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Gender Inequality, Technological Progress,
and the Demographic Transition

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Gender Inequality, Technological Progress, and the Demographic Transition

Nguyen Thang Dao[1] and Julio Dávila[2]

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Abstract

This paper proposes a new mechanism linking technology, the gender gap in education, and fertility in a growth model in order to explain the long run transition from stagnation to modern sustained growth, through the demographic transition, and the accompanying improvements in gender equality in education and income. The mechanism includes three main components. First, increases in the level of technology not only increase the return to human capital but also reduce women's time in doing housework, leaving women with more time for child care and labor-force participation, since technological progress creates labour-saving products for doing housework. Second, the decreases in women's time devoted to housework in the future make households today invest relatively less in education for their sons in order to invest more in education for their daughters because the marginal return to female education is higher than that to male education, therefore, improving the gender equality in education. Third, the better gender equality in education, in turn, accelerates the technological progress. This positive feedback loop generates a demographic transition accompanied with accelerated economic growth.

Keywords: Technological progress, Gender inequality in education, Demographic transition, Fertility, Human Capital

JEL Classification: 11, J13, J16, O11, O40

[1] Mercator Research Institute on Global Commons and Climate Change (MCC), Torgauer Straβe 12-15, 10829 Berlin, Germany. Email: daonguyen@mcc-berlin.net.

[2] CORE, Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium. Email: julio.davila@uclouvain.be.

1. Introduction

Recently, there has been a renewed interest, both from the theoretical and empirical viewpoints, in the links between gender inequality, fertility, and growth to explain some stylized facts of the development processes of societies, such as shown in Galor and Weil (1996), Klasen (2002), Lagerlof (2003), Klasen and Lamanna (2009), Doepke and Tertilt (2009), De la Croix and Vander Donckt (2010), and Diebolt and Perrin (2013a,b) among others. Some of these stylized facts widely observed across societies are: (i) a negative correlation between fertility and female-to-male education ratio; (ii) a positive correlation between per-capita income and female-to-male education ratio; (iii) a negative correlation between female-to-male earning ratio and fertility; (iv) a decline over time in the human capital and earnings gaps between male and female workers; (v) an increase over time in the female labor-force participation, and (vi) a demographic transition in societies as they enter the regime of modern sustained economic growth.

Gender gaps versus fertility and per-capita income growth
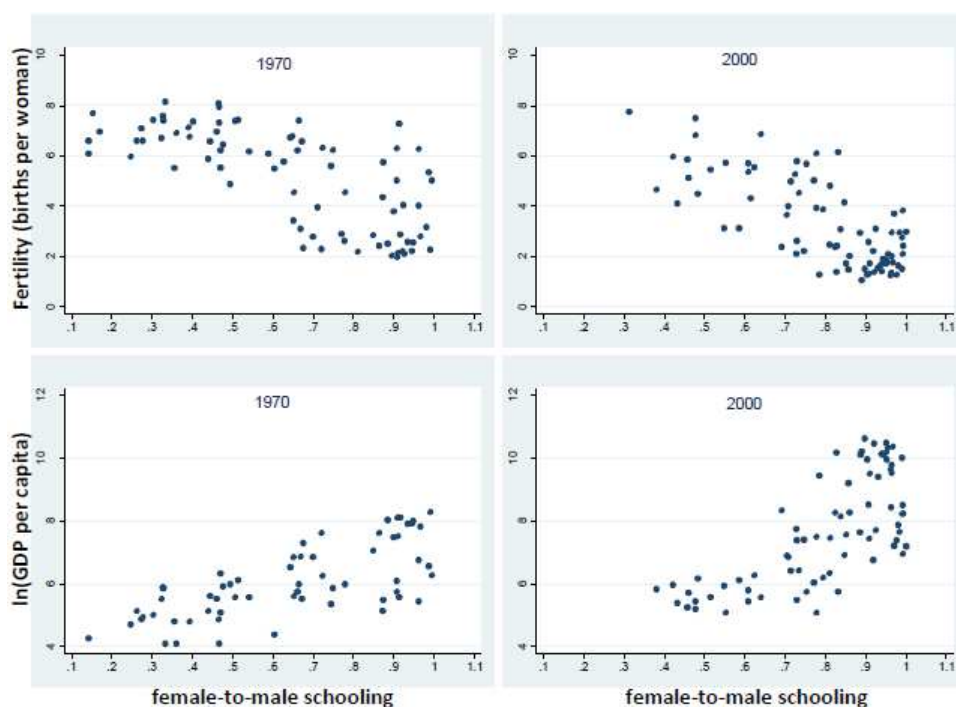
Figure 1. Cross-country plots of fertility and per-capita income against gender equality in education. Source: World Bank (2013a, b).

The Figure 1 above provides the cross-79-country (almost all developing countries) plot, in the years 1970 and 2000, of per-capita income (in logarithms, US dollars) and the fertility rate, against gender inequality in education, measured as the number of schooling years of women over that of men. As the figure shows, the gender equality in education looks strongly

positively correlated with per-capita income, and negatively correlated with fertility, so that the closing of the relative gap in earnings between women and men occurs with the decline in fertility. One can reasonably argue that such a gap in earnings is due to the gap in the education that women and men receive. This stylized fact is also reflected in the upper graphs in Figure 1 above.

The negative correlation between gender inequality in income and fertility has been well explained by Galor and Weil (1996), Lagerlof (2003), and others. Like this previous research, we argue in this paper that an increase in female relative wage increases the opportunity cost of raising children more than the household income, which makes fertility decline (when the level of technology is sufficiently high). However, this paper revisits this stylized fact, in a different growth framework, to point to a mechanism leading also to the decline in gender inequality in income that has been documented. Lagerlof (2003) treats the relative human capital gap between women and men, and hence the relative wage gap between them, as exogenous, and considers the impact of different relative gender wage gaps on the divergence in fertility and the long run growth across societies. Galor and Weil (1996) argue that the decline over time in the gender wage gap is due to the accumulation of physical capital, since physical capital is more complementary to women's labor than men's. We show in this paper that, moreover, the technological progress plays a crucial role by making the gender relative gap in human capital decline, and hence improves too the gender equality in relative income.

Gender gap in human capital

Figure 2 below uses literacy rates to stand for human capital. The literacy rate is not the only possible measurement for human capital. However, the evolution of these rates for men and women can be a proxy for the evolution of human capital. In particular, figure 2 intuitively conveys the decline in the gender gap in education, and hence in human capital.
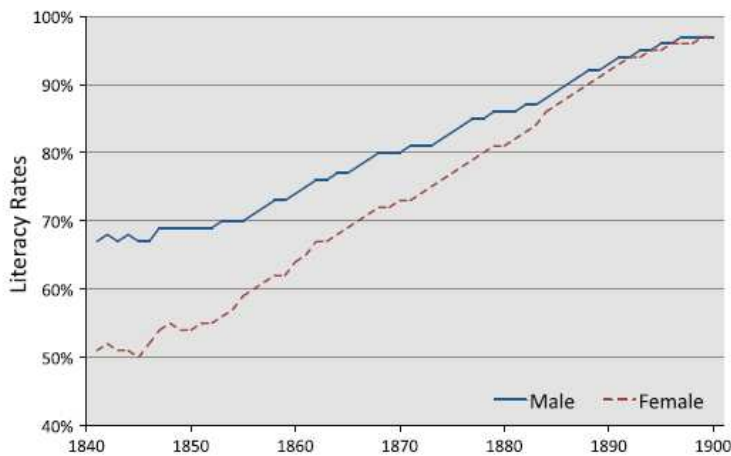

Figure 2. The decline in human capital gap: England 1840 - 1900. Source:[3] Cipolla (1969)

[3]Quoted in Galor (2012)

Most of papers in the literature study the effects of gender inequality in education or human capital on economic growth, as well as conversely the effect of economic growth on gender inequality in education. A list of selected papers includes Barro and Lee (1994), Dollar and Gatti (1999), Klasen (2002), Lagerlof (2003), and Klasen and Lamanna (2009), among others. However, the question addressed here is why gender inequality in human capital decreases during the development process? And how this dynamics of human capital relates to the other stylized facts?

Labor-force participation of women

The increase over time in female labor-force participation, along with the decrease in fertility, due to a decrease in the gender wage gap, is examined theoretically in Galor and Weil (1996), Lagerlof (2003), and Bloom et al. (2009). Greenwood et al. (2005) explain also the increase in female labor-force participation by a technological progress creating labor-saving products for doing housework in a model with constant fertility. This paper builds also on the impact of technological progress on women's time devoted to housework as proposed in Greenwood et al. (2005). However, this paper differs from Greenwood et al. (2005) in that it endogenizes technological progress and analyzes the decrease in gender inequality in education due to the impact of technological progress on women's time devoted to housework, while explaining the demographic transition during the development process.

Evolution of income growth and population growth in Western Europe

The population and output of Western Europe has experienced three distinct regimes going from Malthusian stagnation, through the demographic transition, to modern sustained growth. After thousands of years in Malthusian stagnation characterized by very low growth rates both in per-capita income and population, the economy entered the phase of demographic transition in which the growth rates of both per-capita income and population increased simultaneously. Later on, the growth rate of per-capita income still increased while the growth rate of population fell, as the economy entered the regime of modern sustained growth (see figure 3). This stylized fact is explained by Galor and Weil (2000) and Galor and Moav (2002) by means of unified growth theory models, and recently confirmed empirically by Becker et al. (2010, 2011). Like these other papers, this paper also highlights the role of human capital in the technological progress and the demographic transition, as well as the interactions between them during the development process. Most of the previous literature overlooks, however, the gender issue and its interactions with technology and fertility, while this paper does.
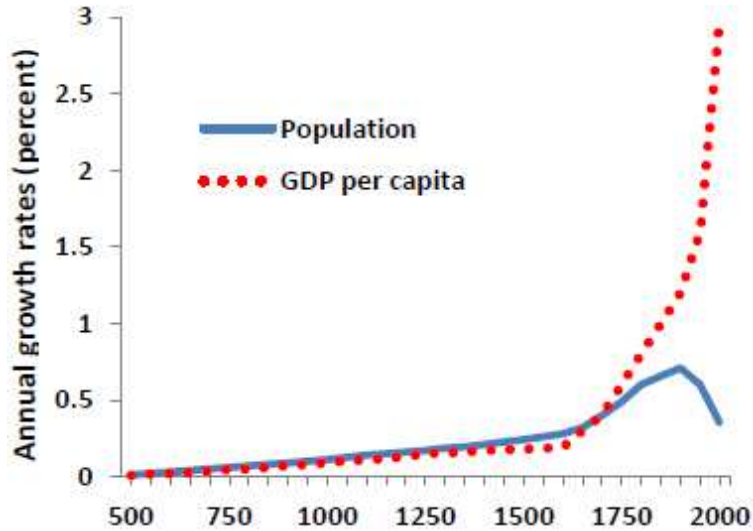
Figure 3. The growth rates of per-capita income and population of Western Europe in the three regimes.
Source: Madison (2001)

The closest paper to this one may be Diebolt and Perrin (2013b), which proposes a unified growth theory model to explain the development process considering gender inequality as well. Basically, Diebolt and Perrin (2013b) also stress the importance of human capital accumulation for economic growth and the positive effect of technological progress on skilled human capital through an increase in the return to education. Nonetheless, the most important differences between Diebolt and Perrin (2013b) and this paper, leading to different explanations for the development process from stagnation to modern sustained growth, are: (i) Diebolt and Perrin (2013b) assume that individuals invest in education for themselves when they are adults, while in this paper we assume that individuals receive educational investment from their parents when they were children; and (ii) Diebolt and Perrin (2013b) consider the *bargaining power* of the wife, which depends on both incomes of the wife and the husband, determining the equality in human capital between the wife and the husband, while this paper considers the *positive effect of technological progress on the potential female labor supply* which makes the households increase the share in educational investment for their daughters. Therefore, the mechanism driving the transition from stagnation to modern sustained economic growth in Diebolt and Perrin (2013b) differs from the one proposed in this paper. In their model, technological progress triggers the female empowerment which, over time, induces women to invest more in human capital for themselves, contributing to human capital accumulation, and hence fostering economic growth. In parallel, the higher female human capital increases the opportunity cost of raising children, making the fertility decline. The mechanism in this paper, however, is that the technological progress increases the potential female labor supply which makes the households increase the share in educational investment for their daughters. The resulting improved equality in human capital between women and men, in turn, accelerates technological progress.

5

This feedback generates both the demographic and economic transitions to modern sustained growth. However, in Diebolt and Perrin (2013b), the growth rate of population, proxied by fertility, always declines over time along with technological progress and the bargaining power of the wife, while it is a fact, *captured* in this paper, that the growth rate of population increases during the early stages of development (before the demographic transition). And since Diebolt and Perrin (2013b) assume that adult individuals spend time for educating themselves to increase their human capital, although their model generates a decline over time in fertility with an increase in human capital, an explanation for the simultaneous increase in female labor-force participation is still absent. In addition, Diebolt and Perrin (2013b) assume the human capital formation is linear in education investment (other factors given, the marginal return to education investment is always constant when education investment exceeds a fixed cost), while we assume in this paper that the human capital formation is an increasing and concave function of education investment. Finally, the positive effect of gender equality in education on technological progress comes in Diebolt and Perrin (2013b) from a positive externality of women's human capital on their children's human capital formation, while in our model this positive effect comes from the higher marginal return to female education in human capital formation.

Although there has been a huge empirical literature considering the relationship between gender inequality and economic growth, the theoretical literature on this issue seems rather limited. In addition, to the best of our knowledge, no published paper so far has explained all stylized facts above in one single theoretical model. Most papers in the related literature only explain the combinations of some of the stylized facts above. This paper, therefore, aims at contributing a simple growth model capturing technological progress, gender inequality in education, fertility and the complex interaction between these issues to explain all the stylized facts listed above. Specifically, it shows that the demographic transition to modern sustained growth, the decline over time of the human capital and earning gender gaps, and the simultaneous increase of the labor-force participation of women are inevitable outcomes of the development process when the driving force for technological progress is the average human capital.

Finally, the iterature has recently focused on explaining the reversal of education between genders, particularly for developed countries (Goldin et al. 2006, Chiappori et al. 2009, Becker et al. 2010, Hazan and Zoabi 2012). Such a reversal of education between genders is an interesting stylized fact that has been observed for the last few decades in almost all developed countries, where increases in female education and labor force participation go hand in hand with the outsourcing of household chores to immigration cheap labour and the provision of adequate public and private kindergardens. This way, women have more opportunities to invest in their own education in order to improve their competivity in the labor market. The paper at hand, however, does not intend to capture this stylized fact. Instead, we focus on the

long run transition from stagnation through the demographic transition and beyond, where the household behaviour is characterized by an education gap typically in favor of the male, and the fertility of household depends crucially on this gap.

The rest of paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model. Section 4 analyses the effects of technological progress on gender inequality in education, fertility, and female labor-force participation. The competitive equilibria and the dynamical system governing them are identified in section 5. Section 6 analyses the development process to explain the stylized facts. Section 7 concludes the paper.

2. Related literature

Cubers and Reignite (2012) provide an excellent review of the literature on gender inequality and economic growth, both in theory and empirics. They consider gender inequality in its many aspects. From their review it follows that most papers in the literature find gender inequality to be harmful for economic growth, and that economic growth helps to improve gender equality.

Among previous theoretical works considering gender inequality in growth models, the most cited may be Galor and Weil (1996). The authors advance a theory to interpret the feedback loop between gender gap, fertility, and growth by which higher wages for women reduce fertility by raising the opportunity cost of children. The lower fertility, in turn, raises the level of per capita physical capital. In Galor and Weil (1996), the only difference between men and women, resulting in a gender wage gap, is that men have more physical strength than women. Since physical capital is more complementary to mental labor than to physical labor, women's wages increase relatively to that of men due to capital accumulation. As a consequence, the model exhibits multiple steady state equilibria. In one of them fertility is high while output and capital per worker are low, and hence women's relative wage is low. The other is characterized by low fertility, high output and capital per worker, hence a high relative wage for women. They conclude that countries with a high initial level of capital per worker will converge to a high income level equilibrium with low fertility and high relative wages for women. The opposite would be true for countries with a low initial level of capital per worker. Cavalcanti and Tavares (2007) use the model in Galor and Weil (1996), allowing for public spending, to argue that the increase in income per capita and decline in fertility are accompanied by: (i) an increase in the share of government expenditure in total output; and (ii) an increase in women's labor force participation. The approach in Galor and Weil (1996), however, assumes that women and men have identical human capital implying male and female are endowed the same educational investment, whereas differences in education are observed widely in many countries, with females typically receiving less education than

males.

Lagerlof (2003) examines the links between gender inequality in human capital and long run economic growth. The author points to a threshold of relative equality in human capital between women and men beyond which (i.e. for relatively high equality) the economy can exhibit sustained growth, otherwise the economy converges to a non-growing steady state, a Malthusian trap. Indeed, Lagerlof (2003) shows that inequality in human capital can result in high fertility, low economic growth, and continued gender inequality in providing human capital for males and females, thus generating a poverty trap that calls for public intervention. That paper, however, assumes that (for whatever reason) men have more human capital than women and the relative inequality in human capital between them is fixed, i.e. it treats this relative inequality as an exogenous variable. Actually, gender inequality differs across cultures, and its size changes endogenously along with technological progress.

Greenwood et al. (2005) provide a channel through which economic growth affects positively gender equality in employment. They argue that technological progress in the household sector is embodied in the form of new labor-saving consumer durables which free up women's time devoted to housework, making them increase their labor force participation. In their approach, however, the authors explicitly assume that the technological progress and gender gap are exogenous, and overlook fertility as well as education investment.

Doepke and Tertilt (2009) propose an interesting mechanism of a positive effect of growth on gender equality. The authors investigate men's incentive to share power with their wives. They argue that, from a man's perspective, he wants his wife to have no rights. But he cares about his daughters's marital bargaining power vis-ï¿œ-vis his son-in-law because he is altruistic to his children. In Doepke and Tertilt (2009), an expansion of wives' legal rights increases human capital investment for their children, helping them to find quality spouses which is also in the preference of the fathers. Therefore, the father gains from the increased power of his children's future mothers-in-law because his children will have quality spouses. That is to say, men face a trade-off between the rights they want for their own wives and the rights of other women in society. This trade-off shifts over time because of the changing role of human capital driven by technological progress. The authors show that when the returns to education are low, men will vote for the regime in which all family decisions are made solely by the husband. When technological progress changes the importance of human capital, men may vote for a regime under which decisions are made jointly by husband and wife. De la Croix and Vander Donckt (2010) propose a model, which is also based on the intrahousehold bargaining between man and woman, capturing several aspects of gender inequality (such as the survival gap, the wage gap, the social and institutional gaps, and the educational gap) to analyze their impacts on demographic and economic outcomes for least developed countries. The authors point out that a key measure to ease these countries out of a poverty trap is to promote survival probabilities of female and infant, which make women more likely to be

active in the market, leading to female education to be more important. A better female education increases the bargaining power of women in the households' decision process, hence decreasing fertility and improving the quality of children, as well as fosters economic growth. One can find more analytical works relating to woman's rights and marital bargaining power in Basu (2006), Fernandez (2009), Doepke and Tertilt (2011), Doepke et al. (2012), and more recently Diebolt and Perrin (2013a, b).

In parallel to theoretical studies, a huge empirical literature has also examined the complex relationship between gender inequality and economic growth. The availability of comprehensive international datasets has allowed the emergence of a large number of time series, cross-section, and panel data empirical studies of this topic. An early study by Barro and Lee (1994) reported a "puzzling" finding that gender inequality in education might increase economic growth. The authors find that when they include male and female primary and secondary schooling in the regression, the coefficient associated with female schooling is negative. However, more recent papers have shown that the opposite actually holds, i.e. gender inequality in education reduces economic growth (Dollar and Gatti 1999, Forbes 2000, Knowles et al. 2002, Klasen 2002, Abu-Ghaida and Klasen 2004, Klasen and Lamanna 2009). These papers also explain why Barro and Lee (1994) found the opposite effect, and show that more careful econometric techniques support that gender inequality in education inhibits economic growth.[4]

Dollar and Gatti (1999) study the effects of gender gaps in education, health, and life expectancy, the legal and economic equality in society and marriage, and the degree of women's empowerment on economic growth. In contrast to Barro and Lee (1994), they find that the coefficient associated with female education is positive, whereas that associated with male education is negative but statistically insignificant. However, the positive effect of female education on growth is nonlinear: an increase in female education has no effect on economic growth for countries with very low female education. But, in countries with relatively high female education, increasing it spurs economic growth. Dollar and Gatti (1999) also provide a strong evidence that increases in per capital income lead to improvements in gender equality in education and health care. And, they conclude that societies that have a preference for not investing in girls pay a price for it in terms of slower growth and lower income.

By using cross-country data 1960 - 2000 and panel regression, Klasen (2002) and Klasen and Lamanna (2009) show that gender inequality in education directly affects economic growth by lowering the average level of human capital and indirectly affects economic growth through its impact on population and investment. In contrast to Barro and Lee (1994), they find that the negative coefficient associated with female education disappears when the multicolinearity problems are taken into account and dummy variables of regions are added. Interestingly,

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[4]Many authors show that Barro and Lee (1994) identified the absence of regional dummy variables, particularly for Latin America and East Asia, making their estimation biased. Their biased finding may also be related to multicolinearity. In most countries, female and male education are closely correlated, making it difficult to estimate their individual effects. Large standard errors for male and female education in Barro and Lee (1994) and the sudden reversal of this finding in other specifications is a strong evidence of this problem. For more discussion of these issue, see Dollar and Gatti (1999), Forbes (2000), Klasen (2002), and Klasen and Lamanna (2009).

these two papers estimate lower and upper bounds on the effect of gender inequality on economic growth. The findings of these papers differ significantly from Dollar and Gatti (1999) in that they find that the negative effect of gender inequality in education not only appears among countries with relative high female education, but also among countries with low female education. They justify the causes for this difference by the fact that they use a larger time-series dataset (1960 - 2000 rather than 1975 - 1990), they use a different measure of human capital (the total years of schooling rather than the share of the adult population with secondary education), and claim that a multicolinearity problem was overlooked in Dollar and Gatti (1999).

Along with these empirical studies above, many other papers also find a negative effect of gender inequality in education on economic growth. An incomplete list of papers includes Hill and King (1995), Tzannatos (1999), Lorgelly and Owen (1999), Forbes (2000), Knowles et al. (2002), and Abu-Ghaida and Klasen (2004), etc.

3. The model

3.1. Preferences and constraints

In any period $t \in \mathbb{N}$, the economy consists of L_t identical households. Each household is composed by one man and one woman. We use indexes m and f to denote for sexes male and female respectively. Each member of the household is endowed one unit of time. Households allocate their time to labor supplied to the market to earn income, to child-rearing, and to housework. As in Becker (1985, p.52), we assume that in a household, the woman is fully responsible for child-rearing and doing other housework.[5] The time devoted to raising children and doing housework cannot be used to work in the market. Households born at date $t - 1$ have preferences over their consumptions at date t, the number of their children, and the income that each couple of their children can earn when they are adults as follows

$$u_t = \gamma \ln \left(n_t w_{t+1} \left[h_{t+1}^m + (1 - \varphi_{t+1}) h_{t+1}^f \right] \right) + (1 - \gamma) \ln c_t \tag{1}$$

where c_t is household consumption in period t; n_t is the number of children (since the basic unit of analysis in this model is a couple then n_t is in fact the number of couples of children that each couple t has)[6]; w_{t+1} is the return to human capital in period $t + 1$; h_{t+1}^m and h_{t+1}^f are the human capital each male and female children is endowed with by their parents

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[5]It is widely assumed that working in the market requires not only human capital but physical strength of workers also, specially in developing economies, so that men have an advantage in supplying their labor to the market compared to women. Moreover, traditional social conventions still make women responsible for child care and other housework in developing countries, and to some extent in developed countries too. At any rate, introducing time for men to do housework does not change the qualitative analysis as long as men take care of the housework less than women do.

[6]Here we also assume that the gender birth ratio (male over female) is 1 : 1 which is closed to the natural gender ratio 1.05 : 1.

respectively; and $\varphi_{t+1} = \varphi(A_{t+1})$, which depends on the level of technology A_{t+1} at $t+1$, is the time that women will spend for housework in period $t+1$. We assume here that the time for doing housework is *inevitable* and households in period t perfectly foresee that their daughters will spend that time doing housework in the period $t+1$, while households do not have perfect foresight on the fertility choice of their offspring. Therefore, households consider $w_{t+1} \left[h_{t+1}^m + (1 - \varphi_{t+1}) h_{t+1}^f \right]$ as the potential income that each couple of their offspring can earn when adults.

Greenwood et al. (2005) provide a theoretical framework to argue that technological advancements in the household sector play a crucial role in liberating women from housework. It is obvious that, along with technological progress, the appearance of household sector products such as washing machines, vacuum cleaners, refrigerators, etc., has helped women in saving time when doing housework. The appearance of some other products, such as frozen foods and ready made clothes, due to technological progress, also liberate women from housework. So, it is rather plausible that in general the time devoted to housework is decreasing in the level of technology. Hence, we assume that[7]

$$\varphi'(A) < 0, \ \varphi''(A) > 0 \qquad \text{and} \qquad 0 = \lim_{A \to +\infty} \varphi(A) < \bar{\varphi} = \varphi(0) < 1 \qquad (A1)$$

The human capital formation for a child with sex $i \in \{m, f\}$ is given by

$$h_{t+1}^i = (e_{t+1}^i)^\theta \tag{2}$$

where $\theta \in [\frac{1}{2}, 1)$ for reasons to be apparent in section 4.2, and e_{t+1}^i is the household t's educational investment for a child with sex $i \in \{m, f\}$.

The budget constraint of the household born at date $t - 1$ is therefore

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \leq w_t[h_t^m + (1 - \varphi_t)h_t^f] \tag{3}$$

where $\rho > 0$ is the cost in time required to raise one couple of children physically. Since only the woman takes care of the children in the household, and the time raising children cannot be used to work in the market, then the opportunity cost for raising one couple of children is $\rho w_t h_t^f$.

Since each person in period t is endowed one unit of time, then the time constraint of the woman in period t is

$$\rho n_t + \varphi_t \leq 1 \tag{4}$$

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[7]The assumption $\lim_{A \to +\infty} \varphi(A) = 0$ is a simplification. The analysis does not change qualitatively if we set $\lim_{A \to +\infty} \varphi(A) = \underline{\varphi} \in (0, \bar{\varphi})$.

3.2. Production and technology

In each period t, output can be produced out of human capital according to the following production function

$$Y_t = f(A_t)H_t \tag{5}$$

where Y_t is aggregate output produced in period t; H_t is aggregate human capital supplied to production in period t; $f(A_t) > 0$, $f'(A_t) > 0$, $\forall A_t > 0$, with A_t being the level of technology in period t; and $\lim_{A_t \to +\infty} f(A_t) = \bar{f}$.

The output per household in period t is

$$y_t = \frac{Y_t}{L_t} = f(A_t)\left(h_t^m + [1 - \rho n_t - \varphi(A_t)]h_t^f\right) \tag{6}$$

The return to human capital in period t is

$$w_t = f(A_t) \tag{7}$$

The dynamics of technology is

$$A_{t+1} = (1 + g_t)A_t \tag{8}$$

where g_t is the rate of technological progress between periods t and $t+1$. We assume that g_t depends on the average human capital of the working generation t, i.e.

$$g_t = g\left(\frac{h_t^m + h_t^f}{2}\right) \tag{9}$$

where $g(h) > 0$, $g'(h) > 0$, $\forall h \geq 0$.[8]

3.3. Household's optimization

Households of generation t choose the optimal mixture of quantity and quality of their children and supply their remaining time, after finishing housework, in the labor market to consume

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[8] One might argue that the roles of male and female human capital on technological progress should be weighted by the time that male and female workers participate in production, i.e. $g_t = g\left(\frac{h_t^m + [1 - \rho n_t - \varphi(A_t)]h_t^f}{2}\right)$ instead of $g_t = g\left(\frac{h_t^m + h_t^f}{2}\right)$. As a matter of fact, modeling technological progress this way just slows down the speed of convergence to a modern sustained growth regime, compared to the modeling choice above, but does not change it qualitatively. The proof for this statement is available upon request. However, we should think that the female human capital, which partly determines the level of technology in the next period, not only has an impact on technological progress through female participation in production but also through the time that women spend taking care of their children which can be viewed as having and impact on the children's health, capacity of adaptation to new knowledge, etc. These effects can translate into increases in total factor productivity in the next period. All in all, we therefore, for a sake of simplicity, assume $g_t = g\left(\frac{h_t^m + h_t^f}{2}\right)$ as an adequate modelling choice capturing the above.

their wages so as to maximize their utility. Substituting (2) into (1), the optimization of the representative household is

$$\max_{\substack{c_t, n_t > 0 \\ e_{t+1}^m, e_{t+1}^f \geq 0}} \gamma \ln \left(n_t w_{t+1} \left[(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta \right] \right) + (1 - \gamma) \ln c_t \tag{10}$$

subject to

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \leq w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) \tag{11}$$

$$\rho n_t + \varphi(A_t) \leq 1 \tag{12}$$

for given w_t, h_t^m, h_t^f, A_t, a perfectly foreseen w_{t+1}, and parameters γ, θ, ρ.

Solving this problem (see Appendix A1 and A2), we get the optimal choice of a household

$$n_t = \min \left\{ \frac{\gamma(1 - \theta)}{\rho} \left(\frac{h_t^m}{h_t^f} + 1 - \varphi(A_t) \right), \frac{1 - \varphi(A_t)}{\rho} \right\} \tag{13}$$

$$c_t = \begin{cases} (1 - \gamma)w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{(1-\gamma)w_t h_t^m}{1 - \gamma + \gamma\theta} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{14}$$

$$e_{t+1}^m = \begin{cases} \frac{\theta \rho w_t h_t^f}{(1-\theta)(1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{\gamma\theta\rho w_t h_t^m}{(1 - \gamma + \gamma\theta)[1 - \varphi(A_t)](1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{15}$$

$$e_{t+1}^f = \begin{cases} \frac{\theta \rho w_t h_t^f}{(1-\theta)(1 + [1 - \varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{\gamma\theta\rho w_t h_t^m}{(1 - \gamma + \gamma\theta)[1 - \varphi(A_t)](1 + [1 - \varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{16}$$

From (13) we find that when the time constraint of the woman is not binding, then the fertility is increasing in the human capital gap between the husband and the wife, and consumption of the household is increasing linearly in the potential income of household (i.e. net of the costs for doing housework) and educational investment for each child are increasing linearly in the potential income of the women. When the time constraint of the women is binding instead, i.e. the women spends full time for child-rearing and doing housework, then the consumption of the household, and educational investment for male and female children are increasing linearly in the real income of the household, i.e. the income of the husband.

13

4. Gender inequality, fertility, and technology

Before examining the dynamical system characterizing the competitive equilibria of the economy, it is interesting to analyse the impact of technological progress on gender inequality in education and fertility. This analysis will help us to understand better the simultaneous evolution of gender inequality in education and fertility along as technology improves.

4.1. Gender inequality in education

We define the following measure of gender inequality (female over male) in education in period t

$$\mu_t = \frac{e_t^f}{e_t^m} \tag{17}$$

so that a complete equality in education across genders appears when $\mu_t = 1$, and an education bias toward males (females) when $\mu_t < (>)1$.

From (15) and (16), we have

$$\mu_t = \frac{1 + [1 - \varphi(A_t)]^{\frac{1}{1-\theta}}}{1 + [1 - \varphi(A_t)]^{\frac{1}{\theta-1}}} = [1 - \varphi(A_t)]^{\frac{1}{1-\theta}} \equiv \mu(A_t) < 1 \tag{18}$$

$$\mu'(A_t) = \frac{1}{\theta - 1}[1 - \varphi(A_t)]^{\frac{\theta}{1-\theta}} \varphi'(A_t) > 0 \tag{19}$$

$$\mu''(A_t) = \frac{1}{\theta - 1}\left[\frac{\theta}{1 - \theta}[1 - \varphi(A_t)]^{\frac{2\theta-1}{1-\theta}} \varphi'(A_t)^2 + [1 - \varphi(A_t)]^{\frac{\theta}{1-\theta}} \varphi''(A_t)\right] < 0 \tag{20}$$

so that the gender equality in education is strictly increasing and concave in the level of technology. Under the assumption $(A1)$ we have

$$\lim_{A_t \to +\infty} \mu(A_t) = 1 \quad \text{and} \quad \mu(0) = (1 - \bar{\varphi})^{\frac{1}{1-\theta}}$$
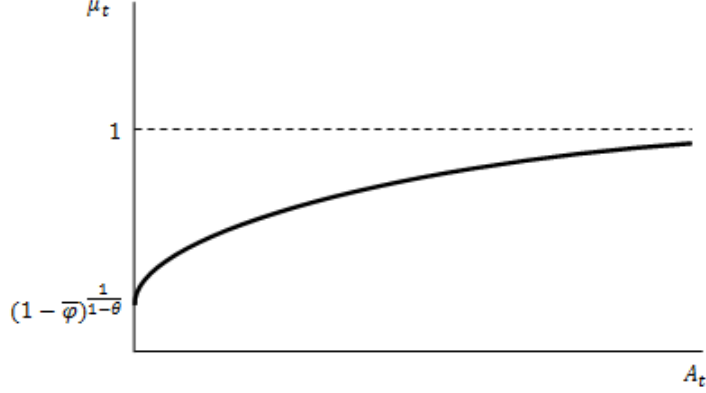
Figure 4. Gender inequality in education against technology

From (15) and (16), we also have

$$
e_{t+1}^m + e_{t+1}^f = \begin{cases} \dfrac{\theta \rho w_t h_t^f}{1-\theta} & \text{if } \rho n_t + \varphi(A_t) < 1 \\[3ex] \dfrac{\gamma \theta \rho w_t h_t^m}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)]} & \text{if } \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{21}
$$

From (18) we find that the gender inequality in education is always biased towards males, i.e. male children receive more educational investment than female children do. This is because when children become adults in the period $t+1$, women have to spend a fraction of time to do housework while men do not. The time devoted to housework does not earn any income, so for a given amount for education investment per one couple of children as in (21), a rational household in period t invests less education on their daughter and invest more on their son. Interestingly, the inequality in education is decreasing in the level of technology in period $t+1$. The reason is rather intuitive when higher level of technology in period $t+1$ reduces the time that women devote to housework then they have a chance to increase their participation to the labor force, leading their parents in period t, by utility optimization behavior, to invest more in education for their daughter to increase labor productivity. As a result, educational investments for male children decrease. Consequently, the inequality in education decreases along with the increase in technology.

Let us now examine the impact of gender inequality in education on the growth rate of technological progress.

Proposition 1: *In the overlapping generations economy above, in any period t, the better the gender equality in education, the higher the growth rate of technological progress. This also implies that the growth rate of technological progress, g_t, gets maximum when complete gender equality in education prevails.*

Proof: In effect, let us denote

15

$$\Sigma_t = e_t^m + e_t^f$$

which is determined in period $t-1$ and independent of the gender inequality in period t.

The growth rate of technology between period t and $t+1$ is defined in (9), i.e.

$$g_t = g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2}\right) = g\left(\frac{(\Sigma_t - e_t^f)^\theta + (e_t^f)^\theta}{2}\right)$$

so that

$$\frac{\partial g_t}{\partial e_t^f} = g'\left(\frac{(\Sigma_t - e_t^f)^\theta + (e_t^f)^\theta}{2}\right)\frac{\theta}{2}\left[(e_t^f)^{\theta-1} - (\Sigma_t - e_t^f)^{\theta-1}\right]$$

Since $g'(\cdot) > 0$ and $\frac{1}{2} \le \theta < 1$ then

$$\frac{\partial g_t}{\partial e_t^f}\begin{cases} > 0 & \Leftrightarrow & e_t^f < \Sigma_t/2 \\ = 0 & \Leftrightarrow & e_t^f = \Sigma_t/2 \\ < 0 & \Leftrightarrow & e_t^f > \Sigma_t/2 \end{cases} \tag{22}$$

Since from (17) and (18), it holds

$$\mu_t = \frac{e_t^f}{\Sigma_t - e_t^f} < 1 \quad \forall t \quad \Rightarrow \quad e_t^f < \Sigma_t/2 \quad \forall t$$

So an increase in education of women e_t^f (when $e_t^f < \Sigma_t/2$), i.e. a better gender equality in education, leads to a higher growth rate of technological progress. And (22) also implies that in the period t, the growth rate of technological progress g_t is maximum when a complete gender equality in education prevails, i.e. $\mu_t = 1$.

Q.E.D.

Proposition 1 is consistent with empirical results Klasen (2002) showing a positive effect of gender equality in education on economic growth. Indeed, Proposition 1 shows a positive effect of gender equality in education on the growth rate of technology which, in turn, affects positively the income growth. This is because in any period t the marginal return to education is higher for the female children than for male children. Hence, for a given amount of educational investment for children, investing more on female children until complete gender equality appears would increase the average human capital for the economy. As a result, the higher the average human capital, the higher the growth rate of technological progress. The empirical evidence for the statement in Proposition 1 and its mechanism can be found in Klasen (2002).[9]

---

[9]Klasen (2002) shows that gender inequality in education directly affects economic growth by lowering the average level of human capital and indirectly affects economic growth through its impact on population and investment.

4.2. Fertility and labor-force participation of women

From equation (18) it follows that

$$\frac{h_t^m}{h_t^f} = \frac{(e_t^m)^\theta}{(e_t^f)^\theta} = [1 - \varphi(A_t)]^{\frac{\theta}{\theta-1}} \tag{23}$$

Substituting (23) into (13), we have

$$n_t = \min \left\{ \underbrace{\frac{\gamma(1-\theta)}{\rho} \left([1 - \varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t) \right)}_{n_a(A_t)}, \underbrace{\frac{1 - \varphi(A_t)}{\rho}}_{n_b(A_t)} \right\} \tag{24}$$

For the existence of a solution to $n_a(A_t) = n_b(A_t)$, i.e. the existence of a threshold for the technological level below which women will spend full time doing housework and raising children, we assume that

$$\varphi(0) = \bar{\varphi} > 1 - \left[\frac{\gamma(1-\theta)}{1 - \gamma(1-\theta)} \right]^{1-\theta} \qquad (A2)$$

The assumption $A2$ requires that at some low level of technology, the time women devote to housework is sufficiently high to prevent them from supplying labor to the market. If this assumption did not hold, the time constraint of women would be never binding for all $A_t > 0$, as derived from (24). That is to say, women would always supply labor to the market regardless how low the level of technology is, and the fertility would be always decreasing in the level of technology. Nevertheless, in the early stages of development, fertility is typically observed to be increasing with the level of technology, and women supply labor to the market when the return to their labor was sufficiently high.

Lemma 1:

(i) Under assumptions A1 and A2, there exists a unique $A^* > 0$ such that

$$n_a(A^*) = n_b(A^*)$$

(ii) At this A^*, the two pieces characterizing the optimal levels of c_t, e_{t+1}^m, and e_{t+1}^f take the same value, making optimal choices continuous.

Proof:

(i) In effect we consider the equation

$$n_a(A_t) = n_b(A_t)$$

that is to say

$$\gamma(1-\theta)\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}}+1-\varphi(A_t)\right)=1-\varphi(A_t)$$

or equivalently

$$\varphi(A_t)=1-\left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta}$$

Under assumption $A1$, $\varphi(A_t)$ is an invertible function. So that, under assumption $A2$, there exists a unique $A^*>0$ solving $n_a(A_t)=n_b(A_t)$, namely

$$A^*=\varphi^{-1}\left(1-\left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta}\right)$$

(ii) The proof for this statement is straightforward by substituting

$$\gamma(1-\theta)\left(\frac{h_t^m}{h_t^f}+1-\varphi(A^*)\right)=1-\varphi(A^*)$$

into equations (14), (15), (16), and evaluating them at $A_t=A^*$.

Q.E.D.

So, under Lemma 1, we can rewrite (24) as follows

$$n_t=n(A_t)=\begin{cases}\frac{\gamma(1-\theta)}{\rho}\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}}+1-\varphi(A_t)\right)\equiv n_a(A_t)\text{ if }A_t\geq A^*\\[2mm]\frac{1-\varphi(A_t)}{\rho}\equiv n_b(A_t)\qquad\qquad\qquad\qquad\text{ if }A_t\leq A^*\end{cases}\tag{25}$$

Equation (25) implies that in any period t women participate in the labor market if, and only if, the contemporary level of technology A_t is sufficiently high (i.e. $A_t>A^*$), otherwise women will spend their full time doing housework and raising children. So, A^* is thus the highest level of technology for which the women do not work in the market.

To see the impact of technology on female participation in the labor market, note first that

$$n_a'(A_t)=\frac{\gamma(1-\theta)}{\rho}\left(\frac{\theta}{1-\theta}[1-\varphi(A_t)]^{\frac{1}{\theta-1}}-1\right)\varphi'(A_t)<0,\qquad\forall A_t$$

since $\theta\in[\frac{1}{2},1)$ as mentioned in section 3.1,

$$n_a''(A_t)=\frac{\gamma(1-\theta)}{\rho}\left[\frac{\theta\varphi'(A_t)^2}{(1-\theta)^2}[1-\varphi(A_t)]^{\frac{2-\theta}{\theta-1}}+\left(\frac{\theta}{1-\theta}[1-\varphi(A_t)]^{\frac{1}{\theta-1}}-1\right)\varphi''(A_t)\right]>0$$

$$\lim_{A_t\to+\infty}n_a(A_t)=\frac{2\gamma(1-\theta)}{\rho}\quad\text{and}\quad n_a(0)=\frac{\gamma(1-\theta)}{\rho}\left([1-\bar\varphi]^{\frac{\theta}{\theta-1}}+1-\bar\varphi\right)$$

18

while

$$n'_b(A_t) = \frac{-\varphi'(A_t)}{\rho} > 0 \quad \text{and} \quad n''_b(A_t) = \frac{-\varphi''(A_t)}{\rho} < 0$$

$$\lim_{A_t \to +\infty} n_b(A_t) = \frac{1}{\rho} > \frac{2\gamma(1-\theta)}{\rho} \quad \text{and} \quad n_b(0) = \frac{1-\bar{\varphi}}{\rho} < n_a(0)$$

Moreover, the time devoted to the labor market of women is

$$\mathcal{L}(A_t) = \begin{cases} 1 - \rho n_a(A_t) - \varphi(A_t) & \text{if} \quad A_t \geq A^* \\ \\ 0 & \text{if} \quad A_t \leq A^* \end{cases} \tag{26}$$

with $\forall A_t \geq A^*$,

$$\mathcal{L}'(A_t) = -\rho n'_a(A_t) - \varphi'(A_t) > 0 \quad \text{and} \quad \mathcal{L}''(A_t) = -\rho n''_a(A_t) - \varphi''(A_t) < 0$$

and

$$\lim_{A_t \to +\infty} \mathcal{L}(A_t) = 1 - 2\gamma(1-\theta)$$

Figure 5 below summarizes the impact of technology on fertility and female participation to the market.
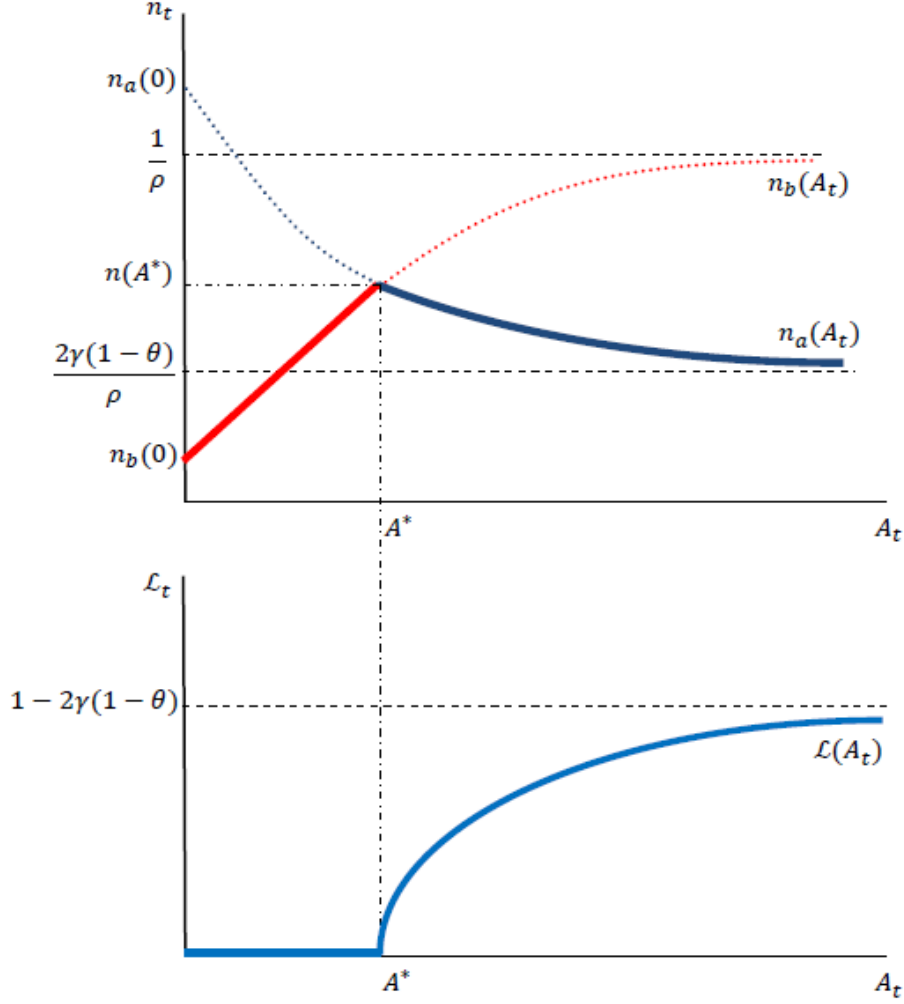
Fig 5. Fertility (upper graph) and female labor-force participation (lower graph) against technology

When the technological level is low enough (i.e. $A_t < A^*$), women spend full time doing housework and raising children. As the technological level increases but is still low, the time for doing housework decreases and the time for raising children increases, then in this case an increase in the technological level leads to an increase in fertility. When technological level exceeds the threshold A^*, women do housework and raise children part time, and participate the labor market in their remaining time. Technological progress makes the human capital gap between men and women decrease, as expressed in (23), and thus reduces the relative earning gap between men and women. This reduction in the relative earning gap implies an increase in the earnings of women. This increase in the earnings of women leads to an increase in the opportunity cost of raising children. From (21) we find that the educational investment for one couple of children increases in the potential earnings of women when the technological level is large enough (i.e. $A_t > A^*$). So when the cost of raising children physically increases, households will trade less quantity for higher quality children. Therefore, along with the decline in fertility and increase in educational investment due to the increase in technology,

the labor-force participation of women increases.

5. Competitive equilibrium dynamics

The competitive equilibria of this economy are characterized, in any period t, by (i) the household's utility maximization under its budget constraint, (ii) the aggregate output equating the total return to human capital, (iii) the dynamics of the technological level, and (iv) the dynamics of population. Therefore, a competitive equilibrium dynamics $\{c_t, n_t, e_{t+1}^m, e_{t+1}^f, Y_t, w_t, A_{t+1}, L_{t+1}\}_{t \in \mathbb{N}}$ is determined by the following system of equations

$$
c_t = \begin{cases}
(1-\gamma)w_t[(e_t^m)^\theta + [1-\varphi(A_t)](e_t^f)^\theta] & \text{if} \quad A_t \geq A^* \\
\\
\frac{1-\gamma}{1-\gamma+\gamma\theta}w_t(e_t^m)^\theta & \text{if} \quad A_t \leq A^*
\end{cases}
$$

$$
n_t = \begin{cases}
\frac{\gamma(1-\theta)}{\rho}\left[\frac{(e_t^m)^\theta}{(e_t^f)^\theta} + 1 - \varphi(A_t)\right] & \text{if} \quad A_t \geq A^* \\
\\
\frac{1-\varphi(A_t)}{\rho} & \text{if} \quad A_t \leq A^*
\end{cases}
$$

$$
e_{t+1}^m = \begin{cases}
\frac{\theta\rho w_t(e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \geq A^* \\
\\
\frac{\gamma\theta\rho w_t(e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \leq A^*
\end{cases}
$$

$$
e_{t+1}^f = \begin{cases}
\frac{\theta\rho w_t(e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \geq A^* \\
\\
\frac{\gamma\theta\rho w_t(e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \leq A^*
\end{cases}
$$

$$
Y_t = f(A_t)L_t\left((e_t^m)^\theta + [1 - \rho n_t - \varphi(A_t)](e_t^f)^\theta\right)
$$

$$
w_t = f(A_t)
$$

$$
A_{t+1} = \left[1 + g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2}\right)\right]A_t
$$

$$
L_{t+1} = n_t L_t
$$

for given initial conditions A_0, L_0, e_0^m, e_0^f.

The competitive equilibrium can be fully characterized by the following reduced system describing the equilibrium dynamics of the level of technology A_{t+1} and educational investments for male and female children e_{t+1}^m, e_{t+1}^f,

$$A_{t+1} = \left[1 + g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2} \right) \right] A_t \tag{27}$$

$$e_{t+1}^m = \begin{cases} \dfrac{\theta \rho f(A_t)(e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \geq A^* \\[3ex] \dfrac{\gamma \theta \rho f(A_t)(e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \leq A^* \end{cases} \tag{28}$$

$$e_{t+1}^f = \begin{cases} \dfrac{\theta \rho f(A_t)(e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \geq A^* \\[3ex] \dfrac{\gamma \theta \rho f(A_t)(e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \leq A^* \end{cases} \tag{29}$$

for a given initial condition A_0, e_0^m, and e_0^f.

To prove the convergence, which is stated in Proposition 2 below, of the dynamic system (27)-(29) , we have to prove the following Lemma 2.

Lemma 2: *For a dynamic equation $x_{t+1} = a_t x_t^\alpha$ with $\alpha \in (0,1)$, $x_0 > 0$, and $\lim_{t \to +\infty} a_t = a > 0$, then*

$$\lim_{t \to +\infty} x_t = a^{\frac{1}{1-\alpha}}.$$

Proof: See Appendix A3.

Proposition 2: *The overlapping generations economy as set up above, with any initial conditions $A_0 > 0$, e_0^m, and e_0^f, will converge to a regime of sustained growth characterized by a constant growth rate of technology, a constant fertility rate, constant education investments, and complete gender equality in education.*

Proof: In effect, for this economy, according to (27) and as long as e_t^m or e_t^f are positive the level of technology increases unboundedly over time because the driving force for technological progress is positive investment in human capital. The technological progress appears even

when the average human capital is very small. Therefore, it is straightforward from (22) and the Lemma 2 that

$$\lim_{t \to +\infty} e_t^f = \left[\frac{\theta \rho \bar{f}}{2(1-\theta)} \right]^{\frac{1}{1-\theta}} \equiv \bar{e}$$

From $\lim_{t \to +\infty} A_t = +\infty$ and $\lim_{A_t \to +\infty} \frac{e_t^f}{e_t^m} = 1$ (as stated in the subsection 4.1), e_t^m also converges to $\bar{e}$.

From (25), the fertility rate will converge to a constant rate,

$$\bar{n} = \lim_{t \to +\infty} n_a(A_t) = \frac{2\gamma(1-\theta)}{\rho}$$

And the technology will grow at a constant rate

$$\bar{g} = g \left(\left[\frac{\theta \rho \bar{f}}{2(1-\theta)} \right]^{\frac{\theta}{1-\theta}} \right)$$

Q.E.D.

The statement in Proposition 2 is consistent with the stylized facts of the developed world, where a modern sustained growth regime prevails characterized by unbounded economic growth, low and decreasing fertility rate, and high human capital.

6. Analysis

It would be interesting to link the theoretical results of this paper to the development process of western Europe characterized by three distinct regimes, (i) the Malthusian regime where both per-capita income and the growth rate of population were very low; (ii) the demographic transition where both increased simultaneously; and later (iii) the modern sustained growth regime where the growth rate of population falls while per-capita income still grows. During the development process of modern sustained growth, the labor-force participation of women increases along with a decline in gender inequalities in education and income.

Consider an economy in the early stage of development with very low initial level of technology and low human capital for both men and women. The technological level is low enough (i.e. $A_0 < A^*$) for women to have to spend their full time raising children and doing housework. The low technological level prevents women from participating in the labor market in two ways. First, it directly requires a large fraction of women's time to do housework. Second, it indirectly creates a gender inequality in education that makes women receive less educational investment from their parents, hence the opportunity cost of raising children physically

becomes cheap, so that households prefer to increase their number of children rather than supplying the woman's labor to the market. Since, in this period, the income of a household is very low due to low human capital and low technological level, the households' educational investments for their children are very small. Therefore, human capital are very low for both male and female children. Moreover, the growth rate of technology is very small as well because of low average human capital, which is the driving force for technological progress. Consequently, households invest very little in education for their daughters because they anticipate the large fraction of time that their daughters have to devote to housework due to the low level of technology in the next period. Since men always supply their labor inelastically to the market, while women have to spend a large fraction of their time for housework, households allocate a large fraction of education investment to their sons, making the education of their sons and daughters very unequal. In addition, in this stage of development, since housework requires a very large fraction of women's time, then the remaining time for raising children is very small, making the population growth rate low. Because the driving force of technological progress is the average human capital and technology grows even when the average human capital is small, over time technological level increases, while still being low (i.e. $A_t < A^*$), making the time for housework to decrease, and women to have more time to take care of children. In this stage, although the technological progress can increase the earnings of women, i.e. increase the opportunity cost of raising children physically, households increase the number of their children. Consequently, the fertility rate increases.

As the technological level progresses, fertility increases and reaches the maximum when the technological level reaches A^*, generating a demographic transition. When the technological level is high enough (i.e. $A_t > A^*$), women will participate the labor market and their participation increases due to two effects of technology. First, the technological progress helps women save time in doing housework, leaving them more time for childcare and labor-force participation. Second, the technological progress improves the return to human capital, thus increases the earnings of women, meaning that the opportunity cost of raising children physically increases. The model shows that the fertility decreases due to the increase in the technological level. We also know that the educational investment for one couple of children, from (28) and (29), when $A_t > A^*$ is

$$e_{t+1}^m + e_{t+1}^f = \frac{\theta \rho f(A_t)(e_t^f)^\theta}{1 - \theta}$$

so that this educational investment increases with respect to the level of technology. In this period, households trade low quantity for high quality of their children. The increases in the level of technology make households invest less in education for male children in order to invest more in education for female children because the return to female education is higher than that to male education. Consequently, the gender equality in education improves over

time, accelerating the technological progress. This feedback loop makes the economy enter the regime of sustained economic growth.

7. Conclusion and prospects for further research

This paper develops a growth model capturing technological progress, gender inequality in education and fertility, and the complex interaction between these issues to explain some stylized facts characterizing the development process. Particularly, the paper proposes a mechanism linking technology, gender inequality and fertility to shed some light on the evolution of societies from stagnation, through demographic transition, to modern sustained growth. The improvement in gender equality in education, income, as well as the increases in female labor-force participation are *inevitable* outcomes of this development process when the driving force for technological progress is average human capital. The paper also shows that technological progress may increase female labor-force participation not only by liberating women from doing housework due to the appearance of time-saving household-sector products, but also by leading households to reduce fertility due to an increase in the return of human capital, and hence an increase in the cost of raising children. In addition, technological progress also makes households trade quantity for higher quality children.

This paper, in general, explains some salient features of the long transition from stagnation to the stage of sustained growth of the developed world. However, in almost all the developed world nowadays, the gap in education between genders has been reversing. Goldin et al. (2006) show that by 2002, the enrollment rates to college of women exceeded that of men in 15 out of 17 OECD countries while in the mid-1980s the enrollment rates of women were lower than those of men in 13 out of these 17 countries. Before 1980s, the enrollment rates of women had never exceeded those of men. This reversal in education between genders is an important phenomenon that attracts recent research in development. Chiappori et al. (2009), Becker et al. (2010), and Hazan and Zoabi (2012) provide different novel mechanisms to explain this fact. In addition, Hazan and Zoabi (2014) explore the U-shape relationship between fertility and women education (in US) which may reflect a new pattern of fertility. This new pattern, indeed, does not necessarily contradict the stylized fact of a negative correlation between fertility and female-to-male education ratio which is explained in this paper, but it seems worthy to be investigated in the unified growth theory framework. We leave the explanation of this and other phenomena for further research in the framework provided by the unified growth theory.

Appendix

A1. Solving the household's optimization problem

$$\max_{\substack{c_t,n_t>0 \\ e_{t+1}^m,e_{t+1}^f \geq 0}} \gamma \ln \left(n_t w_{t+1} \left[(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta \right] \right) + (1-\gamma) \ln c_t$$

subject to

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \leq w_t(h_t^m + [1 - \varphi(A_t)]h_t^f)$$

$$\rho n_t + \varphi(A_t) \leq 1$$

The Kuhn Tucker conditions are

$$\begin{pmatrix} \frac{1-\gamma}{c_t} \\ \frac{\gamma}{n_t} \\ \frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta+[1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} \\ \frac{\gamma[1-\varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta+[1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ \rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f \\ n_t \\ n_t \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \lambda_4 \begin{pmatrix} 0 \\ \rho \\ 0 \\ 0 \end{pmatrix}$$

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) \leq 0$$

$$-e_{t+1}^m \leq 0$$

$$-e_{t+1}^f \leq 0$$

$$\rho n_t + \varphi(A_t) - 1 \leq 0$$

$$\lambda_1 \left[c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) \right] = 0$$

$$\lambda_2 e_{t+1}^m = 0$$

$$\lambda_3 e_{t+1}^f = 0$$

$$\lambda_4[\rho n_t + \varphi(A_t) - 1] = 0$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$$

For these conditions, it is straightforward to show that $\lambda_1 > 0$, because

$$\lambda_1 = \frac{1 - \gamma}{c_t} \neq 0$$

since $\gamma \in (0, 1)$ and $c_t > 0$. That is to say the budget constraint is binding.

Since $\theta - 1 < 0$, then it is also straightforward from conditions

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \lambda_1 n_t - \lambda_2$$

$$\frac{\gamma[1 - \varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \lambda_1 n_t - \lambda_3$$

that $e_{t+1}^m > 0$ and $e_{t+1}^f > 0$ to guarantee the left-hand-sides (both numerators and denominators) to be determined. Therefore, $\lambda_2 = \lambda_3 = 0$. That is to say, the positivity constraints of education investments for male and female children are never binding at the optimal solution.

Now we consider two cases: (i) the time constraint of the woman is not binding; and (ii) it is binding.

- (i) If $\rho n_t + \varphi(A_t) < 1$, then $\lambda_4 = 0$, so that

$$\frac{1 - \gamma}{c_t} = \lambda_1 > 0$$

$$\frac{\gamma}{n_t} = \frac{1 - \gamma}{c_t}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \tag{30}$$

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1 - \gamma}{c_t}n_t \tag{31}$$

$$\frac{\gamma[1 - \varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1 - \gamma}{c_t}n_t \tag{32}$$

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) = 0 \tag{33}$$

27

From (31) and (32) we have

$$\frac{(e^m_{t+1})^{\theta-1}}{[1-\varphi(A_{t+1})](e^f_{t+1})^{\theta-1}} = 1$$

$$\Rightarrow \quad e^f_{t+1} = [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}} e^m_{t+1} \tag{34}$$

From (30) we have

$$\frac{1-\gamma}{c_t} n_t = \frac{\gamma}{\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}} \tag{35}$$

Substitute (34) and (35) into (31) we have

$$\frac{\theta}{(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e^m_{t+1}} = \frac{1}{\rho w_t h^f_t + (1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e^m_{t+1}}$$

So, we obtain

$$e^m_{t+1} = \frac{\theta \rho w_t h^f_t}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} \tag{36}$$

Hence,

$$e^f_{t+1} = \frac{\theta \rho w_t h^f_t}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} \tag{37}$$

From (30), (33), (36) and (37) we have

$$\frac{1}{\gamma}\left(\rho w_t h^f_t + \frac{\theta \rho w_t h^f_t}{1-\theta}\right) n_t = w_t(h^m_t + [1-\varphi(A_t)]h^f_t)$$

Hence,

$$n_t = \frac{\gamma(1-\theta)}{\rho}\left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t)\right) \tag{38}$$

and

$$c_t = (1-\gamma)w_t(h^m_t + [1-\varphi(A_t)]h^f_t) \tag{39}$$

So, the solution is characterized by four equations (36)-(39).

- (ii) If $\rho n_t + \varphi(A_t) = 1$, so that

$$\frac{1 - \gamma}{c_t} = \lambda_1 > 0$$

$$n_t = \frac{1 - \varphi(A_t)}{\rho} \tag{40}$$

$$\frac{\gamma \rho}{1 - \varphi(A_t)} = \frac{1 - \gamma}{c_t}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) + \lambda_4 \rho \tag{41}$$

$$\frac{\gamma \theta (e_{t+1}^m)^{\theta - 1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1 - \gamma}{c_t} \frac{1 - \varphi(A_t)}{\rho} \tag{42}$$

$$\frac{\gamma [1 - \varphi(A_{t+1})] \theta (e_f^{t+1})^{\theta - 1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1 - \gamma}{c_t} \frac{1 - \varphi(A_t)}{\rho} \tag{43}$$

$$c_t + \frac{1 - \varphi(A_t)}{\rho}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) = 0 \tag{44}$$

From (42) and (43) we have

$$\frac{(e_{t+1}^m)^{\theta - 1}}{[1 - \varphi(A_{t+1})](e_{t+1}^f)^{\theta - 1}} = 1$$

$$\Rightarrow \quad e_{t+1}^f = [1 - \varphi(A_{t+1})]^{\frac{1}{1 - \theta}} e_{t+1}^m \tag{45}$$

Substitute (45) into (42) we have

$$\frac{\gamma \theta}{(1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1 - \theta}})e_{t+1}^m} = \frac{1 - \gamma}{c_t} \frac{1 - \varphi(A_t)}{\rho}$$

$$\Rightarrow \quad c_t = \frac{1 - \gamma}{\gamma \theta} \frac{1 - \varphi(A_t)}{\rho}(1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1 - \theta}})e_{t+1}^m \tag{46}$$

Substitute (45) and (46) into (44) we have

$$\frac{1 - \varphi(A_t)}{\rho} \times$$

$$\left\{ \frac{1 - \gamma}{\gamma \theta}(1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1 - \theta}})e_{t+1}^m + \rho w_t h_t^f + (1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1 - \theta}})e_{t+1}^m \right\}$$

$$= w_t(h_t^m + [1 - \varphi(A_t)]h_t^f)$$

$$\Leftrightarrow \frac{1 - \gamma + \gamma \theta}{\gamma \theta}(1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1 - \theta}})e_{t+1}^m = \frac{\rho w_t h_t^m}{1 - \varphi(A_t)}$$

So, we obtain

$$e_{t+1}^m = \frac{\gamma\theta\rho w_t h_t^m}{(1 - \gamma + \gamma\theta)[1 - \varphi(A_t)](1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1-\theta}})} \tag{47}$$

Hence,

$$e_{t+1}^f = \frac{\gamma\theta\rho w_t h_t^m}{(1 - \gamma + \gamma\theta)[1 - \varphi(A_t)](1 + [1 - \varphi(A_{t+1})]^{\frac{1}{\theta-1}})} \tag{48}$$

Substitute (47) into (46) we have

$$c_t = \frac{(1 - \gamma)w_t h_t^m}{1 - \gamma + \gamma\theta} \tag{49}$$

Finally, substituting (47), (48), and (49) into (41), we have

$$\lambda_4 = \frac{\gamma(1 - \theta)}{1 - \varphi(A_t)} - \frac{h_t^f}{h_t^m}(1 - \gamma + \gamma\theta) \tag{50}$$

Hence, in this case, the optimal solution is

$$c_t = \frac{(1 - \gamma)w_t h_t^m}{1 - \gamma + \gamma\theta}$$

$$n_t = \frac{1 - \varphi(A_t)}{\rho}$$

$$e_{t+1}^m = \frac{\gamma\theta\rho w_t h_t^m}{(1 - \gamma + \gamma\theta)[1 - \varphi(A_t)](1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1-\theta}})}$$

$$e_{t+1}^f = \frac{\gamma\theta\rho w_t h_t^m}{(1 - \gamma + \gamma\theta)[1 - \varphi(A_t)](1 + [1 - \varphi(A_{t+1})]^{\frac{1}{\theta-1}})}$$

A2. Checking the SOCs for the maximization problem of household

Since the optimization problem is not convex, then for the FOCs to be sufficient conditions to characterize a (local) maximum to the optimization problem, we have to check the sufficient SOCs. We know from Appedix A5.1 that the positivity constraints of education investments for male and female children are never binding at the solution, while the budget constraint is always binding at the solution, and the time constraint of women can be binding or nonbinding. So we have to check the bordered Hessian matrix in two cases: (i) the time constraint of women is not binding; and (ii) it is binding.

- (i) If $\rho n_t + \varphi(A_t) < 1$, the bordered Hessian matrix of the problem appears as

$$\bar{H}^i = \begin{pmatrix} 0 & 1 & E & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & B & C \\ n_t & 0 & 0 & C & D \end{pmatrix}$$

where B, C, and D are defined below (note that for lightening notations, we denote $e_{t+1}^m = e_m$, $e_{t+1}^f = e_f$, and $\varphi(A_{t+1}) = \varphi$).

$$B = \frac{\partial^2 u_t}{\partial e_m^2} = \gamma\theta\frac{(\theta-1)e_m^{\theta-2}\left[e_m^\theta + (1-\varphi)e_f^\theta\right] - \theta e_m^{2\theta-2}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2}$$

$$= \frac{-\gamma\theta e_m^{\theta-2}\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right]}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$D = \frac{\partial^2 u_t}{\partial e_f^2} = \gamma\theta(1-\varphi)\frac{(\theta-1)e_f^{\theta-2}\left[e_m^\theta + (1-\varphi)e_f^\theta\right] - (1-\varphi)\theta e_f^{2\theta-2}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2}$$

$$= \frac{-\gamma\theta(1-\varphi)e_f^{\theta-2}\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$C = \frac{\partial^2 u_t}{\partial e_m \partial e_f} = \frac{-\gamma\theta^2(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$E = \rho w_t h_t^f + e_m + e_f > 0$$

Now we prove two following properties:

Property 1: $BD > C^2$.
In effect, $BD > 0$, $C^2 > 0$, and

$$\frac{BD}{C^2} = \frac{\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right]\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{\theta^2(1-\varphi)e_m^\theta e_f^\theta}$$

$$= \frac{(1-\theta)e_m^{2\theta} + [(1-\theta)^2 + 1](1-\varphi)e_m^\theta e_f^\theta + (1-\theta)(1-\varphi)^2 e_f^{2\theta}}{\theta^2(1-\varphi)e_m^\theta e_f^\theta}$$

$$= \frac{(1-\theta)e_m^{2\theta} + (1-\theta)(1-\varphi)^2 e_f^{2\theta}}{\theta^2(1-\varphi)e_m^\theta e_f^\theta} + \frac{2(1-\theta)}{\theta^2} + 1 > 1$$

i.e. $BD > C^2$ ∎

Property 2: $2C > B + D$.

We know that $C < 0$, $B < 0$, and $D < 0$, then the property 2 is equivalent to

$$\frac{B+D}{2C} > 1$$

In effect,

$$\frac{B+D}{2C} =$$

$$\frac{e_m^{\theta-2}\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right] + (1-\varphi)e_f^{\theta-2}\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{2\theta(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}}$$

$$= \frac{e_m^{2\theta-2} + (1-\varphi)^2 e_f^{2\theta-2}}{2\theta(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}} + V$$

where $V = \frac{(1-\theta)[e_f^\theta e_m^{\theta-2} + e_f^{\theta-2}e_m^\theta]}{2\theta e_m^{\theta-1}e_f^{\theta-1}} > 0$.

Applying the trivial inequality, $e_m^{2\theta-2} + (1-\varphi)^2 e_f^{2\theta-2} \geq 2(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}$, we have

$$\frac{B+D}{2C} \geq \frac{1}{\theta} + V > 1$$

i.e. $2C > B + D$ ∎

The sufficient SOCs for a maximum in this case are

$$\left|\bar{H}_2^i\right| = \begin{vmatrix} 0 & 1 & E \\ 1 & \frac{\gamma-1}{c_t^2} & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} \end{vmatrix} = \frac{1-\gamma}{c_t^2}E^2 + \frac{\gamma}{n_t^2} > 0$$

$$\left|\bar{H}_3^i\right| = \begin{vmatrix} 0 & 1 & E & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 \\ n_t & 0 & 0 & B \end{vmatrix} = B\left|\bar{H}_2^i\right| + \frac{(\gamma-1)\gamma}{c_t^2} < 0$$

$$\left|\bar{H}_4^i\right| = \left|\bar{H}^i\right| = \begin{vmatrix} 0 & 1 & E & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & B & C \\ n_t & 0 & 0 & C & D \end{vmatrix} = -\begin{vmatrix} 1 & E & n_t & n_t \\ 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ 0 & 0 & B & C \\ 0 & 0 & C & D \end{vmatrix} + \frac{\gamma-1}{c_t^2}\begin{vmatrix} 0 & E & n_t & n_t \\ E & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & B & C \\ n_t & 0 & C & D \end{vmatrix}$$

$$= \left[\frac{\gamma}{n_t^2} + \frac{1-\gamma}{c_t^2}E^2\right](BD - C^2) + \frac{(1-\gamma)\gamma}{c_t^2}(2C - B - D) > 0$$

under properties 1 and 2 above.

So the solution to the agent's problem in this case is a maximum indeed.

- (ii) If $\rho n_t + \varphi(A_t) = 1$, the bordered Hessian matrix of the problem appears as

$$\bar{H}^{ii} = \begin{pmatrix} 0 & 0 & 1 & E & n_t & n_t \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & 0 & B & C \\ n_t & 0 & 0 & 0 & C & D \end{pmatrix}$$

The sufficient SOCs for a maximum in this case are

$$\left|\bar{H}_3^{ii}\right| = \begin{vmatrix} 0 & 0 & 1 & E & n_t \\ 0 & 0 & 0 & \rho & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 \\ n_t & 0 & 0 & 0 & B \end{vmatrix}$$

$$= B\rho^2 - n_t^2 \begin{vmatrix} 0 & 0 & \rho \\ 0 & \frac{\gamma-1}{c_t^2} & 0 \\ \rho & 0 & -\frac{\gamma}{n_t^2} \end{vmatrix} = \left(B - n_t^2 \frac{1-\gamma}{c_t^2} \right) \rho^2 < 0$$

$$\left| \bar{H}_4^{ii} \right| = \left| \bar{H}^{ii} \right| = \begin{vmatrix} 0 & 0 & 1 & E & n_t & n_t \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & 0 & B & C \\ n_t & 0 & 0 & 0 & C & D \end{vmatrix}$$

$$= -\rho^2 \begin{vmatrix} 0 & 1 & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ n_t & 0 & B & C \\ n_t & 0 & C & D \end{vmatrix} = \rho^2 \begin{vmatrix} 1 & n_t & n_t \\ 0 & B & C \\ 0 & C & D \end{vmatrix} + \rho^2 \frac{1-\gamma}{c_t^2} \begin{vmatrix} 0 & n_t & n_t \\ n_t & B & C \\ n_t & C & D \end{vmatrix}$$

$$= \rho^2 \left[BD - C^2 + \frac{1-\gamma}{c_t^2}(2C - B - D) \right] > 0$$

under properties 1 and 2 above.

So the solution to the agent's problem in this case is also a maximum indeed.

A3. Proof of Lemma 2.

In effect, since $\lim_{t\to+\infty} a_t = a > 0$ then $\forall \varepsilon \in (0, a)$, $\exists T_0$ such that $\forall t \geq T_0$, we have

$$a - \varepsilon \leq a_t \leq a + \varepsilon$$

Define

$$X_0 = Y_0 = Z_0 = x_{T_0}$$

and

$$X_{t+1} = a_{T_0+t} X_t^\alpha, \quad Y_{t+1} = (a+\varepsilon)Y_t^\alpha, \quad Z_{t+1} = (a-\varepsilon)Z_t^\alpha$$

We know that

$$\lim_{t\to+\infty} Y_t = (a+\varepsilon)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \lim_{t\to+\infty} Z_t = (a-\varepsilon)^{\frac{1}{1-\alpha}}$$

34

We also have

$$Z_1 = (a - \varepsilon)X_0^\alpha \leq X_1 = a_{T_0}X_0^\alpha \leq (a + \varepsilon)X_0^\alpha = Y_1$$

and

$$Z_2 = (a - \varepsilon)Z_1^\alpha \leq (a - \varepsilon)X_1^\alpha \leq X_2 = a_{T_0+1}X_1^\alpha \leq (a + \varepsilon)X_1^\alpha \leq (a + \varepsilon)Y_1^\alpha = Y_2$$

$$\vdots$$

and so on, by induction we have

$$Z_t \leq X_t \leq Y_t, \ \forall t.$$

Hence,

$$(a - \varepsilon)^{\frac{1}{1-\alpha}} = \lim_{t \to +\infty} Z_t \leq \lim_{T \to +\infty} \left(\inf_{t \geq T} X_t \right) \leq \lim_{T \to +\infty} \left(\sup_{t \geq T} X_t \right) \leq \lim_{t \to +\infty} Y_t = (a + \varepsilon)^{\frac{1}{1-\alpha}}$$

That is to say

$$(a - \varepsilon)^{\frac{1}{1-\alpha}} \leq \lim_{T \to +\infty} \left(\inf_{t \geq T} X_t \right) \leq \lim_{T \to +\infty} \left(\sup_{t \geq T} X_t \right) \leq (a + \varepsilon)^{\frac{1}{1-\alpha}}, \ \forall \varepsilon \in (0, a)$$

Hence,

$$\lim_{\varepsilon \to 0^+} (a - \varepsilon)^{\frac{1}{1-\alpha}} \leq \lim_{T \to +\infty} \left(\inf_{t \geq T} X_t \right) \leq \lim_{T \to +\infty} \left(\sup_{t \geq T} X_t \right) \leq \lim_{\varepsilon \to 0^+} (a + \varepsilon)^{\frac{1}{1-\alpha}}$$

i.e.

$$a^{\frac{1}{1-\alpha}} \leq \lim_{T \to +\infty} \left(\inf_{t \geq T} X_t \right) \leq \lim_{T \to +\infty} \left(\sup_{t \geq T} X_t \right) \leq a^{\frac{1}{1-\alpha}}$$

which implies

$$\lim_{t \to +\infty} X_t = a^{\frac{1}{1-\alpha}}$$

i.e.

$$\lim_{t \to +\infty} x_t = a^{\frac{1}{1-\alpha}}.$$

Q.E.D.

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