

Stochastic calculus in a semi-Markov environment and financial applications

Since the seminal work of Black and Scholes, the importance of realistic models for asset pricing has been an increasing concern. Many alternatives have been proposed that try and capture the statistical features of real-world observations. The model we study is part of the class of so-called regime-switching models. The idea is to let the parameters of the model evolve in time according to an underlying (often considered as hidden) process.

Much of the literature has concentrated on Markov switching models. In this case, it is well known that the distribution of the sojourn time in each state is exponential. We believe this is quite unrealistic. To solve this problem, we suppose our underlying process is a semi-Markov process. This allows to have a sojourn time distribution that depends on the state being visited and the state being visited next and that is not necessarily exponential.

Of course, this extra flexibility comes at a cost. First, the fact we add extra uncertainty leads to an incomplete market (this is true even if we only allow for Markov switching) and second we lose the Markovian structure of the problem that is so convenient in the classical theory.

The first part of our work is mainly theoretical in nature and centred on the pricing of contingent claims in a semi-Markov regime-switching model. We present the mathematical formulation of the problems and deal with the loss of the Markovian structure. We also deal with the market incompleteness by a precise characterization of the minimal entropy martingale measure in this framework.

The second part will be about estimating such models based on discrete-time observations. This is a difficult problem and is very much work in progress.

The third part will be about simulating these models to see what kind of results we get compared with more standard approaches. Numerical simulations can be performed in the direction of monte-carlo simulations or by numerically solving some system of integro-differential equations. We will then compare with real-world data and other approaches.

It would also be interesting to apply our models in some other financial situations such as interest rate models or commodity derivatives, etc.