



5.00 credits	30.0 h	Q1
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Teacher(s)	Vrins Frédéric ;
Language :	English
Place of the course	Louvain-la-Neuve
Prerequisites	<p>Mathematics, informatics, probability and statistics at Bachelor level. In particular, the corresponding UCL courses are</p> <ul style="list-style-type: none"> • Mons : MQANT1110 (Mathématiques de Gestion I), MQANT1113 (Statistiques et Probabilité), MQANT1109 (Informatique de gestion) • LLN : LINGE1114 (Analyse), LINGE1113 (Probabilité), LINGE1225 (algorithmique et programmation en économie et gestion) <p>In addition, this course is reserved for students with a bachelor's degree in business engineering or students with equivalent quantitative method skills</p>
Main themes	<ol style="list-style-type: none"> 1. Part I: Basic probability concepts (probability space, sigma-fields, random variables, distribution, statistics and sampling via Monte Carlo). 2. Part II : Stochastic processes and related concepts. 3. Part III : random walks and Brownian motion. 4. Part IV : stochastic calculus (stochastic integrals, stochastic differential equation, Ito's lemma, Girsanov theorem)
Learning outcomes	<p>At the end of this learning unit, the student is able to :</p> <p><i>During their programme, students of the LSM Master's in management or Master's in Business engineering will have developed the following capabilities'</i></p> <ol style="list-style-type: none"> 2.2. Master highly specific knowledge in one or two areas of management : advanced and current research-based knowledge and methods. 2.4. Activate and apply the acquired knowledge accordingly to solve a problem. 1 3.1. Conduct a clear, structured, analytical reasoning by applying, and eventually adapting, scientifically based conceptual frameworks and models, to define and analyze a problem. 3.5. Produce, through analysis and diagnosis, implementable solutions in context and identify priorities for action. 6.1. Work in a team : Join in and collaborate with team members. Be open and take into consideration the different points of view and ways of thinking, manage differences and conflicts constructively, accept diversity.

<p>Evaluation methods</p>	<p>Continuous evaluation (projects with implementation in R or Python)</p> <ul style="list-style-type: none"> • Date: : <i>specified at the beginning of the course (typically scheduled 2 weeks before exam)</i> • Type of evaluation: <i>Report (teamwork, 20% of final grade) and Individual assessment (following the oral exam, during the examination session; 20% of final grade)</i> <p>Evaluation week</p> <ul style="list-style-type: none"> • Oral: <i>No</i> • Written: <i>No</i> <p>Examination session</p> <ul style="list-style-type: none"> • Oral: <i>Yes</i> • Written: <i>No</i> • Comments: The final examination is made of two parts : <ul style="list-style-type: none"> • <i>1h preparation of questions (exercises + theory) followed by a 10 to 15 min discussion with the professor (60% of final grade)</i> • <i>10 min discussion with the teaching assistant to assess the individual contribution of the student in the group project (20% of final grade).</i> <p>Complementary information about the project</p> <ul style="list-style-type: none"> • <i>The grade of the project (both the group and individual parts) will be set to 0 for the students who would not present the individual oral examination.</i> • <i>Students are allowed to pass only one of the two oral examinations.</i> • <i><u>In case of failure at the first session:</u> the grade of the report of the project will be automatically transferred. The same holds for the individual part of the project, provided that it was successful in the first session. Otherwise, the student must retake the individual part during the second session.</i> • <i><u>In case of failure in second session:</u> If both parts of the project were successful, the student gets a dispense for the project for the next academic year. Otherwise, the student must enroll in the project of the new academic year. It is the responsibility of the student to make sure (s)he joins a group ! <u>Pay attention to the announcement on Moodle !</u></i> <p>Important note about plagiarism and the use of generative AI <i>By submitting an assignment for evaluation, you assert that:</i></p> <ul style="list-style-type: none"> • <i>it accurately reflects the facts and to do so you need to have verified the facts, especially if they originate from generative AI resources;</i> • <i>all your sources that go beyond common knowledge are suitably attributed. Common knowledge is what a knowledgeable reader can assess without requiring confirmation from a separate source;</i> • <i>you have respected all specific requirements of your assigned work, in particular requirements for transparency and documentation of process, or have explained yourself where this was not possible.</i> <p><i>If any of these assertions are not true, whether by intent or negligence, you have violated your commitment to truth, and possibly other aspects of academic integrity. This constitutes academic misconduct.</i></p>
<p>Teaching methods</p>	<ul style="list-style-type: none"> • Ex-cathedra course • Optional exercise sessions • Group project (in R or Python), consisting of pricing and risk-managing a derivatives product. The main objective of the projects is to make the concepts more concrete, which facilitates the learning processes. <p>Students may be asked to prepare some courses before joining the classes. Some optional homeworks may also be proposed, in order for the students to be able to assess the level of their understanding.</p>
<p>Content</p>	<p>The purpose of this course is to introduce the key mathematical concepts to understand the mechanics of derivatives pricing, both in discrete and continuous time. The language used in this course is quite formal and technical, but the equations are always put in perspective to highlight the intuition hidden behind.</p> <ul style="list-style-type: none"> • Introduction. A short introduction explains what <i>derivative products</i> are and why their valuation is fundamentally different from that of other assets, such as stocks. The principles of <i>law of one price</i> and <i>valuation by absence of arbitrage opportunities</i> or <i>by replication</i> are presented. Pricing by replication is exemplified on a simple linear derivative (forward contract on a stock), and the difficulty to apply this method to non-linear payoffs is underlined. This explains why valuation models are needed for such products, highlights the key features that these models should display, and provides the motivation for the course. • Part I. Reminder about basic probability theory (events, probabilities, random variables, distributions, expectations, conditional expectations, etc). Most of these concepts have been introduced in Bachelor's courses but they will be revisited in a slightly more formal way. This part also aims to introduce new notions (such as sigma-fields, Radon-Nikodym variables and changes of probability measures) that will be needed when moving to financial applications.

	<ul style="list-style-type: none"> • Part II. We show that the price of European options can be determined using a replication argument in discrete time. To this end, we consider a random walk on a binomial tree (Cox-Ross-Rubinstein) and show that the payoff can be replicated by trading dynamically in cash and the underlying asset. Next, we observe that the resulting expression takes the form of the conditional expectation of the discounted payoff under some appropriate probability measure. Taking the limit of this expression as the time step tends to zero yields the celebrated Black-Scholes-Merton price for European calls and puts. • Part III. We explain how the limit price found above can be related to a dynamic replication strategy in continuous time. This requires to extend the above discrete-time notions to continuous-time. The Brownian motion and stochastic differential equations are introduced, as well as fundamental results like Ito's lemma and Girsanov's theorem. This leads to the Black-Scholes-Merton partial differential equation (PDE). We show that the solutions to this class of PDE indeed coincide with the limit prices found in the end of Part II. • Part IV. We conclude the course with the hedging and risk management of options, discussing the so-called "greeks". We also explain the replication errors that may arise in practice, due to discrete-time rebalancing and transaction costs. <p>Although such a course can be taught at a relatively "high level", the philosophy in this module is to go deep in the details, unveiling the financial and technical assumptions underlying the models. The purpose is not to review a broad list of derivatives products but is, instead, to have a profound understanding of the models (who connects finance to mathematics) by analysing the pricing procedure applied on simple options.</p> <p>These skills will be extensively used in LLSMS2226 (credit and interest rates risk)</p>
<p>Inline resources</p>	<ul style="list-style-type: none"> • On-line pedagogical tool : http://derivativespricing.uclouvain.be/ • <i>Electronic version of the companion book</i> : https://www.cambridge.org/highereducation/books/derivatives-pricing/D315080AC34EB3ACCF32F460D33E43F7#overview
<p>Bibliography</p>	<ul style="list-style-type: none"> • Vrins, Derivatives Pricing, Cambridge University Press, 2025 • Hassler, Stochastic Processes and Calculus: an elementary introductions with applications, Springer 2016. • Mikosh, Elementary Stochastic Calculus (with Finance in view), Wolrd Scientific, 1998. • Joshi, Concepts and Practice of Mathematical Finance, Cambridge University Press, 2003. • Shreve, Stochastic calculus for Finance I & II, Springer 2004.
<p>Faculty or entity in charge</p>	<p>CLSM</p>

Programmes containing this learning unit (UE)				
Program title	Acronym	Credits	Prerequisite	Learning outcomes
Master [120] : Business Engineering	INGE2M	5		
Master [120] in Economics: General	ECON2M	5		
Master [120] : Business Engineering	INGM2M	5		