The financing of first pillar pension

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Table of contents

Context and motivation

- Financing of pensions (first pillar) in a stochastic framework
- Two extreme concepts

	Pay-As-You-Go (unfunded)	**Funded pension**
Mechanism	Active people finance retired people’s pensions	Contributions are invested on financial market for one’s own future pension
Annual rate of return	Growth rate of the salary mass	Financial Return

- Two ways of constructing the equilibrium of the scheme

	Defined Contributions (DC)	**Defined Benefits (DB)**
Mechanism	You know what you’re paying	You know what you’ll receive at pension

Context and motivation (2)

- Ex. Belgian state pensions = PAYG, DB (salaries, length, status)
- Demographics :
  - ➢ Fact : ↓ fertility rates (less children) and ↑ life expectancy (older people)
  - ➢ Consequences :
    - ageing populations (dependency ratio : from 40,5% in 2000 to 44,7% in 2010 and 68,5% in 2030)
    - ↓ growth rate of the salary mass
    - pure PAYG systems are not viable in the long term
  - ➢ One of the most important macro economic issue for the next years.
- Solutions ?

Notional Accounts

- Classification

	PAYG	Funding
DB	Classical social security	Classical employee benefit plan
DC	NDC plans	Pension saving accounts

- PAYG scheme
- Defined contribution
  - Individual fictive account
  - Notional (fake) interest rate
  - At pension, the capital is converted into an annuity (constant or not)

Notional Accounts (2)

- Three parameters
  - R(t), the annual notional rate of return
  - a(t), the annuity divisor (accumulated notional capital → annual pension)
  - RP(t), the growth rate of the pensions

$$P_1(t) = \frac{1}{a(t)}.\sum_{l=t-n}^{t-1}\left[c(l).\prod_{i=l+1}^{t}R(i)\right]$$

$$P_j(t) = P_{j-1}(t-1).RP(t)$$

- Ex. Sweden : R(t) = growth of average nominal wage
  - Source of instability
  - Constant readjustments
  - Natural choice for these three parameters which guarantees automatic financial stability (ability of a pension plan to adjust to financial and demographic shocks without legislative intervention)

Notional Accounts (3)

- Case 1. Similar pension for everyone at time t

$$RP(t) = exp\left(\gamma_t\right).\frac{DR(t-1)}{DR(t)}$$

	Choice for $R(i)$	Implication for $a(t)$
1	Total growth of the wages mass : $\frac{Y(i).s(i)}{Y(i-1).s(i-1)}$	$\sum_{k=1}^{n} \frac{O(t)}{Y(t-n+k-1)}$
2	Same value as $RP(i) = exp\left(\gamma_i\right).\frac{DR(i-1)}{DR(i)}$	$\sum_{k=1}^{n} DR(t-n+k-1)$

- Case 2. No simplification

$$RP(t) = \frac{C(t) - O_1(t).P_1(t)}{O_2(t).P_1(t-1) + \sum_{j=3}^{m} \left\{ O_j(t).P_1(t-j+1).\prod_{k=t-j+2}^{t-1} RP(k). \right\}}$$

Notional Accounts (4)

- Application : Modeling in progress …

  - Demographics (cfr Lee)
    - Stochastic version for projecting the Leslie Matrix

$$\mathbf{N}_t = \mathbf{X}_t\mathbf{N}_{t-1} + \mathbf{I}_t$$

    - with behind
      - a Lee Carter model for the mortality rates $\ln \hat{\mu}_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt}$
      - a constrained Lee Carter model for fertility rates $\hat{f}_x(t) = \gamma_x + \delta_x.\lambda_t + \varepsilon'_{xt}$

  - Financial (Hull and White with one factor)

$$dr(t) = \left(\theta(t) - a.r(t)\right).dt + +\sigma_r.dW_r(t)$$

  - Results : soon !

Mixing PAYG and funding

	Pay-As-You-Go (unfunded)	**Funded pension**
Mechanism	Active people finance retired people's pensions	Contributions are invested on financial market for one's own future pension
Annual rate of return	Growth rate of the salary mass	Financial Return

- Idea : PAYG and funding schemes are dealing with different risks and have therefore specific advantages and drawbacks
  - What about mixing them ?
  - We therefore seek for an equilibrium between those two "extreme" pension schemes

Mixing PAYG and funding (2)

- Main questions
  - ➢ Can we obtain a theoretical justification for diversification between pay as you go and funding ?
  - ➢ What is the optimal level of diversification ?
  - ➢ Influence of the correlation between demography and financial markets?
- Hypothesis
  - ➢ Macro economic point of view
  - ➢ OLG two periods' set up
    - x_0 :affiliation age to the pension plan (working period)
    - x_r : retirement age
  - ➢ Defined contributions

Mixing PAYG and Funding - Deterministic

- Pay as you go : $L(x_0;t).\Pi.S(t) = L(x_r;t).P(t)$
  - Replacement rate : $RR(t) = \Pi.\frac{1}{p(x_0;1)}.(1+d)$
- Funding : $L(x_0;t-1).\Pi.S(t-1).(1+i) = L(x_r;t).P(t)$
  - Replacement rate : $RR(t) = \Pi.\frac{1}{p(x_0;1)}.\frac{1+i}{1+g}$
- With :
  - L(t; x) : the number of people aged x at time t
  - i : rate of return of the financial investment
  - g : rate of increase of the mean salary
  - d : demographic rate of increase of the active population
- Decision : Samuelson rule : $1+d > \frac{1+i}{1+g}$ → PAYG

Mixing PAYG and Funding - Stochastic

- Deterministic case :
  - No diversification
  - Only rates of return matters as no risk
- Now :
  - Stochastic processes
  - The cotisation is split in two parts :
    - funding part (a)
    - PAYG part (1-a)
  - Replacement rate for one surviving person aged x_r at time t :

$$RR(t) = \Pi.\frac{1}{p(x_0;1)}.\left[a.\frac{1+i}{1+g} + (1-a).(1+d)\right]$$

  - Process of interest : $X = a.F + (1-a).D$
    - Hyp : longevity risk is independant of the others
    - All other processes may be correlated

Mixing PAYG and Funding – Stochastic (2)

- Aim : determine the the optimal part which should be invested in the contribution system (a) in order to optimize the pension benefit received at retirement time.
- Porfolio Theory (risk return approach)
  - Choice of an utility function :

$$U[X] = E[X] - \frac{\gamma}{2}.V[X]$$

  - Solution

$$a^{SOL} = \frac{V[D] - Cov[F;D]}{V[F-D]} + \frac{1}{\gamma}.\frac{E[F-D]}{V[F-D]} = a^* + \frac{1}{\gamma}.\Delta$$

    - First part : same for everyone (minimum variance)
    - Second part : linked to
      - the coeficient of risk aversion of the individual
      - link between the financial and demographic expectations and risks

Mixing PAYG and Funding – Stochastic (3)

- Example: lognormal model

$$dD = \mu_d D.dt + \sigma_d.D.\omega_d(t)$$

$$dG = \mu_g G.dt + \sigma_g.G.\omega_g(t)$$

$$dI = \mu_i I.dt + \sigma_i.I.\omega_i(t)$$

  - Correlated brownian motions
  - $a^{SOL} = a^* + \frac{1}{\gamma}.\Delta$

$$a^* = \frac{e^{2\mu_d+\sigma_d^2}.\left(e^{\sigma_d^2}-1\right) - e^{(\mu_f+\mu_d)+\frac{1}{2}(\sigma_f^2+\sigma_d^2)}.\left(e^{\sigma_{fd}}-1\right)}{e^{2\mu_f+\sigma_f^2}.\left(e^{\sigma_f^2}-1\right) + e^{2\mu_d+\sigma_d^2}.\left(e^{\sigma_d^2}-1\right) - 2.e^{(\mu_f+\mu_d)+\frac{1}{2}(\sigma_f^2+\sigma_d^2)}.\left(e^{\sigma_{fd}}-1\right)}$$

$$\Delta = \frac{e^{\mu_f+\frac{1}{2}\sigma_f^2} - e^{\mu_d+\frac{1}{2}\sigma_d^2}}{e^{2\mu_f+\sigma_f^2}.\left(e^{\sigma_f^2}-1\right) + e^{2\mu_d+\sigma_d^2}.\left(e^{\sigma_d^2}-1\right) - 2.e^{(\mu_f+\mu_d)+\frac{1}{2}(\sigma_f^2+\sigma_d^2)}.\left(e^{\sigma_{fd}}-1\right)}$$

Mixing PAYG and Funding – Stochastic (4)

- Diversification effect :
  - If 0 < a < 1
  - a* depends on the Corr(F;D) correlation between the risks
    - Corr = 0 (independance case) → diversification $a* = \frac{V[D]-Cov[F;D]}{V[F-D]}$
    - Corr <0 → diversification is garanteed
    - Corr > 0 (most probable) → we cannot say anything
  - Six cases :

Case	$E[F] > E[D] \Leftrightarrow \Delta > 0$	$E[F] < E[D] \Leftrightarrow \Delta < 0$
$a^* < 0$	[1] $a_{OPT} \in [0;1]$	[2] $a_{OPT} = 0$
$a^* > 1$	[3] $a_{OPT} = 1$	[4] $a_{OPT} \in [0;1]$
$a^* \in [0;1]$	[5] $a_{OPT} \in [a^*;1]$	[6] $a_{OPT} \in [0;a^*]$

    - No diversification cases (2 and 3) : only funding or only PAYG (extreme, kind of « arbitrage » cases)
    - Every strategy cases: cases (1 and 4)
    - In between cases (5 and 6)

Thank You !