Estimation des lois de puissance en physique solaire

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Objectif. Les lois de puissance (distribution de type Pareto) sont omniprésentes en physique solaire. La valeur de l'indice de la loi de puissance peut fournir des informations importantes sur les mécanismes physiques sous-jacents. L'amplitude des éruptions solaires est supposée suivre une loi en puissance. Nous proposons d'étudier les relations entre la fréquence et l'amplitude de ces éruptions et de comparer différentes méthodes pour l'estimation de l'indice de la loi de puissance. Le but du mémoire est de pouvoir conseiller les physiciens solaires sur la ou les méthodes les plus appropriées pour analyser ce type de données.

Background. Power laws are ubiquitous in solar physics. The presence of a power law (and the deviations from it) provides important information about the underlying physics of the measured events. Parameters derived from observational power laws are compared to theoretical predictions in order to validate the theory.

Power laws are important because they suggest scale invariance: small and large events have the same properties because they are initiated by the same underlying physical process. In the case of solar flares (sudden brightenings on the surface of the sun), for instance, a strong correspondence is found between the power-law behavior of the X-ray radiation received from solar microflares and from major flares and indeed solar physicists believe all flares are triggered by one and the same process, called magnetic reconnection.

The importance of correctly estimating the scaling parameter for solar flares is illustrated by the argument that the solar flare energy distribution must have a power-law index larger than 2 in order for nanoflares to explain the enigmatic coronal heating problem. A power law this steep implies that a large number of small events with low energies occurring simultaneously would supply sufficient energy to heat the solar corona. Therefore, many solar physicists have spent considerable effort trying to determine the exact scaling parameter for solar flares, resulting in a wide range of estimations of the index, which left the question undecided.

This shows that the precise estimation of a power law model is a delicate and subtle problem. Obviously, studying power-law distributions over several orders of magnitudes requires large amounts of measurements (events) and appropriate methodology. Solar physicists are often less trained in sophisticated statistical methods and resort to a graphical approach of the problem: they bin their data on a logarithmic scale and estimate the power law index with a linear regression fit to this data. It is clear that this method is inaccurate, especially in the case of low sample sizes. In recent years however, the maximum likelihood estimator, which gives more accurate results, has gained in popularity. Several other methods exist, but are rarely applied in solar physics.