Spare parts inventory pooling games

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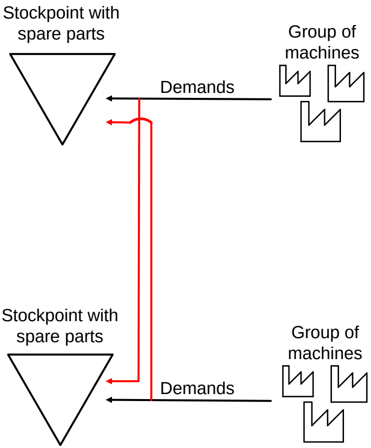
Outline

Spare parts

- Equipment-intensive high-tech industries
  - Example: airlines, nuclear power plants
- High availability needed
- Random (infrequent) failures of critical components
  $\Rightarrow$ Can cause the machine to go down
- Spare parts are kept on stock
  - To combat costly downtimes
  - Typically very expensive with low demand rates

Spare parts inventory pooling

- Inventory pooling:
  Demand at a stockpoint that is out of stock is satisfied from another stockpoint
- This can improve availability and reduce costs



Spare parts inventory pooling games

Situation: Several independent companies that

- separately stock spare parts;
- may reduce expected costs by pooling inventory

Problem: Joint costs have to be allocated amongst the participating companies in a pooling group.

- Can this be done in a fair, stable way?
- Is the *core* of the associated cooperative game non-empty?

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Cooperative game theory and the core

- Player set $N = \{1, 2, \ldots, n\}$
- Cost function $c : 2^N \to \mathbb{R}$
    - So coalition $M \subseteq N$ alone faces costs $c(M)$
- Cooperative cost game: (N, c)
- Allocation $(x_i)_{i \in N} \in \mathbb{R}^N$ distributes $c(N)$ over all players.

Definition

Core: set of all allocations that are

- Efficient: $\sum_{i \in N} x_i = c(N)$
- Stable: for all $M \subseteq N$, $\sum_{i \in M} x_i \leq c(M)$

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The research question

Does a spare parts inventory pooling game have a non-empty core?

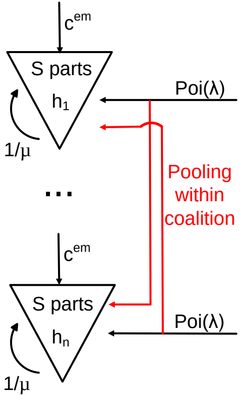
Literature overview

Related topics:

- Spare parts inventory pooling
  - Axsäter (1990), Alfredsson and Verrijdt (1999), Grahovac and Chakravarty (2001), Paterson et al. (2009) ...
- Cooperative inventory games
  - Hartman and Dror (1996), Slikker et al. (2005), Anily and Haviv (2007), Özen et al. (2008), ...
- Spare parts inventory pooling games
  - Zhao et al. (2005, 2006), Wong et al. (2007), Kilpi et al. (2009).

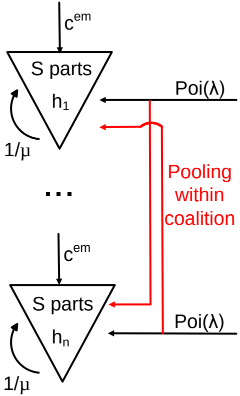
The base setting - notation

- Base setting: companies (almost) identical (symmetry assumptions will be relaxed later)
- Set of companies: $N = \{1, 2, \ldots, n\}$
- Demands: Poisson(λ)
- Repairs: Expected lead time $1/\mu$
- Stock-out: emergency procedure; costs c^{em}
- Base stock level: S
- Holding cost rate: h_i



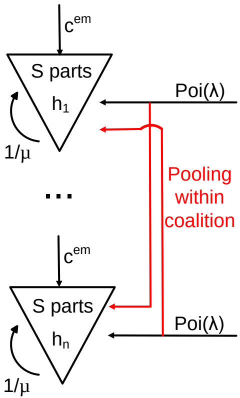
Cooperation between companies

- Assumptions:
  - Negligible distance between stockpoints
  - Full pooling is applied
  - Each company is interested in the expected infinite horizon costs per time unit.
  - The chosen base stock level is fixed



Stock-out probability and cost function

- Consider coalition $M \subseteq N$, i.e. $m = |M|$ companies.
- Steady-state probability of having 0 parts on hand in the coalition:
  Erlang loss function:
  $\pi_0(mS, \rho) = \frac{\rho^{mS}/(mS)!}{\sum_{y=0}^{mS} \rho^y / y!}$ with $\rho = \frac{m\lambda}{\mu}$
- Cost function:
  $c(M) = \sum_{i \in M} h_i S + \pi_0(mS, \rho) \cdot m\lambda \cdot c^{em}$



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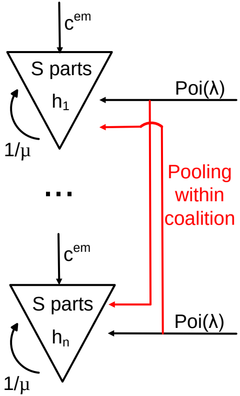

An example game

Example

$N = \{1, 2, 3\}$, $S = 1$, $\lambda = 5$, $\mu = 25$, $h_i = 4000$, $c^{em} = 13000$.
$\pi_0(1, \frac{5}{25}) \approx 0.17$, $\pi_0(2, \frac{10}{25}) \approx 0.05$, $\pi_0(3, \frac{15}{25}) \approx 0.02$.

Coalition size |M|	Characteristic costs c(M) (rounded)
1	14800
2	15000
3	15900

Core$\neq \emptyset$; e.g. allocation $x_i = 5300$ $\forall i \in N$ is a core element.
But the game is not concave!

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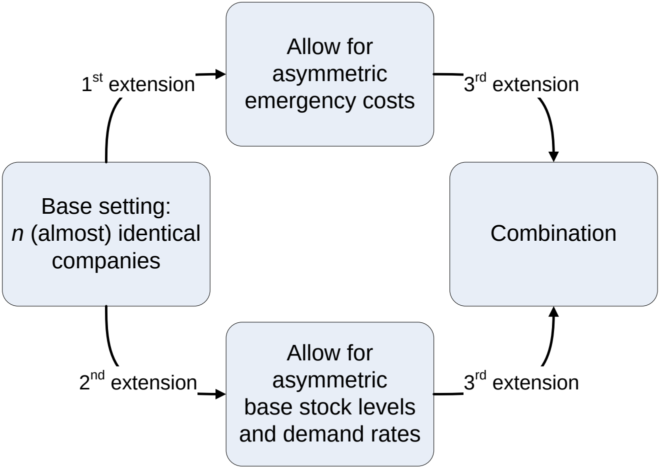
Outline of the analysis



Base Setting

Theorem

- *Situation:* $(N, S, \lambda, \mu, (h_i)_{i \in N}, c^{em})$.
- *Core of associated game is non-empty.*

Proof.

- Consider a cost allocation with No Transfer Payments:
  $NTP_i = h_i S + \pi_0 \left(|N| S, \frac{|N|\lambda}{\mu} \right) \cdot \lambda c^{em}, \; i \in N$
- $NTP \in Core(N, c)$. □

Asymmetric Emergency Costs

Theorem

- *Situation:* $\left(N, S, \lambda, \mu, (h_i)_{i \in N}, (c_i^{em})_{i \in N}\right)$.
- *Core of associated game is non-empty.*

Proof.

- Consider a cost allocation with No Transfer Payments:
  $NTP_i^E = h_i S + \pi_0 \left(|N| S, \frac{|N| \lambda}{\mu} \right) \cdot \lambda c_i^{em}, \, i \in N$
- $NTP^E \in Core(N, c)$. □

Asymmetric Demand Rates and Base Stock Levels

Theorem

- *Situation:* $(N, (S_i)_{i \in N}, (\lambda_i)_{i \in N}, \mu, (h_i)_{i \in N}, c^{em})$
- $c(M) = \sum_{i \in M} h_i S_i + \pi_0 \left(\sum_{i \in M} S_i, \sum_{i \in M} \frac{\lambda_i}{\mu} \right) \cdot \sum_{i \in M} \lambda_i c^{em}, M \subseteq N$
- *Core of associated game is non-empty.*

Asymmetric Demand Rates and Base Stock Levels

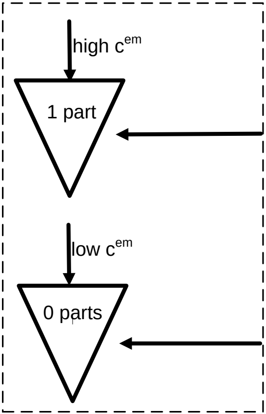
Proof.

- For a setting with identical λ but allowing asymmetric S:
  - Use convexity and subadditivity properties of the Erlang loss function $\Longrightarrow$ Shapley-Bondareva conditions hold
  - A game in such a setting has a non-empty core.
- Take a setting allowing asymmetric λ *and* S.
- Split each company into sub-companies to construct a game with identical λ but allowing asymmetric S.
- Take a core element from this constructed game.
- Transform it into a core element for the setting allowing asymmetric λ *and* S. □

Combination of asymmetries

- Counterexamples with empty cores for situations with
  - Asymmetric emergency costs *and* base stock levels
  - Asymmetric emergency costs *and* demand rates
- Main problem: full pooling approach.

Intuition: Non-optimal Full Pooling



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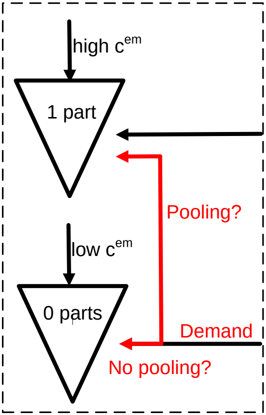

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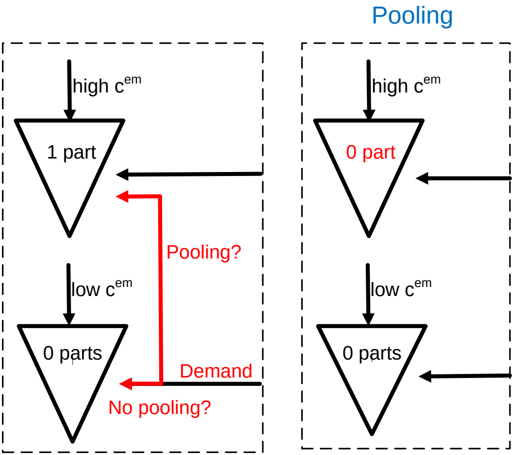

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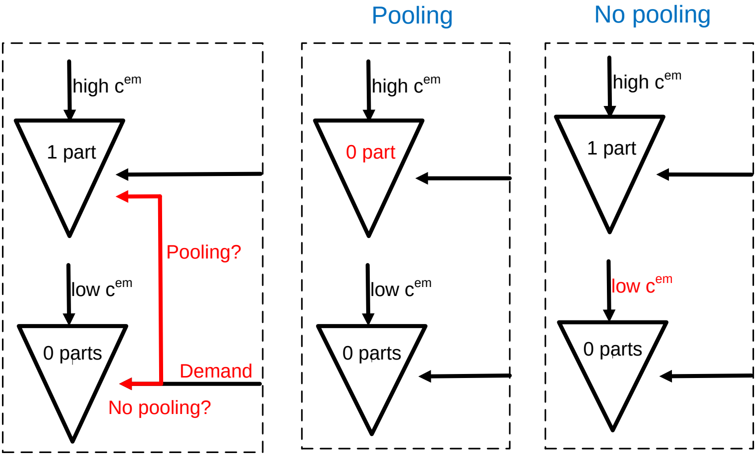

Summary

- **Setting**: several independent high-tech companies that
  - stock repairable spare parts of the same item
  - can pool their inventory
- **Main results**: Core $\neq \emptyset$
- For the base setting and for settings allowing
  - non-identical emergency costs.
  - non-identical base stock levels and demand rates; or
- But not always for a combination of asymmetries!
- **Future/current research**: extensions
  - Non-zero lateral transshipment costs, optimized base stock levels, partial pooling approach, two-echelon structure, ...