The copula-graphic estimator in censored nonparametric location-scale regression models

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Outline

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Motivating example

Consider data on survival times of patients after heart transplantation,

- Y = time until death (caused from heart failure)
- C = time until death (caused from other reasons)
- X = age of a patient

we observe:

- $T = \min(Y, C)$
- $\Delta = \mathbb{1}(Y < C)$
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Consider the model:

- $(T_i = \min(Y_i, C_i), X_i, \Delta_i)$ are iid vectors

$$Y = m(X) + \sigma(X)\varepsilon$$

- $m(X) =$ location functional
- $\sigma(X) =$ scale functional
- $X \perp\!\!\!\perp \varepsilon$
- Y and C are Copula dependent

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Survival time Y and censoring time C are copula dependent i.e.

$\forall x$ there is function C_x such that,
$P(Y > y, C > c|X = x) = C_x(\bar{F}(y|x), \bar{G}(c|x))$

where

- $\bar{F}(y|x) = P(Y > y|X = x)$
- $\bar{G}(c|x) = P(C > c|X = x)$
- C_x is Archimedean Copula, i.e.
  there exist generating function $\phi_x(\cdot)$ s.t.
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Find estimator for $F(y|x) = P(Y \leq y|X = x)$

Our assumptions:

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Previous work:

Van Keilegom and Akritas, 1999

- $Y = m(X) + \sigma(X)\varepsilon$, $X \perp\!\!\!\perp \varepsilon$
- For given X, $Y \perp\!\!\!\perp C$

Braekers and Veraverbeke, 2005

- Relation between X and Y is completely nonparametric
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Estimation

For simplicity reasons assume that $\sigma(x) = 1$

$$Y = m(X) + \varepsilon$$

Find estimator for $\bar{F}(y|x) = P(Y > y|X = x)$

We can show that

$$\bar{F}(y|x) = \bar{F}_e(y - m(x))$$

where $\bar{F}_e(y) = P(\varepsilon > y)$

We will estimate $\bar{F}(y|x)$ by estimating $\bar{F}_e(\cdot)$ and $m(\cdot)$

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Estimate error distribution $\bar{F}_e(y) = P(\varepsilon > y)$

Using notation

- $T = \min(Y, C)$
- $\bar{G}_e(c|x) = P(C - m(X) > c|X = x)$
- $\bar{H}_e(y|x) = P(T - m(X) > y|X = x)$

we can get

- $\bar{H}_e(y|x) = \phi_x^{-1}(\phi_x(\bar{F}_e(y)) + \phi_x(\bar{G}_e(y|x)))$

Now by applying ϕ_x on both sides and integrating them with respect to dF_X we get

- $\int \phi_x(\bar{H}_e(y|x))dF_X(x) = \phi(\bar{F}_e(y)) + \int \phi_x(\bar{G}_e(y|x)dF_X(x)$ (1)

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Find estimator $\hat{\bar{F}}_e(\cdot)$ that solves equation (1) and is

- right continous
- step functions
- have jumps at the points $T_i - m(X_i)$ for $\Delta_i = 1$

Assuming that $\bar{G}_e(\cdot)$ satisfies

- $\bar{G}_e(T_i - m(X_i) = \bar{G}_e((T_i - m(X_i)^-), \forall \Delta_i = 1$

we can get explicit form of $\hat{\bar{F}}_e(\cdot)$

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$$\hat{\hat{\bar{F}}}_e(y) = \phi^{-1}\{- \sum_{\substack{T_i - m(X_i) \le y \\ \Delta_i = 1}} \int [\phi_x(\bar{H}_e((T_i - m(X_i))^- | x) - \phi_x(\bar{H}_e(T_i - m(X_i) | x))] dF_X(x)\}$$

- $F_X(x) = P(X \le x) \longrightarrow$
  $\hat{F}_X(x) = \frac{1}{n}\sum_{i=1}^{n} 1(X_i \le x)$
- $\phi(u) = \int \phi_x(u) dF_X(x) \longrightarrow$
  $\hat{\phi}(u) = \int \phi_x(u) d\hat{F}_X(x)$
- $\bar{H}_e(y|x) = P(T - m(X) > y | X = x) \longrightarrow$
  $\hat{\bar{H}}_e(y|x) = \sum_{i=1}^{n} W_{ni}(x, h_n) 1(T_i - m(X_i) > y)$

  where $W_{ni}(x, h) = \frac{K((x - X_i)/h)}{\sum_{j=1}^{n} K((x - X_j)/h)}$ are Nadaraya Watson weights

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$$m(x) = \int_0^1 F^{-1}(s|x)J(s)ds$$

where

- $\int_0^1 J(s)ds = 1$
- $J(s) \geq 0$

We will estimate $\hat{m}(\cdot)$ by plugging in pre-estimator $\tilde{F}(\cdot|x)$ which is modification of Braekers and Veraverbeke, 2005.

Which gives us $\hat{m}(\cdot)$ and $\hat{\tilde{F}}_e(\cdot)$

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