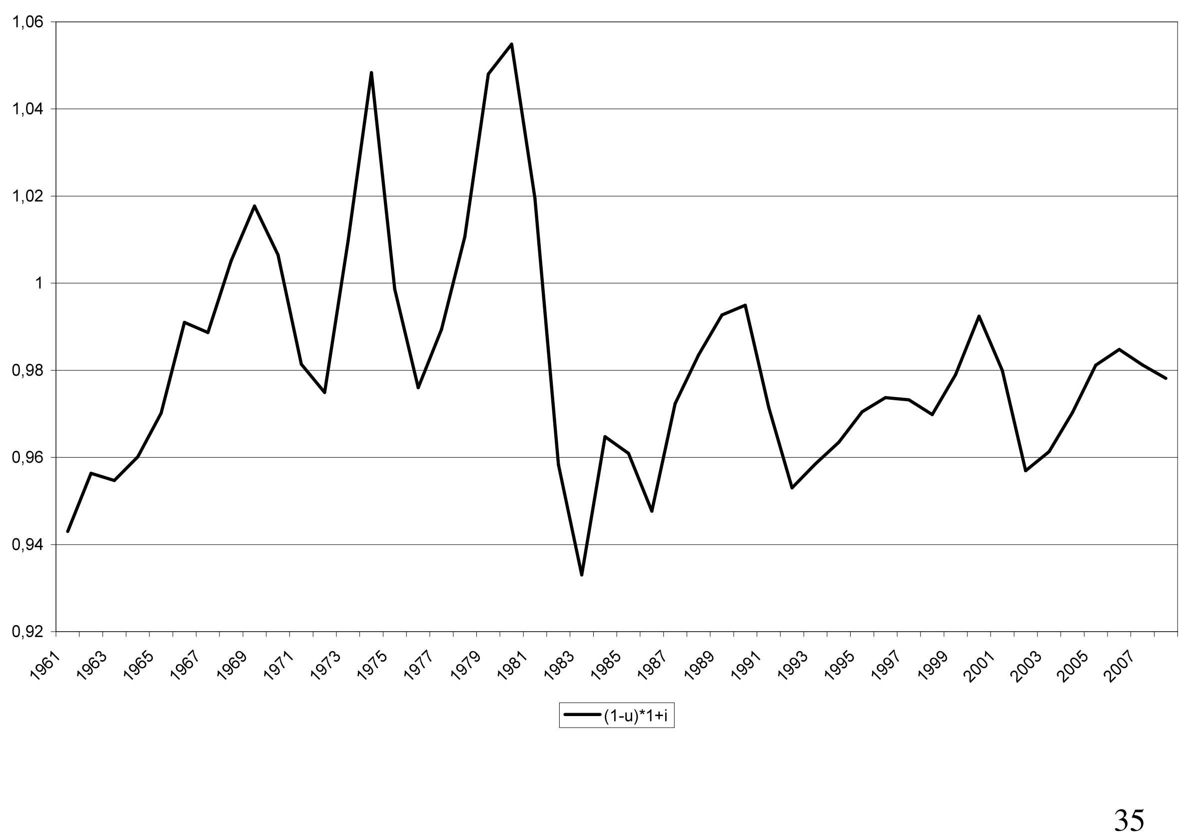


Figure 3 (1 - $\bar{u}$). (1 + i) (United States)



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