Decomposition Methods for the Optimization of Survivable Telecommunications Networks

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Abstract

Nowadays, survivability in telecommunication networks is a crucial problem in networks optimization. Given a set of requirements, which correspond to single or multicommodity flows, the survivable network design problem (*SNDP*) consists in determining a minimum cost network topology and capacity assignment such that all traffic demands are satisfied, especially in the case of link or node failure. In the literature, it is generally proposed that a reserve network is added to the main one in order to provide an additional capacity used only in the case of failure. Because capacity in telecommunications is a key resource, keeping excess capacity is expensive for telecommunication companies, but the inability to transfer data, even for a limited time, could be seen as unacceptable to the user, and consequently have an indirect impact on the profitability of the company. In this research, we are primarily interested in proposing a Linear Programming model, taking into account, on the one hand, specificity of failure occurrences, and on the other hand, an optimal traffic re-allocation or rerouting (global vs local) strategies. We also investigate the relevance and the performance of applying decomposition techniques (like the Benders Decomposition) to this kind of problems.

1 General Context

In the past decade, the international telecommunication business has changed almost beyond recognition. Regulatory and technological changes have transformed a once cozy cartel of monopoly carriers into a fiercely competitive market characterized by plummeting prices and razor-thin margins. The shock of market liberalization (particularly the European "Big Bang" reform of the late 1990s) has worked its way through the industry, and carriers have adapted to their new environment, albeit reluctantly. Large staff reductions and network rationalization have enabled carriers to survive, if not to thrive.

This unavoidable disturbance in the telecommunications world has forced the operating companies to change the way they approach their business. Client satisfaction has become consequently an essential criteria for success and that is why network survivability is a major preoccupation leading to potential advantages. Actually, the explosion and future growth of new services, of which telephony represents but a small fraction (see Figure 1 hereafter), is attracting a new, more demanding clientele. The competition between operating companies, now more intense due to market deregulation, requires that an ever increasing attention be given to the quality and cost of services.

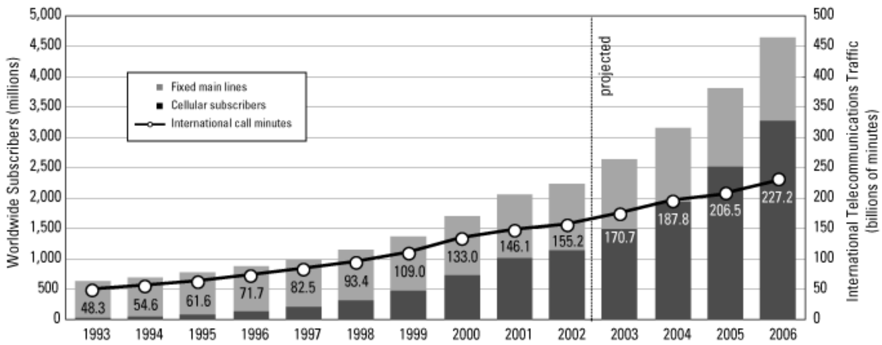

Figure 1: Evolution of the worldwide subscribers

Client requirements include high network availability, low tariffs levels, and finally good network reliability. If the rupture of a single telephone line is not a catastrophic event for an operator, the inability to transfer data, even for a limited period of time, could be unacceptable to the user. Even a small risk of a failure in the operator's network could discourage a client from making use of the services of that operator. Failures however cannot be entirely eliminated, whether it be a question of equipment (switching, router, transmission...) at a network node, or a physical break in a connection link.

However operating companies, due to the growth of competition and consequently due to the need to limit costs, cannot invest in sufficient excess capacities, to allow them to ensure a maximum level of client satisfaction. Operating companies try therefore to ensure that the networks will be survivable, which means that enough capacity is installed over it, so that all traffic interrupted by a failure may be rerouted. Nevertheless this implies huge capital investment for their network operators.

Their major dilemma, and also all the aim of this research, is to find the right trade-off between capacity allocation and service level, or as Wu [54] said :"*...Reducing network protection cost, while maintaining an acceptable level of survivability, has become an important challenge for network planners and engineers...*".

2 Research Objectives

As in the PhD of Christelle Wynants [55], this research will focus on a particular set of network flow problems: the network synthesis (*NS* for short) or dimensioning problem which consists in determining the amount of capacity to install on each link of a given network in order to satisfy some known traffic requirements at minimum cost. Here, we will talk more precisely about the survivable network design problem (*SNDP*) which is a particular class of the *NS* problems. As for the *NS*, a classical *SNDP* instance is characterized by a graph, a cost structure and a set of requirements. Adding capacity to links induces some costs which depend on the application. First of all, we will approximate these installation costs by a linear function dependant on the amount of capacity installed. The particular survivability aspect includes the optimization of the network for the non-failure case and all single link failures.

We will thus formally represent the network as a graph $G = (N, E)$, where N is the set of nodes and E the set of non-directed potential edges linking the different elements of the node set[1]. Because each edge between node i and j can be used in both directions (from i to j and from j to i), we define A, the set of directed potential arcs with the following property: $|A| = 2|E|$. The basic idea is to find the minimum network capacity which will be sufficient in each case (failure and non-failure) to send information for each pairs "origin-destination". We will illustrate this idea in the following simple example.

Figure 2: Example of a multicommodity network with 2 demands

Consider the simple network represented in Figure 2. This network is composed of 8 nodes (A,B,C,D,E,F,G,H), 10 potential links and 2 different demands (10 units from H to A and 5 units from B to G)[2].

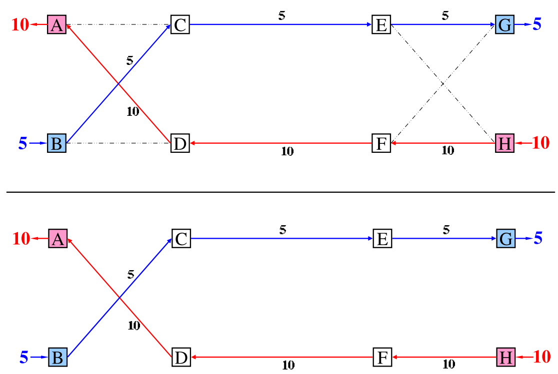

Figure 3: Solution without failure

Assume also that the optimal topology of this network leads to the situation represented in Figure 3. We can see in this figure that among the 10 potential edges, only 6 are used in the optimal topology, leading to two different

[1]In our application to the sector of telecommunications, and more precisely to the internet world, the different nodes are the routers and the computers whereas the edges represent the link between these routers.

[2]In this simple example, we ignore the cost structure associated with the links.

paths (B-C-E-G for the demand of 5 units from B to G, and H-F-D-A for the demand of 10 units from H to A)[3]. If we consider an edge cost of 1 € by unit transported, this solution leads to a total cost of 45 €[4] which minimizes the total cost without failure occurrence.

Now, assume that only the two central edges (CE) and (DF) can break down one at a time. Using the topology of Figure 3 in case of failure, leads certainly to the optimal solution in terms of cost but on the other hand this topology cannot be totally effective, because it is impossible to route all demands without delay and bad service quality.

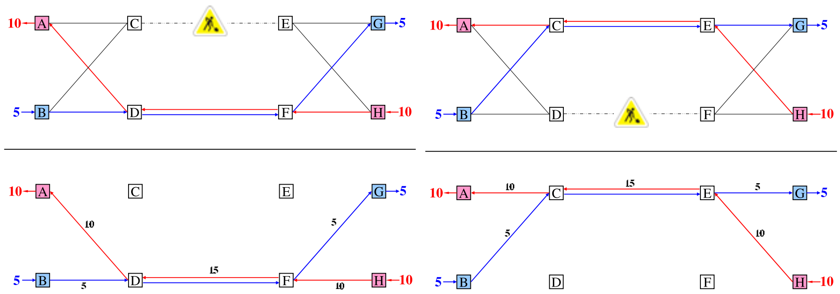

Figure 4: Solution with one edge failure at a time

As an alternative approach, an intuitive solution would be to put for each potential edge of the network, a capacity equal to the sum of the different demands. In this case, this intuitive solution leads to a total cost of 150 € because we have 10 potential edges and the sum of the demands is equal to 15 units. This solution is very effective in terms of service level because it works in any failure situation. But in terms of cost, it is very expensive.

Another approach, (and the one we shall adopt) is to compute for each failure case a new optimal network topology.[5] We see easily in Figure 4 that the two configurations are quite different. Because the network configuration has to work for the two different cases of failure, we have to use a topology

[3]Note that, in the picture, the capacity of each edge is indicated beside each edge used.

[4]45 is just equal to the sum on the edge of the capacity used.

[5]In the Figure 4, the left part represents the failure of edge (CE) whereas the right part represents the failure of edge (DF).

like the one represented in Figure 5, which is an aggregate solution. From an economic point of view, this solution leads to a cost of 90 €. This cost is greater than the one illustrated in the Figure 3 but it is the minimum cost which leads to a 100% service level.

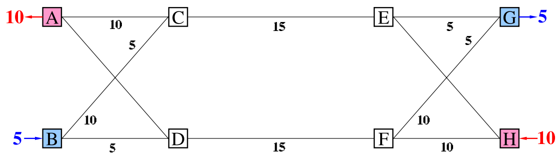

Figure 5: Optimal solution with failures

In this simple example it is relatively easy to imagine the aim of our research. The problem is that in reality, we do not use small networks with only 8 nodes and 10 edges. For example, and as expressed in [31], a realistic France Telecom's Main Interconnection Network counts 60 nodes and 120 links. With this kind of network, it is more complicated to find the optimal solution for all scenarios of failure because the number of variables and constraints of the model explode, and the problems are often large-scale. That is why we will introduce in this work the concept of decomposition.

3 Literature Review

In this section we will focus first of all on a literature review of the network design problems, which is the root category of the type of problems we are working on in this paper. After that, we will introduce some decomposition concepts.

3.1 Network Design Problems

Network optimization problems which, among other issues, include topological optimization and optimum network dimensioning, arise in many important areas of applications, such as telecommunications [21, 36], transportation systems, distribution and logistics [8], flexible and automated manufacturing systems, facility location decisions, warehouse and plant location, vehicle fleet planning applications, energy systems...For our research, we have decided to focus on the telecommunications sector.

But even at this sector level, research perspectives are diffuse [4, 7, 9, 19, 38].

An important class of problems that have been studied is the class of capacited problems, in which the network edges have capacity constraints and in which the aim is to find optimal flow due to this constraints. In [1] Agarwal at al. deal with the design of capacited multicommodity networks with multiple facilities, as in [29, 37, 43, 44, 54]. Depending of the types of facilities[6] available to design the network, this problem may have several versions. On the other hand, some others researches [11, 15, 22, 35, 45] focus on uncapacited problems and try to define the right capacity which leads to the minimum total cost.

Another branch of the tree concerns the way the cost function will be modelled. Some studies [35] use a linear cost function which means that the transfer cost of an edge increase linearly with the quantity of data using this edge. Others use two cost components, one fixed cost for the link installation or activation and one variable cost dependent of the quantity of data using this edge. Minoux in [19, 37, 38] prefers to use a discrete, nonconvex, discontinuous and step increasing cost function on the links which is, according to him, appropriate to capture two major aspects of actual cost structure in telecommunication network engineering:

- the discreteness of capacity and cost increase on the links as well as at the nodes.
- the so-called "economy of scale" phenomenon, which is basically that the cost per unit capacity installed decreases with the maximum capacity provided by an equipment.

Finally, after the considerations on uncapacited or capacited and cost function, an important research area in these last few years seems to be survivability. As already stated, the internet has seen tremendous growth in the past decade and has now become the critical information infrastructure for both personal and business applications. It is expected to be always available as it is essential to our daily commercial, social and cultural activities. Service disruption for even a short time can be catastrophic in the world of e-commerce, causing economic damage as well as tarnishing the reputation of a network service provider. In addition, many emerging services such as Voice over IP and virtual private

[6]Facilities with a single fixed capacity, i.e. T1 facility which provides a transmission rate of 1.5 megabits/second. In addition to a high capacity facility such as T1, a facility of unit capacity is also available (Two-facility loading problem). Finally, several facilities of different capacities and costs are available.

networks require stringent service availability and reliability. Unfortunately, failures are fairly common in the everyday operation of a network due to various causes such as maintenance, faulty interfaces, and accidental fiber cuts. Moreover, it has been observed that most failures are transient: 50% last less than a minute. Hence there is a growing demand for failure resilient methods that ensure high service availability and reliability despite transient link failure. As expressed in [32], the survivability analysis was first discussed in the context of military command, control and communication (C^3) systems in 1970s.

The current practice in dimensioning survivable networks is to design two networks: a "base network" used in normal operation with no failure and a so-called "reserve network" dedicated to rerouting traffic in case of failure [29, 31, 35, 36]. In other words, capacities on the reserve network are not used to route traffic in normal situations and capacities on the base network are not used to reroute traffic in case of failure. Research also focuses on the rerouting techniques. Two different rerouting methods may be used: local or global rerouting. With local rerouting, each failure induces a new local demand between the end points of the failed link with a value equal to the total flow initially routed on this failed link. However with global rerouting [31], each demand affected by the failure is rerouted from its proper origin to its destination [30]. Others issues in survivability are also interesting. Altinkemer [4] cites some studies linked with the survivability concept like the one of Eric Rosenberg [46] who investigates capacity requirements for node and arc survivability in a rectilinear grid communication network when a failure occurs. The study of Soni and Pirkul [48] which is also cited in the Altinkemer's paper, tries to re-establish connectivity during a link or node failure by adding new constraints where each case is simple enough to solve.[7] In [5], the authors tackles the problems using another approach. They say that usually it is not easy to reorient flows quickly enough to take advantage of alternate routes. They define a so-called "residual flow" as the flow that was not using any of the destroyed nodes and/or arcs, in other words, this residual flow seems to be the global flow not affected by the failure. The objective of their research is therefore to determine a flow such that the residual flow value is maximized.

As the scope of applications is large, the scope of methods used for the

[7] As we can see later, this concept is quite similar to our decomposition principle.

resolution of this kind of problems is also diversified [9]. Some studies choose Branch and Cut techniques [11, 22, 36], Column Generation in [47], Column and Cut Generation in [43], Lagrangian and Linear Relaxation in [55], Tabu Search in [23, 55], Simulated Annealing in [25] and, as our work, Benders Decomposition in [10, 21, 34, 35, 45].

In this research we will focus on the survivable network design problem (*SNDP*) and we will consider, in the first step of the research, a linear cost structure. We study also the impact of using on one hand local rerouting and on the other hand a more global approach. But the major contribution consists in the resolution of such a type of problems by the Benders decomposition.

3.2 Decomposition Methods

Since the original work in 1960 of Dantzig and Wolfe, the fathers of the well known simplex algorithm, the idea of decomposition, by which a large problem is decomposed into smaller problems, has existed as an attractive approach to large-scale linear programming. These two authors proposed the first version of the decomposition techniques known in the literature. Without going into detail, (the reader can find more explanations of the theory in [33]), suppose that the basic model is a linear programming problem with two sets of constraints as follows:

$$\begin{array}{lrcl} \max & cx & & \\ s.t. & A_1 x & \leq & b_1 \\ & A_2 x & \leq & b_2 \\ & x & \geq & 0 \end{array} \tag{1}$$

where $c \in \mathbb{R}^n, A_1 \in \mathbb{R}^{m \times n}, b_1 \in \mathbb{R}^m, A_2 \in \mathbb{R}^{q \times n}, b_2 \in \mathbb{R}^q$ are given constants and $x \in \mathbb{R}^n$ is a vector of variables. This kind of linear program with the block-angular structure arises from the optimization of systems consisting of coupled subsystems. Applied to these block-angular structures, the elegant decomposition principle introduced by Dantzig and Wolfe, implies that the entire problem can be solved by using a coordinated sequence of independent subproblems corresponding to the subsystems. In others words, the fundamental idea is to solve (1) by interaction between two optimization problems, one of which is subject to the first set of constraints and the other subject to the second set of constraints.

Two years later, in 1962, J.F Benders devised a clever approach for exploiting the structure of mathematical programming problems with so-called complicating variables (variables which, when temporarily fixed, render the remaining optimization problem considerably more tractable). For the class of problems specifically considered by Benders, fixing the values of the complicating variables reduces the given problem to an ordinary linear program, parameterized, of course, by the value of the complicating variables vector. The algorithm he proposed for finding the optimal value of this vector employs a cutting-plane approach. Linear programming duality theory such as Farkas lemma was also employed to derive the natural family of cuts characterizing the representations. Benders decomposition thus leads consequently to certain improvement in terms of algorithm convergence and also in terms of rapidity, which justifies the fact that it has been used in a significant number of applications [6, 8, 10, 21, 26, 34, 35, 45].

Mathematically, the original problem can be written as follows:

$$\begin{array}{lrcl} \max & c_1x_1 + c_2x_2 & & \\ s.t. & A_1x_1 + A_2x_2 & \leq & b \\ & x_1, x_2 & \geq & 0 \end{array} \tag{2}$$

The Benders decomposition idea is to rewrite this original problem parameterizing one group of variables, for example the x_2 variables. This leads to the following relaxed problem:

$$\begin{array}{lrcl} \max & c_1x_1 + w_p(x_1) & & \\ s.t. & x_1 & \geq & 0 \end{array} \tag{3}$$

Let x_1^* be the optimal solution of (3) and $w_p(x_1^*)$ be the optimal solution of the following subproblem in which :

$$\begin{array}{rll} w_p(x_1^*) = \max & c_2x_2 & \\ s.t. & A_2x_2 & = b - A_1x_1^* \\ & x_2 & \geq 0 \end{array} \tag{4}$$

The dual version of the subproblem can be formulated as:

$$w_d(x_1^*) = \min \quad y_2^T(b - A_1x_1^*)$$

$$
\begin{array}{lll}
s.t. & y_2^T A_2 & \geq c_2 \\
 & y_2 & \geq 0
\end{array} \tag{5}
$$

The idea is thus to solve the dual subproblem and owing to duality proprieties conclusions which say that the optimal dual solution is always a higher bound of the primal one, we can write the following new constraint:

$$
w_d(x_1^*) = y_2^T(b - A_1 x_1^*) \geq w_p(x_1^*) \tag{6}
$$

for any $y_2 \geq 0$ such that $y_2^T A_2 \geq c_2$.

Consequently, the optimal solution of the dual subproblem leads to the addition at each iteration, of one or more new constraints (also called Benders Cuts) to the relaxed problem (3), also called "Master Program":

$$
\begin{array}{rrcl}
\max & c_1 x_1 + w_p(x_1) & & \\
s.t. & y_2^{T*} A_1 x_1 + w_p(x_1) & \leq & y_2^{T*} b \\
 & x_1 & \geq & 0
\end{array} \tag{7}
$$

The algorithm can be seen as a two stage optimization model in which we fix firstly the value of one variables group and then solve the Master Program. The optimal solution of this MP become the parameters of the subproblems. We solve then the subproblems and find the optimal solution which become, in turn, the new parameters of the MP... The Dantzig-Wolfe decomposition algorithm was thus called "*Primal Decomposition Algorithm*", and is characterized by a division of the problem between two groups of constraints, whereas the Benders decomposition algorithm took the name of "*Dual Decomposition Algorithm*", and is based, on the opposite, on a variables division.

4 Research Plan

In this section we use the well-known multicommodity network flow problem (*MCNF*) as starting model. As said before, our contribution is to introduce the occurrence of failure. We can deal with this latter problem in two different ways. Firstly, and as used in **model 1**, we use an Independent Approach for the failures. In others words, each edge can break down separately, and for each edge we will solve independently a different problem in which one particular edge is suppressed. This approach is thus equivalent to solving k different problems, (where k is equal to the number of edges), and these problems have different independent solutions. The optimal solution is simply the one in which we will take for each edge of the network the maximum capacity allowed by the solution of the different problems.

The other way to deal with failure case is to apply Global Optimization. In fact, we will examine in the results section, that the Independent Approach leads some times to a network topology with excess cost, because it does not links the different failure scenarios together. This Global Optimization will be the subject of the **model 2**.

Because these problems are large-scale problems, one contribution of our research is, as said in section 3, to use the Benders Decomposition. We will thus apply this principle to model 2 and find a new formulation (called **Benders Approach**) in which the number of variables and constraints decrease considerably compared to our model 2.

Finally, we can show that all the commodities with a common destination can be aggregated leading to a substantial decrease in the number of variables and constraints. We try to include this considerations in our **model 3**.

4.1 Models

For the following models, we assume that the cost structure is linear. Also, we do not consider fixed cost on the edge in order to use a linear program and not a mixed integer one. When an edge is down , we assume that the two associated directed arcs are consequently down. It is mandatory that in the network considered, there exist at least two different paths between two nodes. This condition is crucial if we want to reroute correctly some flows when a particular edge is down, without loss of service level. All the following models use a node-arc formulation. Last but not least, we consider only one edge failure at a time. A graph $G = (N, E)$ is used to represent the network where N is the set of network nodes, E is the set of potential undirected network edges, $A = \{(i, j), (j, i) | \{i, j\} \in E\}$ is the set of directed network arcs and $D \subset N \times N \times \mathbb{R}$ is the set of network demands or commodities. For each candidate edge $ij \in E$ we also use parameter c_{ij} which is the cost associated with edge ij.

4.1.1 Model 1: Independent Approach

The independent approach uses almost the same formulation as the *MCNF* (e.q. [2]). A new set $S = E$ is defined and represents the different failure scenarios. The decision variables are firstly the F_{ij}^{odf-mn},$\forall (i,j) \in A$, $\forall (o,d,f) \in D$ and $\forall mn \in S$ which represent the flow using arc (i,j) for the demand (o,d,f) of size f from the source node o to destination node d in case of edge mn failure. The second decision variable are the y_{ij}^{mn} which represent the capacity of edge ij in case of edge mn failure. Finally y_{ij} is the final capacity of edge ij taking into account all the different failure scenarios. The model can be written as:

Objective Function

$$\min \sum_{mn \in S} \sum_{ij \in E} c_{ij} y_{ij}^{mn} \tag{8}$$

Capacity Definition Constraints

$$\sum_{odf \in D} F_{ij}^{odf-mn} + \sum_{odf \in D} F_{ji}^{odf-mn} \leq y_{ij}^{mn}, \forall ij \in E, \forall mn \in S \tag{9}$$

Flow Conservation Constraints

$$\sum_{j \in N} F_{ji}^{odf-mn} + \sum_{j \in N} F_{ij}^{odf-mn} = \begin{cases} f & if \quad i = d \\ -f & if \quad i = o \\ 0 & else \end{cases} , \forall i \in N, \forall odf \in D, \forall mn \in S \tag{10}$$

Failure Definition Constraints

$$y_{mn}^{mn} = 0, \forall mn \in S \tag{11}$$

Final Capacity Constraints

$$y_{ij} \geq y_{ij}^{mn}, \forall ij \in E, \forall mn \in S \tag{12}$$

Non-negativity Constraints

$$F_{ij}^{odf-mn} \geq 0, \forall ij \in A, \forall odf \in D, \forall mn \in S \tag{13}$$

$$y_{ij}^{mn} \geq 0, \forall ij \in E, \forall mn \in S \tag{14}$$

$$y_{ij} \geq 0, \forall ij \in E \tag{15}$$

where the objective function (8) minimizes the total cost of reserving capacity on edge in each scenario. Capacity Definition Constraints (9) and Flow Conservation Constraints (10) are constraints commonly used in *MCNF*. Constraint (11) introduces the failure in the network. Constraint (12) defines the final capacity of the edges equal to the maximum capacity of all the different scenarios of failure. Finally, constraints (13), (14), (15) are non-negativity constraints. This approach is independent because in the objective function, we optimize the sum on the scenario set of the independent capacities. Optimal solution is given by the value of $y_{ij} = \max_{mn \in S} y_{ij}^{mn}$. From the particular structure of the objective function, this model can be decomposed by failure scenario in order to solve k independent small problems where k is equal to the number of failure scenarios, in others words $k = |S|$.

4.1.2 Model 2: Global Approach

Another alternative is to model the problem as a Global Optimization. This new model is almost similar than the previous one:

Objective Function

$$\min \sum_{ij \in E} c_{ij} y_{ij} \tag{16}$$

Capacity Definition Constraints

$$\sum_{odf \in D} F_{ij}^{odf-mn} + \sum_{odf \in D} F_{ji}^{odf-mn} \leq y_{ij}, \forall ij \in E, \forall mn \in S \tag{17}$$

Flow Conservation Constraints

$$\sum_{j \in N} F_{ji}^{odf-mn} + \sum_{j \in N} F_{ij}^{odf-mn} = \begin{cases} f & if \quad i = d \\ -f & if \quad i = o \\ 0 & else \end{cases}, \forall i \in N, \forall odf \in D, \forall mn \in S \tag{18}$$

Failure Definition Constraints

$$F_{mn}^{odf-mn} = F_{nm}^{odf-mn} = 0, \forall odf \in D, \forall mn \in S \tag{19}$$

Non-negativity Constraints

$$F_{ij}^{odf-mn} \geq 0, \forall ij \in A, \forall odf \in D, \forall mn \in S \tag{20}$$

$$y_{ij} \geq 0, \forall ij \in E \tag{21}$$

One main difference between model 1 and model 2 is the formulation of the objective function. In (40) the summation on the set of scenarios has disappeared. Now we determine directly in (41) the global capacity because the variable y_{ij} is computed taking into account all the different scenarios of failure. In other words constraints (41) link all together the different failure scenarios and ensure the fact that the approach is global. And consequently the constraint called "*Final Capacity Constraints*" (12) as the variable y_{ij}^{mn} in model 1 are not necessary in this Global model.

4.1.3 Benders Approach

Model 2 is a large size model because of the important number of variables (mainly the variables F_{ij}^{odf-mn}) and also the important number of constraints. On the other hand, model 2 is composed of two groups of variables, the capacity variables y_{ij} linking all the failure scenarios and the flow variables F_{ij}^{odf-mn}. These two conditions (large size model and two groups of variables) motivate our choice in favor of using Benders Decomposition for solving this particular model 2. That is why we have to divide the global problem into Master Problem and SubProblems. The Master Problem which is a relaxed problem can be modelled as:

Master Problem:

Objective Function

$$\min \sum_{ij \in E} c_{ij} y_{ij} \tag{22}$$

Non-negativity Constraints

$$y_{ij} \geq 0, \forall ij \in E \tag{23}$$

This MP is function only of the y_{ij} variables, the rest of the model 2 (which is function of the variables F_{ij}^{odf-mn}) becomes the SubProblems for a scenario of failure $\overline{mn}$ fixed:

SubProblems for $\overline{mn}$ fixed:

Objective Function

$$\min \sum_{ij \in E} \sum_{odf \in D} 0 \cdot F_{ij}^{odf-\overline{mn}} \tag{24}$$

Capacity Definition Constraints

$$\sum_{odf \in D} F_{ij}^{odf-\overline{mn}} + \sum_{odf \in D} F_{ji}^{odf-\overline{mn}} \leq y_{ij}, \forall ij \in E \tag{25}$$

Flow Conservation Constraints

$$\sum_{j \in N} F_{ji}^{odf-\overline{mn}} + \sum_{j \in N} F_{ij}^{odf-\overline{mn}} = \begin{cases} f & if \quad i = d \\ -f & if \quad i = o \\ 0 & else \end{cases}, \forall i \in N, \forall odf \in D \tag{26}$$

Failure Definition Constraints

$$F_{mn}^{odf-\overline{mn}} = F_{nm}^{odf-\overline{mn}} = 0, \forall odf \in D \tag{27}$$

Non-negativity Constraints

$$F_{ij}^{odf-\overline{mn}} \geq 0, \forall ij \in A, \forall odf \in D \tag{28}$$

$$\tag{29}$$

We compute now the dual version of the SubProblems, in which the variables y_{ij} of the Master Problem become parameters $\overline{y}_{ij}$. We introduce two new decision variables linked to the two main kind of constraints (the Capacity Definition Constraints and the Flow Conservation Constraints) in the primal version of the SubProblems. Let $\sigma_{ij}^{\overline{mn}}$ be the marginal profit[8] given by one more unit of capacity available on edge ij in case of the failure of edge $\overline{mn}$, $\forall ij \in E$. Let $\pi_i^{odf-\overline{mn}}$ be the value of the shortest path from node o to node i, for demand odf when edge $\overline{mn}$ is down, $\forall i \in N$ and $\forall odf \in D$. The new SubProblems are:

[8] Note that this new variables are directly linked with the constraints of Capacity Definition of the primal SubProblems whereas the other new variables $\pi_i^{odf-\overline{mn}}$ are directly linked with the constraints of Flow Conservation of the primal SubProblems

SubProblems for $\overline{mn}$ fixed: Dual Version

Objective Function

$$\max \sum_{odf \in D} ((\pi_d^{odf-\overline{mn}} - \pi_o^{odf-\overline{mn}})f) - \sum_{ij \in E} \overline{y}_{ij}\sigma_{ij}^{\overline{mn}} \tag{30}$$

Shortest Path 1 Constraints

$$\pi_j^{odf-\overline{mn}} - \pi_i^{odf-\overline{mn}} \leq \sigma_{ij}^{\overline{mn}}, \forall ij \in E, \forall odf \in D, \ (i \neq \overline{m}) \ or \ (j \neq \overline{n}) \tag{31}$$

Shortest Path 2 Constraints

$$\pi_i^{odf-\overline{mn}} - \pi_j^{odf-\overline{mn}} \leq \sigma_{ij}^{\overline{mn}}, \forall ij \in E, \forall odf \in D, \ (i \neq \overline{m}) \ or \ (j \neq \overline{n}) \tag{32}$$

Normalization Sigma Constraints

$$\sigma_{ij}^{\overline{mn}} \leq 1, \forall ij \in E, \ (i \neq \overline{m}) \ or \ (j \neq \overline{n}) \tag{33}$$

Normalization Pi Constraints

$$\pi_o^{odf-\overline{mn}} = 0, \forall odf \in D \tag{34}$$

Correction Constraints

$$\sigma_{ij}^{\overline{mn}} = 0, \forall ij \in E, \ (i \neq \overline{m}) \ and \ (j \neq \overline{n}) \tag{35}$$

Non Negativity Constraints

$$\sigma_{ij}^{\overline{mn}} \geq 0, \forall ij \in E \tag{36}$$

where equation (32) is the same kind of equation than (31), but in place of going from i to j, we follow the edge ij in the opposite way. The normalization constraints (33), (34) exist only to prevent the algorithm from going to infinity. As seen in section 3.2, due to duality, we introduce cuts in the Master Problem which becomes:

Master Problem: Modified

Objective Function

$$\min \sum_{ij \in E} c_{ij} y_{ij} \tag{37}$$

Cuts Constraints

$$\sum_{odf \in D} ((\overline{\pi}_d^{odf-\overline{mn}} - \overline{\pi}_o^{odf-\overline{mn}})f) - \sum_{ij \in E} y_{ij}\overline{\sigma}_{ij}^{\overline{mn}} \leq 0, \forall mn \in S \tag{38}$$

Non-negativity Constraints

$$y_{ij} \geq 0, \forall ij \in E \tag{39}$$

4.1.4 Model 3: Aggregated Approach

Finally we adapt the model 2 taking into account aggregation by destination. Some new sets or parameters have been created. Let $DEST$ be the subset of all different demand destinations. Let $ORIG[i]$ where $i \in DEST$ be the subset of the different demand origins for a special destination. Let F^d be the aggregate quantity we have to send to the destination d and let f_i^d where $d \in DEST$, $i \in ORIG[d]$ be the particular demand quantity we have to send from a specific origin and a specific destination with: $\sum_{i \in ORIG[d]} f_i^d = F^d, \forall d \in DEST$.

Objective Function

$$\min \sum_{ij \in E} c_{ij} y_{ij} \tag{40}$$

Capacity Definition Constraints

$$\sum_{d \in DEST} F_{ij}^{d-mn} + \sum_{d \in DEST} F_{ji}^{d-mn} \leq y_{ij}, \forall ij \in E, \forall mn \in S \tag{41}$$

Flow Conservation Constraints

$$\sum_{j \in N} F_{ji}^{d-mn} + \sum_{j \in N} F_{ij}^{d-mn} = \begin{cases} F^d & if \quad i = d \\ -f_i^d & if \quad i \in ORIG[d] \\ 0 & else \end{cases}, \forall i \in N, \forall d \in DEST, \forall mn \in S \tag{42}$$

Failure Definition Constraints

$$F_{mn}^{d-mn} = F_{nm}^{d-mn} = 0, \forall d \in DEST, \forall mn \in S \tag{43}$$

Non-negativity Constraints

$$F_{ij}^{d-mn} \geq 0, \forall ij \in A, \forall d \in DEST, \forall mn \in S \tag{44}$$

$$y_{ij} \geq 0, \forall ij \in E \tag{45}$$

It is obvious that if all the different demands have different destinations, this model leads to the same performance that model 2, because it is not possible to aggregate anything. Otherwise, as with model 2, we can also apply Benders Decomposition to this new model 3 which leads to the **Benders Aggregated Approach** tested in next section.

4.2 Implementation and Results

The different models presented before have all been implemented in AMPL language, and the code can be found in [12]. In this results section, we want to evaluate the performance of our models.

Topology	IA Cost	GA Cost	Win (%)
3/6/2	160	160	0
6/26/5	15106	11095	26
7/18/9	780	685	12
10/32/16	44446	33316	25
30/150/2	21327	14982	30
30/150/5	52221	32501	38
30/150/7	62126	39971	36
30/150/10	76768	51695	33

Table 1: Cost of the optimal solution.

The first conclusion is the one linked to the difference between model 1, The Independent Approach and model 2, The Global Approach. By doing some tests on different small networks, we want to measure empirically the gain in terms of cost obtained by using The Global Approach rather than the Independent one. Table 1 gives the answer to this question. The first column represents the topology of the network tested. 3/6/2 signifies that the network tested have 3 different nodes, 6 potential arcs (or 3 potential edges) and 2 demands. The second column gives us the cost of the optimal solution found by using the Independent Approach. The third column represents the optimal solution found by using the model 2, the Global Approach. Column four gives us an idea of the potential gain linked to the use of the Global Approach. This Table shows us that the Global model leads to substantial cost savings that increase with the size of the network. This leads to the conclusion that models 2 and 3 and the associated Benders Approach and Benders Aggregated Approach which all the four use Global Optimization, will be chosen if we want to find the most economical solution.

Next, we have to evaluate if the decomposed models (Benders Approach and Benders Aggregated Approach) are better in terms of CPU time than the non-decomposed one (model 2). Table 2 shows us again in the first column the topology of the network. In column two, three, four and five, we find respectively the CPU time needed to find the optimal solution[9] for model 1 (Independent Approach), model 2 (Global Approach), Benders Approach and finally for Benders Aggregated Approach. The last column gives us the gain between Benders Aggregated Approach and model 2, the Global Approach.

Topology	IA	GA	Benders	BendersAgg	Win (%)
3/6/2	0.03	0.01	0.11	0 .11	-
6/26/5	0.11	0.07	1.52	1.22	-
7/18/9	0.09	0.06	1.21	0.57	-
10/32/16	0.23	1.69	5.07	2.55	-
30/150/2	0.79	5.58	26.04	21	-
30/150/5	1.09	322	165.83	82.61	74.34
30/150/7	1.28	452	258.77	76.75	83.01
30/150/10	1.58	889.13	440.39	66.66	92.5

Table 2: CPU time in sec needed to find the optimal solution.

Here, the conclusions are not so obvious as in the case of the previous evaluation. In fact, for the small-scale problems (the first five cases), we see that the Decomposed Approaches take more times to find the optimal solution.

[9]Notice that the solution found by the model 1 is not in reality the optimal solution. Only model 2, 3 and Benders Approach gives us the optimal network topology with failure.

This is mainly due to the fact that the Decomposed programs have to jump from the master problem to the subproblems and have to make some iterations in order to find the optimum. For easy and small cases like the first five, it is clear that they are easier to solve using the Global Approach than using the Decomposed ones. However, in case of large-scale problems, (notice that real problems are more likely to be large-scale problems than small-scale), we find some improvements in terms of CPU time for the two Decomposed programs. In this case, we can conclude that Decomposed programs are more effective for large-scale problems than the Global Approach.

If we compare the second and the fifth column, we see that in every case, it takes less time using the Independent Approach than using the Decomposed Approaches. Thus, in terms of times it seems to be that the Independent Approach gives the best solution whereas in terms of cost, the Decomposed ones are better. People who works in management world have to make the correct trade-off between economy of cost and economy of time.

Finally, we want to evaluate if an aggregation by destination is profitable, in terms of CPU time, for the Benders Decomposition. In Table 2, as said before, we have the performance in terms of CPU time, and in Table 3 we find the performance in terms of number of variables included in each model.

Topology	GA	Benders	BendersAgg
3/6/2	39	30	30
6/26/5	1703	572	338
7/18/9	1467	657	216
10/32/16	8208	2832	592
30/150/2	22575	10200	10200
30/150/5	56325	16950	10200
30/150/7	78825	21450	10200
30/150/10	112500	28200	10200

Table 3: Comparison of variables number in the principal models and approaches.

The two tables are providing similar conclusions for this evaluation. Doing an aggregation by destination decreases the number of variables and consequently the CPU time. In others words, we can say that the Benders Aggregated Approach is the best approach in terms of CPU time and also in terms of cost, for large-scale problems.

5 Contribution and Futures Researches

In this paper we modelled the problem of designing survivable telecommunication networks. We propose to design a single network for both base traffic and restoration. That means that the capacity of our network is better used both in the case of failure or without failure than with methods which optimize a reserve network because they create a parallel network with capacity which is not used in case of non-failure.

Another specific approach of our work is to take advantage of decomposition methods in order to improve the problem solving process. As seen in section 4, the improvements are encouraging.

Moreover, aggregation allows us to decrease the size of the model and leads to additional improvements and better convergence.

In addition to decomposition method which leads to the optimal solution in a very acceptable time, we propose an alternative model (the independent one) which converges faster than the decomposed one, but sometimes with a loss of precision.

The experiments show that our final method is able to solve a large-scale problem by decreasing the CPU time by 90%.

Our future research will focus on the problem of local against global rerouting. Currently, our models do not manage this kind of consideration because for each failure scenario, we compute new flows. Global or local rerouting constraints can probably improve the convergence of our models because only a few demands have to be rerouted and not the totality of the demands set. The question is to know if the decomposition principle could again be used in these cases. Our aim is to adapt the model taking into account local or global considerations, in a way that decomposition methods could again be used. In others words, we have to find a model which could take advantage of the decomposition approach on one hand, but also take advantage of new local or global rerouting constraints.

A critic formulated against Benders Decomposition is that when the model is composed of big subproblems, it can reduce the convergence of the algorithm. In our work we will have to pay attention to the subproblem size, and as much as possible we have to try to decrease it.

Some studies like [30, 35] suggest improvements to better find the right cuts the algorithm has to add to the master program. In fact, it can happen that some cuts are redundant with others.

We can also modify our basic assumptions like the cost function which is assumed to be linear in our current models. In fact in reality the more an edge is used, the more it costs to use it. That is why in some studies piecewise-linear cost functions are used. We can also use other non-linear functions to model the reality.

Currently we use a node-arcs formulation, it can be interesting to test the performance of arcs-paths formulation which is another variant. Using this arcs-paths formulation, we can use an idea expressed in [17, 22], where the authors talk about Equal Commodity Multi-flow (ECM) principle. This ECM splits the commodity flow at each node with more than one outgoing arc belonging to a shortest path from the node itself to the destination.

Finally, using binary variables to model the fixed installation cost of edges, or adding nodes capacity, or nodes failure occurrences, or failure probability function on edges or nodes, or management of more than one failure at a time are others paths of research that we could take into account in the continuation of this work.

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